

# Relational Bundles and Eliminative Ontic Structural Realism

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## 0.0 Introduction

Two arguments frame the contemporary scientific realism debate: the *no-miracles argument* and the *pessimistic meta-induction*. *Scientific realists* accept the former argument contending that any theory other than scientific realism would make the predictive success of science a miracle. *Scientific anti-realists* accept the latter argument asserting that there is a high probability that like our previously accepted mature scientific theories our best current scientific theories will also be falsified (and hence, we ought not commit to their ontologies). Many realists have recently turned to a particular formulation of scientific realism, *Structural Realism* (SR), as a means to accommodate both arguments.

SR comes in two varieties the first of which goes by the moniker *Epistemic Structural Realism* (ESR). Proponents of ESR<sup>2</sup> ask us to commit to the mathematical structure and not to the ontology posited by our best scientific theories. *Ontic Structural Realism* (OSR) incorporates our best insights about scientific realism, namely the aforementioned arguments, while revising our traditional metaphysical suppositions to accommodate them. Proponents of OSR<sup>3</sup> assert that the ontological category of individuals must be discarded or revised. Metaphysical underdetermination<sup>4</sup> of quantum particles motivates OSR over ESR. Call the version of OSR that does away with individuals as a primitive category *Eliminative Ontic Structural Realism* (EOSR). Advocates of EOSR propose a radical reconceptualization of our metaphysical commitments captured in the following motto: "structure is all that there is."<sup>5</sup>

Several problems confront EOSR of which I will attend to four. The first problem stems from the objection that relations require relata.<sup>6</sup> The defenders of EOSR have offered the reply that relations may take other relations as relata. However, such a reply appears to require a commitment to an infinitary ontology and the denial of there being a fundamental level to nature.<sup>7</sup> The second problem is that structures are presently represented using a set-theoretic edifice largely borrowed from first-order model theory. Unfortunately, model-theoretic construals of structure privilege an ontology of individual objects. The eliminative ontic structural realist therefore requires a way to represent structure that aligns with their ontological posits. The third problem is that explicit formulation of a metaphysics of relations has not yet been undertaken. The fourth problem is that arguments for EOSR have thus far failed to motivate that position. It is the purpose of this short essay to adumbrate a *bundle theory of relations* by which the proponents of EOSR may come to address all four problems while attending to a number of major objections regarding the coherence of the position in the first place.

## 1.0 Structure

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<sup>2</sup> See Morganti 2004 and Worrall 1989 for representative positions.

<sup>3</sup> Ladyman and Ross 2007 explicitly defend OSR as does Bain (forthcoming), Esfeld and Lam 2011, French 2011, and Taylor 2011.

<sup>4</sup> A case of *metaphysical underdetermination* is one in which the available evidence is insufficient to identify which metaphysical commitments we ought to hold - i.e. when our metaphysical commitments are undetermined by the evidence for them.

<sup>5</sup> Or perhaps best presented by way of a slight modification of a popular expression – rather than say “everything is interconnected” the position asserts “there are only interconnections, not things”.

<sup>6</sup> See Ladyman and Ross 2007 pp. 154-156 and Frigg 2011 p. 262.

<sup>7</sup> See Frigg 2011 pp. 262-263.

Structure is a basic notion. Structures are patterns.<sup>8</sup> A *structure* is "the abstract form of a system"<sup>9</sup> highlighting the interrelationships among its constituent entities, and ignoring any features that do not affect how they relate to other entities in the system. It is convenient to think of a structure as a collection of entities with relations over those entities.<sup>10</sup> It is therefore expedient to represent a structure as an ordered tuple  $S = \langle \mathcal{A}, R \rangle$  where  $\mathcal{A}$  is a (non-empty) domain and  $R$  a (non-empty) set of relations on  $\mathcal{A}$ .<sup>11</sup> Note that one needn't be restricted to thinking of structures in terms of first-order model theory as we shall soon see.

### 1.1 Surveying Structuralism

Contemporary structuralism is a revival of positions that trace their lineage back to Bertrand Russell, Ernst Cassirer and Henri Poincaré.<sup>12</sup> John Worrall formulated contemporary structuralism<sup>13</sup> in order to synthesize a response to the two most compelling arguments in the debate over realism in the philosophy of science: the *no miracles* argument and the *pessimistic meta-induction*.

