De Everett

FORMULAS

(See Paragraph 1.6 for Notation and Notes)

	Simple Approximation	Discriminate Aiming	Random Aiming	Planned Salvo	Planned Salvo, Simple Shot Case
Hit Probabilities $\rho(m,n,k)$ = probability of k targets out of a total of n with m weapons	Nonexistent	If $\check{p}(1,k) = \xi_k$ $\check{p}(m,k) = \sum_{j=0}^{k} \xi_{k-j}\, \check{p}(m-1,j) \quad (m>1)$ Then $\rho(m,n,k) = \begin{cases} \check{p}(m,k) & (k<n) \\ 1 - \sum_{j=0}^{n-1} \check{p}(m,j) \end{cases}$	$\rho(1,n,k) = \xi_k$ (Recurrence formula) $\rho(m,n,k) = \sum_{j=0}^{k} \left\{ \sum_{\ell=0}^{j} \frac{\binom{n-j}{k-j}\binom{j}{\ell}}{\binom{n}{k-j+\ell}} \xi_{k-j+\ell} \right\} \rho(m-1,n,j)$ --- $\rho(m,n,k) = \binom{n}{k} \sum_{i=0}^{k} \binom{k}{i} (-1)^{i}$ [illegible] ^{m} $\gamma_c = \xi_0 + \frac{c}{n}\xi_1 + \frac{c(c-1)}{n(n-1)}\xi_2 + \ldots$ (Explicit formula)	Indeterminate	$\rho(m,n,k) = q^{m-sk} \sum_{i=0}^{\operatorname{Min}(k,r)} \binom{r}{i}\binom{n-r}{k-i} \times \left(\frac{1}{q} - q^{s}\right)^{i} p^{k-i}$ where p = hit probability of a weapon $q = 1 - p$ $m = sn + r,\ 0 \leqslant r < n$ (s = quotient, r = remainder of $m \div n$)
Cumulative Hit Probabilities P = probability of hitting k or fewer targets Q = probability of hitting k or more targets	Nonexistent	If $\check{P}(1,k) = \sum_{j=0}^{k} \xi_j$ $\check{P}(m,k) = \sum_{j=0}^{k} \xi_{k-j}\, \check{P}(m-1,j) \quad (m>1)$ Then $P(m,n,k) = \begin{cases} \check{P}(m,k) & (k<n) \\ 1 & (k=n) \end{cases}$	$Q(m,n,k) = (n-k)\binom{n}{k} \sum_{i=0}^{k} \frac{(-1)^{i}}{n-k+i} \binom{k}{i} \gamma_{k-i}^{m}$	Indeterminate	P(: no convenient formula. Either add the ρ_k ($P_k = \sum_{j=0}^{k} \rho_k$) or use normal approximation.
Expectation $E(m,n)$ = expected number of targets hit out of total of n with m weapons	$E(m,n) = \min(E_1 m, n)$	If $\mu_1(1,n) = \sum_{j=0}^{n} j\xi_{n-j} \quad (= n - E_1)$ $\mu_1(m,n) = \sum_{j=0}^{n} \xi_{n-j}\, \mu_1(m-1,j) \quad (m>0)$ Then $E(m,n) = n - \mu_1(m,n)$	$E(m,n) = n\left[1 - \left(1 - \frac{E_1}{n}\right)^{m}\right]$	$E(m,n) = n\left[1 - (\wp_1 \ldots \wp_n)^{\frac{m}{n}} \varphi\right]$ where $\wp_k = \sum_{j=0}^{k-1} \xi_j$ $\varphi = \frac{1}{n}\left[(n-\mu)e^{\frac{\mu}{nd}} + \mu e^{-\frac{n-\mu}{nd}}\right]$ $\approx 1 + \frac{(n-\mu)\mu}{2nd^2} \leqslant 1 + \frac{d^2}{8}$ where $\frac{C_j}{d} = \log \frac{1}{\wp_j}$ C_j, d = integers with greatest common divisor of all $(C_1 - c_j) = 1$. $C = \sum_{j=1}^{n} C_j$ and μ = remainder of mC after division by n $(0 \leqslant \mu < n)$.	$E(m,n) = n(1 - q^{s}) + rpq^{s}$

Dispersion $\sigma^2(m,n)$ = dispersion of hits under like circumstances	Nonexistent	If $\mu_2(1,n) = \sum_{j=0}^{n} j^2\,\xi_{n-j}\ (=\sigma_1{}^2 + \mu_1{}^2)$ $\mu_2(m,n) = \sum_{j=0}^{n} \xi_{n-j}\ \mu_2(m-1,j)$ Then $\sigma^2(m,n) = \mu_2(m,n) - \mu_1{}^2(m,n)$	$\sigma^2(m,n) = n(n-1)\gamma_{n-2}^{m} + m\gamma_{n-1}^{m} -$ $- n^2\gamma_{n-1}^{2m}$ Note: $\gamma_{n-2} = \frac{1}{n(n-1)}[n(n-1)\xi_0 + (n-1)(n-2)$ $\xi_1 + (n-2)(n-3)\xi_2 + \ldots]$ $\gamma_{n-1} = 1 - \frac{E_1}{n}$	Indeterminate	$\sigma^2(m,n) = nq^s(1-q^s) + rpq^s(q^s(1+q)-1)$
Limiting Behavior for Large m and n $R = \frac{m}{n}$ $\overline{E}(R) = \lim E(m,n)$ $\overline{\sigma}^2(R) = \lim \sigma^2(m,n)$	$\overline{E}(R) = \min(E_1R, 1)$ or If $R \leqslant \frac{1}{E_1}$ $\overline{E}(R) = E_1R$ If $R > \frac{1}{E}$ $\overline{E}(R) = 1$	If $R < \frac{1}{E_1}$: $\overline{E}(R) = RE_1$ $\overline{\sigma}^2(R) = R\sigma_1{}^2$ Limiting distribution: Normal If $R > \frac{1}{E_1}$: $\overline{E}(R) = 1$ $\lim \sigma^2 = 0$ Limiting distribution: Prob. [hitting all targets] = 1 Prob. [anything else] = 0 Note: Limiting behavior of E(m,n) is same as in simple approximation.	$\overline{E}(R) = 1 - \epsilon$ $\overline{\sigma}^2(R) = \epsilon[1-\epsilon(1+R(E_1-\sigma_1{}^2))]$ where $\epsilon = e^{-RE_1}$ Limiting distribution: Normal	$\overline{E}(R) = 1 - (\wp_1 \ldots \wp_N)^R\ \overline{\varphi}(R)$ where $\overline{\varphi}(R) = (1 - \{Rc\})e^{\frac{1}{d}\{Rc\}} +$ $\{Rc\}e^{-\frac{1}{d}(1-\{Rc\})}$ $\{Rc\}$ = fractional part of Rc.	$E(R) = 1 - q^{[R]} + \{R\}pq^{[R]}$ $\overline{\sigma}^2(R) = q^{[R]}(1-q^{[R]}) + \{R\}pq^{[R]}$ $(q^{[R]}(1+q)-1)$ where [R] = largest integer not exceeding R. $\{R\} = R - [R]$. Limiting distribution: Normal.