Objective vs Subjective probability

We shall use the phrase <u>subjective probability</u> to refer to an estimate based upon incomplete information, i.e. as a measure of the ignorance of the observer. For example if a deck of cards is shuffled, cut, and one card placed face down upon a table, and we ask what is the probability that that card is the ace of spades, then the answer $\frac{1}{52}$ is a <u>subjective</u> probability. In "reality" it is already decided whether or not the card is the ace of spades, and this probability reflects only our ignorance of what actually obtains. We could go further, and inform another individual that the color of the card is black, whereupon he would say that the probability is $\frac{1}{26}$ that the card is the ace of spades. Yet another observer might have surreptitiously had a glimpse of the card as it was placed on the table, and for him the subjective probability would be either 0 or 1.

All of these observers have given <u>correct</u> probabilities of the <u>same</u> event, relative to their information. Subjective probabilities hence have the property that they depend upon the information of the observer (i.e. are changed by increased information) and that different observers will in general ascribe different subjective probabilities to the <u>same</u> event

The concept of an objective probability is somewhat difficult to define operationally, but we may list some requirements. If an event has an objective probability p of occurring, we shall require that this probability is the same for <u>all</u> observers, and that it is <u>not</u> changed by increased information. That is, it cannot be the case that observer 1 states that the objective prob of an event is p_1, while observer 2 states that the objective probability of the <u>same</u> event is $p_2 \neq p_1$, without at least one observer being incorrect. ie either p_1 or p_2 is <u>not</u> an objective probability.

A possible operational definition of objective probability would be at the limit (if such existed) of subjective probabilities as information become more + more perfect. Such a definition leads to an objective probability which obviously satisfies the requirements that it be invariant to changes of observers, + increases of information. However, we shall need only the negative statements about what an objective probability cannot be, and shall investigate the consistency of various formulations of quantum mechanics which interpret their probabilities to be objective.

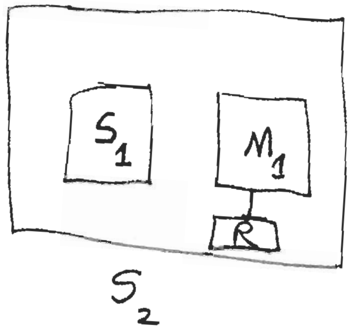
As a starting point, we might investigate the consequences of the following set of postulates:

1. Every physical system S possesses a state function ψ_S which gives the <u>objective</u> probabilities of the results of any measurement performed upon the system.
2. The state function ψ_S of an isolated system changes causally with time (i.e. proceeds deterministically) as long as the system remains isolated.

We shall show that the above postulates are inconsistent by consideration of the following Gedanken experiment. We suppose that we have a system, S_1, and a measuring device M_1, for measuring some property of S_1, which is connected to a recording device which will record the results of the measurement at a classical level, such as the position of a relay arm, and we assume that the measuring device is arranged to make the measurement automatically at some time. We further assume that the entire system S_2, consisting of S_1, M_1 and the recording device are isolated from any external interactions.

Now, the system S_1 possesses a state function ψ_1 wh[illegible] gives the objective probability of the results of the measurem[illegible] (we shall also assume that ψ_1 is not an eigenstate of the measurement, so that we shall have a probability which is neither zero or 1. But if we now consider the total system S_2, before the measurement, it also possesses a state function ψ_2. Furthermore, ψ_2 at any later time is strictly determined by our initial ψ_2, since there are no outside interactions, so that in particular ψ_2 for some time after the measurement has taken place is strictly determined by its value before the measurement. We now consider what this later ψ_2 may say about the ~~results of a determination~~ configurati[illegible] of the recording device. If it gives a probability mixture over the configurations, then clearly these probabilities are of the subjective type, since they refer to something which ~~has actually to~~ actually exists in a pure state, because in reality the configuration of the recording device has already been determined. (just as in the case of the man and the card.) On the other hand If this later ψ_2 gives the exact configuration of the recording device, then clearly the outcome of the measurement was determined before it took place, since the later ψ_2 was strictly determined by the earlier, in which case the probability given by ψ_1 was not objective.

So we see that in any case at least one of the probabilities was subjective, and the postulates as stated are untenable.



R: Recording device (classical level) say position of a cannon ball.

ψ_1 = State function for S_1 before measurement (assumed to be not eigenstate of M_1)

ψ_2 = State function of entire system, including S_1, M_1, and R.

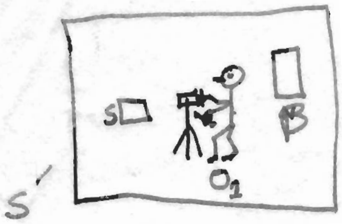
Now, by the hypothesis of state function causality, ψ_2 is determined for all times as long as there is no outside interaction with S_2. The question is, what does ψ_2 say about the future position of the cannon ball?. Clearly if it describes a mixed state, it is not an objective state function, while if it predicts the exact state, then the ψ_1 was not objective, that is its probabilities were not objective, since the whole process was really determined.

There are several ways of removing this inconsistency.

1) Not every physical system possesses a state function. I.e. that even in principle quantum mechanics *cannot* describe the process of measurement itself. This is somewhat repugnant, since it leads to an artificial partition dichotomy of the universe into two distinct classes, systems, and measurements.

2) The wave function of an isolated system is *not* always causally determined, but may suffer abrupt discontinuities from mixed states into probability mixtures of pure states, corresponding to an internal phenomenon which is regarded as a measurement. This is quite tenable, but leaves entirely unknown what is to be regarded as such a measuren
That is, for this interpretation no formalism has been develop
to give the points of discontinuity. . . . .

3] The probabilities occurring in quantum mechanics are *not* objective. That is, they correspond to our ignorance of some hidden parameters. Under such an interpretation the paradox resolves easily since the outside wave function possesses more information i.e. phase factors, etc for the interaction, so that it leads to a causal description.



We consider an observer O_1 in an isolated room about to make a measurement on system S_1 in the room, whose wave function ψ_S is known to him. Subsequen to the measurement he will write the result of his measurement on the blackboard B. Outside of the room, and isolated from it is a second observer O_2 who is in possession of the wave function ψ_1 of the system S' which consists of the entire room containing S, O_1 and his apparatus. O_2 will not interact with S_1 for one year, at which time he will look at the blackboard to determine the result of O_1's measurement. He computes from his wave function what he shall see at this time. Now either he can predict exaclty what he will see on the blackboard, in which case O_1 was deluding himself when he ascribed an objective probability to the event, or else he cannot predict what is on the board, in which case his probabilities are subjective, since the writing on the board is a classicle thing.

Postulates: Every physical system possesses a state function ψ which gives the objective probabilities of the results of any measurement.

The state function ψ of an isolated system is causally determined, that is it satisfies some equation ~~which~~ so that future state functions are uniquely determined by specifying it at one time.

These postulates are inconsistent!

A state function which ascribes a probability distribution over observations at the classical level is NOT objective.