The *no-miracles* argument purports that the success of science would be miraculous if scientific theories were not at least approximately true. The *pessimistic meta-induction* is essentially the argument that "we cannot commit ourselves to the belief in present theories since successful theories throughout the history of science were refuted or abandoned."<sup>14</sup> This argument is taken to motivate ontological anti-realism on the basis that the ontologies of our best current scientific theories cannot be taken at face-value because the theories, from which these ontologies are derived, will too be discarded or abandoned.

Worrall's original proposal, which has since been dubbed ESR, asks us to commit to the structure and not to the content of our best current scientific theories. According to ESR our knowledge of unobservables is restricted to knowledge of the relations in which unobservables stand. Taylor notes that ESR thus comprises the following theses<sup>15</sup>:

- (R) Our scientific theories reveal to us the mind-independent features of nature.
- (SR) Our scientific theories reveal to us the structural features of nature.
- (K) We can come to have structural knowledge of nature.
- (I) We cannot come to know the nature of individuals.

According to ESR, our mature scientific theories are predictively successful because they accurately capture the structure (relations) of the world. Furthermore, ESR respects the *pessimistic meta-induction* and ontological anti-realism by **I**. ESR thus does service to both the *no-miracles argument* and the *pessimistic meta-induction* getting the best of both scientific realism and anti-realism.

ESR has spawned an ontological variety of structuralism: EOSR.

### 1.2 Eliminative Ontic Structural Realism

The argument to EOSR proceeds in two steps. First, I define a metaphysical position  $m$  to be compatible with naturalism *if and only if*  $m$  is motivated by (derived from and hence, compatible with) a contemporary (best) scientific theory. A contemporary scientific theory is a best scientific theory at some time  $t$ . A best scientific

<sup>8</sup> See Resnik 1997 pp. 202-209

<sup>9</sup> See Shapiro 1997 p. 74.

<sup>10</sup> Note that on occasion the defenders of OSR take 'structure' to mean 'relations'. See Ladyman and Ross 2007 pp. 148-154 and Taylor 2011 pp. 98-99.

<sup>11</sup> Properties are considered here to be monadic relations. Such a presentation of structure is considered to be the standard way to present structures.

<sup>12</sup> Russell 2004 p. 14 is a paradigm example of such early structuralist thought.

<sup>13</sup> See Worrall 1989.

<sup>14</sup> See Kantorovich 2006 p. 2.

<sup>15</sup> See Taylor pp. 56-57 and p. 60.

theory is a scientific theory enshrined into scientific practice and institutions as the premier way to explain, predict or describe the behavior of observed phenomena in a particular area of scientific investigation (e.g. psychology, physics, microeconomics).<sup>16</sup>

The justification for this rests on the grounds that I take naturalism to mean that we should be motivated to constrain our philosophical activities, at least those that wish to make the claim toward objective truth, by science; and that science through a process of continual revision and theory change continues to reject certain theories or to produce better ones. Thus, any metaphysical position that fails to derive from or compatible with a current best scientific theory likewise fails to be a metaphysical position motivated by, or compatible with, naturalism.

Now, I will characterize physicalism as being any particular metaphysical position which satisfies (holds) these assumptions<sup>17</sup>:

- (L) The world is comprised of levels.
- (FL) There is a fundamental level.
- (FA) This fundamental level is a collection of irreducible atoms.
- (LI) These atoms interact locally (micro-banging).
- (AO) These atoms are objects (i.e. individuals) which are understood to be something to which properties attach to (bare particulars) and/or which display a primitive essence, *haecceity*, *quiddity*, or substance.
- (P) These objects are physical.

Physicalism, so conceived, fails to be motivated by a best current scientific theory and also fails to be compatible with Quantum Mechanics and hence, fails to be compatible with naturalism. Consider also that physicalism is motivated to provide a realist metaphysics or ontology for natural metaphysics, presumably derived from physics itself.

