Amplification Processes.

In chapter IV we discussed abstract measuring processes, which were considered to be simply a direct coupling between two systems, the object-system and the apparatus (observer). However, in actuality a whole chain of intervening systems linking a microscopic system to a macroscopic observer. Each link in the chain of intervening systems becomes correlated to its predecessor, so that the result is an amplification of effects from the microscopic object-system to a macroscopic apparatus, and then to the observer.
The amplification process depends upon the 
ability of one micro-system (particle, for example) to 
become correlated with the states of an enormous 
number of other microscopic systems, the totality 
of which we shall call a detection system. For example, 
the totality of gas atoms in a Geiger counter, or the 
water molecules in a cloud chamber, constitute such a 
detection system.

The amplification is accomplished by arranging 
the condition of the detection system so that the 
states of the individual micro-systems of the detector 
are metastable, in a way that if one micro-
system should fall from its metastable state it 
would influence the reduction of others. This type 
of arrangement leaves the entire detection system 
metastable against chain reactions which involve 
a large number of its constituent systems. In a
Geiger counter, for example, the presence of a strong electric field leaves the gas atoms metastable against ionization. Furthermore, the products of the ionization of one gas atom in a Geiger counter can cause further ionizations, in a cascading process. The operation of cloud chambers and photographic films is also due to metastability against such chain reactions.

The chain reactions cause large numbers of the micro-systems of the detector to behave as a unit, all remaining in the metastable state, or all discharging. In this manner the status of a sufficiently large number of micro-systems are correlated, so that one can speak of the whole ensemble being in a state of discharge, or not.

For example, there are essentially only two macroscopically distinguishable states for a Geiger counter, discharged or undischarged. The correlation of large numbers of gas atoms, due to the chain reaction effect, implies that either very few, or else very many of the gas atoms are ionized at a given time. Consider the state function \( \Psi^G \) of a Geiger counter, which is a function of all the coordinates of all of the constituent particles. Because of the correlation of the behavior of a large number of the constituent gas atoms, the total state \( \Psi^G \) can always be written as a superposition of two states

\[
\Psi^G = \Psi^D + \alpha \Psi^U
\]

where \( \Psi^D \) signifies a state where only a small number of gas atoms
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are ionized, and \( \Psi^{-2} \), a state for which a large number are ionized.

To see that the decomposition (3.2) is valid, expand \( \Psi^0 \) in terms of individual gas atom stationary states:

\[
\Psi^0 = \sum_{i,j,k} a_{ij} \Psi_i^m \Psi_j^m \Psi_k^m
\]

where \( \Psi_k^m \) is the \( m \)th state of atom \( k \).

Each element of the superposition (3.2)

\[
\Psi_i^m \Psi_j^m \Psi_k^m
\]

must contain a very large number of atoms in ionized states, and a very small number, because of the chain reaction effect. By choosing some medium sized number as a dividing line, each element of (3.2) can be placed in one of the two categories. If we then carry out the sum (3.2) over only those elements of the first category, we get a state (and coefficient)

\[
\alpha_i \Psi_i^{-1} \left[ D \right] = \sum_{i,j,k} a_{ij} \Psi_i^m \Psi_j^m \Psi_k^m
\]

The state \( \Psi_i^{-1} \left[ D \right] \) is then a state where only a large number of particles are ionized. The subscript \( D \) indicates that it describes a Geiger counter which has discharged.
If we carry out the sum over the remaining terms of (3.2) we get in a similar fashion:

\[ a_2 \psi^{-2} [u] = \sum_{j, k} c_{j,k} \psi_j^2 \psi_k^2 \]

where \([u]\) indicates the unchanged condition.

Combining (3.4) and (3.6) we arrive at the desired relation (3.1).

This type of decomposition is also applicable to all other detection devices which are based upon the chain reaction principle (such as cloud chambers, photo plates, etc.)

We consider now the coupling of such devices together into systems (object-systems) for the purpose of a measurement. If it is true that the object-system state \( \Phi \) will at some time trigger the chain reaction so that the state of the counter becomes \( \psi_{[1]} \), while the object-system state \( \Phi \) will not, then it is still true that the initial object-system state \( a_1 \Phi + a_2 \Phi \) will result in the superposition

\[ a_1 \Phi \psi_{[0]}^{-2} + a_2 \Phi \psi_{[u]}^{-2} \]

No matter what the complexity or exact mechanism of a measuring process, the general superposition principle of Chap. 1 remains valid, at any rate, up to the next stage of its development by relying on the observations of amplification processes.
So far, this method of decomposition can be applied to any systems, whether or not they have the chain reaction property. However, in our case more is implied, namely, that the spread of the number of ionized atoms in both $\Phi_1$ and $\Phi_2$ will be small compared to the separation of their averages, due to the fact that the existence of the chain reaction means that either many or else few atoms will be ionized, with the middle ground virtually excluded.

For example, let us suppose that a particle, whose state is a wave packet $\Phi$ of linear extension greater than that of our geiger counter, approaches the counter. Just before it reaches the counter, it can be decomposed into a superposition $\Phi = \alpha_1 \Phi_1 + \alpha_2 \Phi_2$, where $\Phi_2$ has non-zero amplitude only in the region before the counter and $\Phi_2$ has non-zero amplitude elsewhere (so that $\Phi_1$ is a packet which will entirely pass through the counter while $\Phi_2$ will entirely miss the counter). The initial total state for the system particle + counter is thus:

$$\Phi^{[0]} = (\alpha_1 \Phi_1 + \alpha_2 \Phi_2)^{[0]}$$

where $\Phi^{[0]}$ is the initial (assumed to be discharged) state of the counter. But as $\Phi$ traverses the counter, and almost all $\phi$ at a slighter...
But at a slightly later time $\phi$ is transferred to $\phi^2$ by a transition from $\phi^2$ into a discharged state $\phi^2 [0]$ while $\phi$ passes by into a state $\phi^1$ leaving the counter undischarged. On the contrary, the state of the later time is

$$\alpha_1 \phi^1 [0] + \alpha_2 \phi^2 [0]$$

in accordance with (3.6). Furthermore, the relative particle state for $\phi^2 [0]$ is a wave packet emanating from the counter, while the relative state for $\phi^1 [0]$ is a wave with a "shadow" caused by the counter. The counter therefore serves as an apparatus which performs an approximate position measurement on the particle.

No matter what the complexity or exact mechanism of a measuring process, the general statement in III-§3 remains valid, and our abstract discussion is unaffected. It is a vain hope that somehow embedded in the machinery of the amplification process is a mechanism which will prevent the macroscopic apparatus from reflecting the same indeterminacy as its object system.