III. Quantum Formalism Mechanics

Q.M. of Composite Systems

- States of composite system
- Density matrix
- Relative states

- Canonical correlation
- Measured to approximately equal of correlation
- Meta uncertainty principle

- Example: Von Neumann (of how measurement can take place)
- Atomic constitution - amplification processes
  - General principle of superposition
  - Idea of measurement (good; measurement)
  - Idea of more states

Introductory paragraph:
Assume state points in Hilbert space. In this section, we study some consequences of the quantum mechanical formalism of composite systems.
3) Can, if we choose, at any instant put comp system in context and say that $a$ is required by $b$ (or vice-versa), since there is exact correspondence.

4) However, this is not most convenient. We think of a measurement as an interaction, then what is con seen depends upon time of interaction. We should, rather, prefer to think of this measurement as an approximation to finite times of interaction, which approached exact measure, as time $0$. (This leads to ideal that what is con seen in limit is what is measured.)

5) we thus have requirement that newly an interaction again must bring yet closer to ideal exact means. (Seems to imply a repeatability)

(should not be implied if finite $\Delta t$ exists)

6) Thus consider an example of von Neumann, of a

simplified measurement.

7) discussing effect of $\frac{3}{2} \rightarrow \frac{1}{2}$

and ask to agent $A$ suggested locations

for purposes of Neumann eigenstate,
\[ \Sigma \alpha \phi \eta, \]

\[ \Rightarrow \sum_i U |\phi \eta_i\rangle \]

\[ \Rightarrow \sum_i \alpha_i \phi_i \eta_i \]

\[ \Rightarrow \text{unitary matrix} \]

\[ \Rightarrow \text{preserved} \]

\[ \Rightarrow \text{here also (??)} \]

\[ \Rightarrow U \phi \eta = e^{i \phi} U \eta \]

\[ \Rightarrow U_{m,j} = (\phi_i \eta_i) U_{m,n} \]

\[ = (\delta_{m,n} e^{i \phi}) \eta_j \]

\[ = \delta_{m,n} \eta_j \]

\[ \Rightarrow AB \]

\[ \text{interact ans} \]

\[ \Rightarrow \Sigma \alpha \beta \eta \]

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\[ \lim_{t \to \infty} U_t^{-1} \mathbf{A} U_t = \mathbf{A} \quad \text{for all } \mathbf{A} \]

\[ \Rightarrow \quad U_t^{-1} \mathbf{A} U_t = \mathbf{A} \quad \Rightarrow \quad \mathbf{A} U_t = U_t \mathbf{A} \]

\[ \Rightarrow [\mathbf{A}; U_t] = 0 \]

**Requirement in Stationarity**

\[ \Rightarrow \quad \text{back to } \psi = \sum \alpha_i \phi \psi_i \quad \Rightarrow \quad \mathbf{A}_i, \mathbf{B}_i \text{ canonical proj} \]

\[ \Rightarrow \langle \mathbf{A}; \mathbf{B} \rangle \psi_0 = \mathbf{A}^\ast \mathbf{A}_0 \delta \psi_0 \]

More:

\[ \text{what for } U_0 \psi_0 ? \quad (\psi_0, \mathbf{A}^\ast \mathbf{B} \psi_0) = \mathbf{A}^\ast \mathbf{A}_0 \delta \psi_0 \]

\[ = \langle \psi_0, \mathbf{A}^\ast \mathbf{B} U_0 \psi_0 \rangle = \langle U \psi_0, \mathbf{A}^\ast \mathbf{B} U_0 \psi_0 \rangle \]

\[ = \mathbf{A}^\ast \mathbf{A}_0 \delta \psi_0 \]

Therefore, the eigenvalue \( \langle \mathbf{A}^\ast \mathbf{B}_0 \psi_0, U_0 \psi_0 \rangle = \mathbf{C}_{i_0} \mathbf{a}_0 \delta \psi_0 \)

if \( \mathbf{C}_{i_0} = \mathbf{A}_0 \beta_0 \) then \( \mathbf{A}_0 \beta_0 \) canonical

(except don't require since)
\( U = e^{i\Delta B} \sum \frac{(i\Delta B)^m}{m!} = \sum \frac{i^n (\Delta \lambda)(\Delta \nu)^n}{m!} \)

\( (\Delta \lambda)^n \gamma \eta \)

\( (\Delta \lambda)^n (\Delta \nu^2) \)

\( = (\Delta \lambda)^n (\Delta \nu^2) \)

\( \sum i^n \frac{1}{m} \cdot (\Delta \nu^n)^m \)

\( \Rightarrow H = AB \)

\( \Rightarrow H(\phi \eta) = AB(\phi \eta) = \lambda \phi (B \eta) = \frac{\phi \cdot \phi}{\eta} \phi \eta \)

Substitute \( \phi \) eigenstates of Composite operator \( H \)

\( \Rightarrow U \phi \eta = \phi (\overline{\phi} \eta) \)
\[ Y_0 = \Sigma A_i \frac{1}{\xi_i} y_i \]

\[ (\frac{1}{\xi_i}) (U^2 y_i) \]

\[ \Rightarrow \Sigma A_i (U^2 y_i) (U^2 y_i) \]

\[ \Rightarrow \Sigma A_i a \theta \Phi \]

\[ = \Sigma A_i a \theta \Phi \] and still can

\[ \text{with } U^{-1} A U \]

\[ (U^2)^N B U^2 \]

new counterintuition

Def: No interaction if \( U = \bar{U} \bar{U} \)

\[ \text{note } \frac{\partial}{\partial x} = \frac{-i}{\hbar} \frac{\partial}{\partial x} \]