I need only turn to Quantum Mechanics to explicitly falsify condition **LI** through quantum entanglement. **AO** and **FA** are the subject of significant controversy given the empirical success of Quantum Field Theory.<sup>18</sup> Thus,

- (P1) Physicalism is motivated from Classical Mechanics and Physics.
- (P2) Classical Mechanics and Physics are subsumed by Quantum Mechanics and Field Theory.
- (P3) Quantum Mechanics and Field Theory explicitly deny **LI**.<sup>19</sup>

It follows that physicalism stands in opposition to naturalism at least insofar as it is not even compatible with or motivated by a current best scientific theory. Embarrassingly, many physicalist metaphysics do *not* make *any* reference to contemporary physics.<sup>20</sup>

I will turn to the issue of levels and fundamentalism (i.e. **L**, **FA**, and **FL**) shortly. Physicalism, however, as it has traditionally been conceived is clearly incompatible with naturalism. Into this yawning gap, steps OSR.

The next step is motivated by a mixture of parsimony and metaphysical underdetermination.

Proponents of EOSR deny the epistemic limitations of ESR by asserting a revisionary metaphysical claim: our traditional ontological category of individuals is misguided<sup>21</sup> and structures are all that exist. To **R**, **SR**, and **K** the proponents of EOSR add the following thesis:

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<sup>16</sup> This is a reformulation of an argument originally raised by Ladyman and Ross. See Ladyman and Ross 2007.

<sup>17</sup> See Kim 2005.

<sup>18</sup> Consider the the "particles or fields" debate raging in the philosophy of physics.

<sup>19</sup> This is doubly embarrassing for the physicalist given that (P3) is common knowledge – Einstein's "spooky action at a distance". See Einstein, Podolsky, and Rosen 1935 for their seminal EPR paradox paper and Bell 1964 for the explicit falsification of the EPR local realism hypothesis. See Goldstein 1994 for a recent mathematical treatment of non-locality without inequalities.

<sup>20</sup> See Ladyman and Ross 2007 pp 10 and 18. Kim, for instance, does not cite a single recent scientific theory in *Physicalism* 2005.

(S) Structure is all that there is.

Such a move is motivated by quantum mechanics where the status of individuality and object-hood are underdetermined.<sup>22</sup> Consider the case of two quantum particles in a spherically symmetric singlet state.<sup>23</sup> Each particle in the state has the same properties - charge, spin, mass, etc. When the particles are entangled we cannot properly ascribe to them a position in space-time so, we cannot use spatio-temporal properties to discern them. Thus, even though there are *two* particles in the singlet state, the particles appear to have no properties different from each other. That the particles have no properties different from each other runs contrary to the Principle of the Identity of Indiscernibles (PII) which states that no two individuals have all the same properties. This leaves us in a dilemma. Either quantum particles are non-individuals or they are individuals of a kind that violates PII. Given that there isn't a good reason to prefer one view over the other, we are left with a bona-fide case of metaphysical underdetermination.

Quantum particles thereby spell trouble for the realist. How can we be asked to commit to an ontology derived from science if we can't even decide the nature of fundamental particles? EOSR was formulated to side-step the problem by striking the ontological category of individuals from our ontology and committing to a metaphysics of structure.

Is that move justified? Many do not think so and it's not hard to see why. Suppose that  $\mathcal{A}$  and  $\mathcal{B}$  are specific theories about some topic or concept  $X$  and that constitute a particular specification, operationalization, or characterization of  $X$ . Now, suppose further that  $\mathcal{A}$  is undetermined with respect to  $\mathcal{B}$ . Does it follow that we ought to outright stop discussing (or even reject)  $X$ ? No, an intermediate position avails itself to us. We can simply suspend judgment about  $\mathcal{A}$  and  $\mathcal{B}$  while continuing to talk about or explore  $X$ . That is to say, though  $X$  may lack a specific operationalization or specification, it may nevertheless play an essential, practical, or truth-apt role in our theoretical considerations. Further, the concept might even be basic or un-analyzable yet vague and opaque – though the inability to characterize the concept further would a definite strike against it though practical considerations might come into play.

Consider the following simple thought experiment: suppose Sally has heard from Joe that Bob has a Toyota and Jane has also heard from the same Joe that the same Bob has a Chevy. Sally and Jane have equal epistemic standing with respect to the vehicle that Bob has. Enter John who is trying to determine whether Bob has a Chevy or whether he has a Toyota. Sally and Jane are equally good epistemic agents and enter John who relies upon them to determine what type of car that John has. His, John's, evidence is underdetermined – he assigns the same credence, to the same amount of evidence, from equally reliable epistemic agents. Should John, Sally, and Jane simply dispense with talk about Bob's vehicle? No, they simply don't know what make his vehicle is. They can still talk about Bob's car though they don't know its specific character (in this case make).

It gets worse, suppose though that our best theories about objects were not only merely undetermined but actually both explicitly falsified and undetermined. Even in this scenario, though the concept of 'objecthood' may remain ill-motivated and vague or, that is to say, though it would lack in this specific scenario a *disambiguation* – i.e. an *operationalization* owing its lack of an unfalsified theoretical framework it may nevertheless play an essential, useful, or coherent role in our theoretical and epistemic activities (and for other reasons, like the fact that *prima facie* language appears to be object-based in conjunction with the fact that at least some language appears to be truth-apt which provides independent motivation for utilizing the concept of object). Here we grasp that the utility, or practical use, of a concept may sometimes outweigh other theoretical considerations.<sup>24</sup> In other words, the move to eliminating objects is ill motivated from the argument that Ladyman and Ross have given.

However, while unmotivated, their argument has nevertheless inspired the development of a novel way of doing metaphysics though attempts thus far have failed to achieve their aims. Over the remaining course of this paper, I will show how the aim of EOSR can be fulfilled by a *bundle theory of relations*. Such a position gains

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<sup>21</sup>See Chakravartty 2003 pp. 867-868.

<sup>22</sup> See Ladyman and Ross 2007.

<sup>23</sup> See Ladyman and Ross pp. 135-138.

<sup>24</sup> See Carnap 1950.

*independent* motivation from the fact that it is more parsimonious in several key ways (including the number of independent ontological categories, subcategories of entities, and number of ontological distinctions required) over existing theories about fundamental ontological kinds including trope bundle theories.

Does EOSR entail that there aren't things like tables, rocks, and chairs? If tables, rocks, and chairs are understood as individuals, then yes. If tables, rocks, and chairs are understood as patterns or as abstractions from more primitive patterns, then no. James Ladyman and Don Ross hold that while there are not self-subsisting individuals in the world, we nevertheless find it useful to think about entities encountered in the world as objects.<sup>25</sup> But note that objects then play merely a heuristic role in allowing us to track salient patterns.<sup>26</sup> On the EOSR view, our discourse, which includes talk of 'tables', 'rocks', and 'chairs' tracks patterns in the world.

Does EOSR commit its defenders to mathematical Platonism about the physical world - a kind of metaphysical "Pythagoreanism"?<sup>27</sup> Such a concern has been formulated as follows: the "difference between mathematical (uninstantiated) structure and physical (instantiated) structure cannot itself be explained in purely structural terms."<sup>28</sup> What then makes structure physical and not mathematical? Ladyman and Ross respond to this objection by suggesting the abandonment of the abstract/concrete distinction. They motivate the abandonment of the abstract/concrete distinction from the observations that theoretical terms may very well refer to mathematical entities,<sup>29</sup> that ideal (putatively abstract) entities are employed throughout physics, and that the distinction is usually made in terms of either causal power or spatio-temporality.<sup>30</sup> On the last point, Ladyman and Ross state that it is difficult to establish the causal power of certain purportedly concrete entities. For instance, nobody has thus far proposed an acceptable account of causation for the case of a singlet state in the Bohm-EPR experiment which "fails to screen off the correlations between the results in the two wings of the apparatus, and thus fails to satisfy the principle of the common cause."<sup>31</sup>

I agree with the spirit of their rebuttal. However, there are several more options available to the proponent of OSR. One may reject *ante rem* mathematical structuralism and either accept traditional object based mathematical platonism, *in rebus* structuralism, or any otherwise non-realist positions regarding the existence of mathematical objects without disturbing the abstract/concrete distinction. Thus, the objection only succeeds if one envisions mathematical entities to be platonist (i.e. free-standing, *ante rem*, abstract) structures but mathematical platonism itself relies on a number of dubious premises.