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\[ U = e^{-i(H_0 + H_2) x} \]

\[ U = e^{i[H_0 - H_2] x} e^{i[H_0 + H_2] x} = \bar{U} \bar{U} \]

\[ \text{(no OK, for no interaction)} \]

More general \[ U = \bar{C} \frac{1}{\bar{\xi}} \bar{A} \bar{B} \bar{C} \]
\[ \psi = \sum \phi \phi \phi \]

\[ \frac{\psi}{\psi'} = \sum \phi \phi \phi \]

ie \((U^2)\) is operator for time dependence

\[ \psi = U' \psi' \]

\[ \langle \psi \rangle \psi'' = \langle \psi \rangle U \psi = (U^0_a A U \psi^0) \]

\[ = (\psi^0 U^* A U \psi^0) \]

\[ = \langle U^* A U \rangle \psi^0 \]

\[ \Rightarrow A' \psi'' = \psi \]

Now, about \( U A \cdot U \)

ie \( A \cdot \psi = \lambda \psi \)

\[ \Rightarrow U^* A \psi = \lambda U \psi \]

\[ \Rightarrow U A \cdot U^{-1} (U \psi) = \lambda (U \psi) \]

OK
\[ \psi^T \Phi^T = \psi^T \Phi \]

What operators are in this case? \\

must be commuting upwards.

We know that if \( U = U_1 U_2 \) (direct product), then no interaction (definition).

\[ \sum_i \phi_i \Phi_i \rightarrow \sum_i \phi_i \left( \sum_k \alpha_i \beta_k \phi_i \Theta_k \right) \]

since \( \phi \Theta \rightarrow \sum_k \alpha_i \beta_k \phi \Theta_k \)

\[ \text{work} \quad \phi \Phi \rightarrow \sum_i \eta_i \quad \text{all this only for non units} \]

\[ \sum_i \phi_i \Phi_i \rightarrow \sum_k \left( \sum_i \phi_i \beta_i \right) \phi \Theta_k \]

\[ \text{given that} \quad S T A S = D \text{ diagonalizes } A \]

what diagonalizes \( U^{-1} A U \)?

\[ (S^{-1} U) U^{-1} A U (U^{-1} S) = D \]

\[ \therefore \text{answer is} \quad S \text{diagonalizes } A \quad (S^{-1} A S) = D \]

then \( U^{-1} S \text{diagonalizes } U^{-1} A U \)
\[ \mathcal{P} = \sum_m p_m [\psi_m] = \sum_m p_m (\psi_m) \psi_m \]

\[ \Rightarrow S_{i,j} = (\phi_i, p_{i,j}) = (\phi_i, \sum_m p_m (\psi_m, \phi_j) \psi_m) \]

\[ = \sum_m p_m (\psi_m, \phi_j) (\phi_i, \psi_m) \]

\[ = \sum_m p_m (\phi_i, \psi_m) (\phi_j, \psi_m) \]

alternatively:

\[ \text{Trace } PA = \sum_i (\phi_i, PA \phi_i) = \sum_i (\phi_i, A \phi_i) \]

\[ = \sum_i (\phi_i, A \sum_m p_m (\psi_m, \phi_j) \psi_m) \]

\[ = \sum_i p_m (\psi_m, \phi_i) (\phi_i, A \psi_m) \]

\[ = \sum_i p_m (\psi_m, A \psi_m) \]

**Theorem** \[ \sum_i (\psi_i, \phi_i) (\phi_i, \psi_i) = (\psi_i, \psi_i) \]

Proof:

\[ \mathcal{H} = 2 (\phi_i, \psi_i) \phi_i \]

\[ \Rightarrow (\psi_i, \psi_i) = (\psi_i, 2 (\phi_i, \psi_i) \phi_i) \]

\[ = \sum_i (\phi_i, \psi_i) (\psi_i, \phi_i) \]

**Trace** \( S = 1 \)

**Trace** \( PA = \)
\[ p = \sum_m \left[ \psi_m \right] \] is valid for \( \psi \) noticing?

\[
\text{Trace } A = \sum_m (\psi_m A \psi_m)
\]

\[
\text{Exp } A = \sum_m \left( \psi_m A \psi_m \right)
\]

\[
\text{Trace PA} = \sum_m (\psi_m PA \psi_m)
\]

\[
= \sum_m (\psi_m A \sum_m [\psi_m] \phi_m)
\]

\[
= \sum_m \psi_m (\psi_m A [\psi_m] \phi_m)
\]

\[
= \sum_m \psi_m \left( [\psi_m] \phi_m A [\psi_m] \phi_m \right)
\]

\[
= \sum_m \sum_m \left( [\psi_m] \phi_m A [\psi_m] \phi_m \right)
\]

\[
\left[ \psi_m \right] = (\psi_m, \psi_m) \psi_m \quad A \left[ \psi_m \right] \phi_m = [A \psi_m] \phi_m
\]

\[
= \sum_m \sum_m \left( (\psi_m, \phi_m) \psi_m A (\psi_m, \phi_m) \right)
\]

\[
= \sum_m \sum_m \left( (\psi_m, \phi_m) (\psi_m, \phi_m) \right)
\]

\[
\text{Yes!}
\]