Mathematical Platonism is usually characterized as the following three theses:

- (**M**) Mathematical objects exist. I.e.  $\exists x(Mx)$ .<sup>32</sup>
- (**MA**) All mathematical objects are abstract. I.e.  $\forall x(Mx \rightarrow Ax)$ .
- (**MI**) Mathematicalia are independent of the physical world and its intelligent agents. I.e.  $\forall x(Mx \rightarrow \neg Px)$ .

Here, 'objects' may be replaced with 'entities' or 'structures'. I take **MA** to be an uncontroversial thesis.<sup>33</sup> **M** is usually defended by appealing to semantic accounts of truth and reference – i.e. the traditional arguments levied against psychologism (raised perhaps most famously by Frege):

- (P1) Mathematical statements are true.
- (P2) The correspondance theory of truth is true.

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<sup>25</sup> Ibid. p. 239.

<sup>26</sup> Ibid. p. 155.

<sup>27</sup> See Cao 2003, Ladyman and Ross 2007 pp. 157-158, and van Fraassen 2006 pp. 292-294.

<sup>28</sup> See Ladyman and Ross 2007 p. 158.

<sup>29</sup> Consider the wave-function.

<sup>30</sup> See Ladyman and Ross 2007 p. 160.

<sup>31</sup> Ibid.

<sup>32</sup> I use these first-order sentences just for convenience, presumably we are ranging over all entities/objects with our quantifiers.

<sup>33</sup> The early Maddy is a noted exception. See Maddy 1990.

- (P3) If both mathematical statements are true and the correspondance theory of truth is true then, mathematical platonism is true.
- (C) Mathematical Platonism is true.

Mathematical practice also suggests, at least superficially, existence claims which are then used in foundational axiomatic systems which produce statements that are understood to be true. But the so-called “Fregean Argument” relies on the dubious correspondance theory of truth. While it is not the purpose of this paper to enter into the truth debate, there are serious concerns that should make us wary of being quick to endorse such a conception of truth. The correspondance relation has never really been fleshed out to much degree (and is therefore, mysterious), is seemingly empirical untestable, underdetermined (which conception of correspondance is the right one and more importantly, what evidence is there to decide between it and its rivals?) and hence, we have no reason to believe in its existence over say a deflationary conception of truth which may very well offer a more parsimonious explanation and description of truth-talk. Thus, many philosophers have come to believe that Frege’s Argument is unconvincing and carries with it too great a cost found in the theory of truth that it presupposes.

A common strategy employed by mathematical realists in order to defend the respectability of **M** under a naturalist metaphysics was to appeal to the Quine-Putnam Indispensability Argument.

- (P1) We should have ontological commitment to all and only the entities that are indispensable to our best scientific theories.
- (P2) Mathematical entities are indispensable to our best scientific theories.

It purportedly follows that we ought to have ontological commitment to mathematical entities. I note here that the conclusion does not entail mathematical platonism. It simply entails some kind of ontological commitment.<sup>34</sup> Secondly, just what we take entities to be varies depending on whether formalism<sup>35</sup>, intuitionism<sup>36</sup>, or any other sort of mathematical anti-realism are true<sup>37</sup> (though perhaps the first commitment requires one to also take a stand on just what ontological commitment). Each of these arguments can be run against mathematical structuralism by replacing talk of 'objects' with talk of 'structures' or 'entities'. Others have felt the need to directly challenge the purported indispensability of mathematical entities altogether. Recently, Harty Field, who has nominalist leanings, has attempted to outflank the Quine-Putnam Indispensability Argument by demonstrating that numbers are in fact dispensable to scientific activity by providing a nominalist formulation of scientific theories (though many believe this to be a failed attempt).<sup>38</sup> If such an argument were to succeed, it would show the first premise incorrect. However, a successful view along such lines does not appear to be widely accepted. Nevertheless, there is logical space for such a position.

The upshot of both considerations suggests that something like mathematical platonism need not be endorsed (and perhaps shouldn't be). Furthermore, the abstract/concrete distinction has never been adequately justified in the first place. Perhaps the most popular formulation of the abstract/concrete distinction is given by the following four theses:

- (AC) Something is abstract *if and only if* it is not concrete. I.e.  $\forall x(Ax \leftrightarrow \neg Cx)$ .
- (CA) There is nothing that is both concrete and abstract. I.e.  $\neg \exists x(Ax \wedge Cx)$ .
- (PC) All physical things are concrete. I.e.  $\forall x(Px \rightarrow Cx)$ .

Adding to these the uncontroversial thesis that:

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<sup>34</sup> A subject of great controversy in the metaontology debate. Consider Carnap's notion of ontological commitment which does not entail any substantial ontological significance - i.e. "Numbers *really* exist" versus "there are numbers" - endorsed by many neo-Carnapians.

<sup>35</sup> Hilbert's view that mathematical statements are purely syntactic and that the axioms of ZFC set theory and Peano arithmetic are provable from a finite set of axioms.

<sup>36</sup> The view that mathematical entities are mental and that proof requires *constructive* proof.

<sup>37</sup> I believe that this argument may very well have been raised elsewhere though I do not recollect by whom or where.

<sup>38</sup> See Field 1980.

(U) Every entity is either abstract, concrete or both. I.e.  $\forall x(Ax \vee Cx)$

But why this formulation? I have yet to see a compelling argument for this or any other formulation of the abstract/concrete distinction. In fact, most attempts to explicate the distinction appear to simply presuppose that the distinction is true and then attempt to offer some point of demarcation. For example, consider that the traditional distinction between repeatable and non-repeatable entities, one that has on occasion been taken to in part comprise the necessary conditions for the abstract/concrete distinction, does not entail that there is an abstract/concrete distinction merely that there is a repeatable and non-repeatable distinction. It is only stipulated that it is part of the abstract/concrete distinction though no support is ever offered for the latter distinction in the first place.

As noted before, most structuralists agree with the suggestion proposed by Ladyman and Ross. Shapiro takes it that the structuralist perspective is a "healthy blurring"<sup>39</sup> of the distinction between mathematical and ordinary entities. Resnik points out that quantum particles already appear to be quasi-abstract:

"The term 'particle' brings to mind the image of a tiny object located in space-time. But, on what seems to be the consensus view of the puzzling entities of quantum physics, this image will not do. Most quantum particles do not have definite locations, masses, velocities, spin, or other physical properties most of the time."<sup>40</sup>

He elaborates:

"We began by observing that it is usual for philosophers of mathematics to distinguish between supposedly abstract, mathematical objects and supposedly concrete, physical ones by appealing to the spatio-temporal locatability, causal powers, or detectability of the latter. We now see that distinguishing between abstract and concrete objects in this way is obsolete."<sup>41</sup>

Suppose one were to endorse a different set of conditions by which to distinguish between the two kinds - say one that rejects outright **CA**. What then would be the problem with OSR and EOSR? Suppose that one were to take Ladyman and Ross, Michael Resnik, and Stewart Shapiro at face-value. What does a view inspired by our best current scientific theories look like? Something like the following:

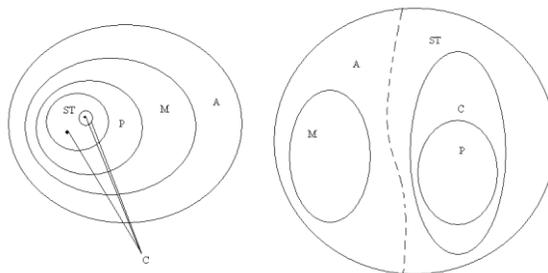


Fig.1<sup>42</sup>

Now, while maintaining **CA** appears to lack any justification, contemporary physics (and particularly interpretations of quantum field theory which treat particles as perturbations of fields lacking any *haecceity* or primitive thinness)<sup>43</sup> lends credence to the idea that mathematical and physical entities are one of a kind.

<sup>39</sup> See Shapiro 1997 p. 256.

<sup>40</sup> See Resnik 1997 p. 102.

<sup>41</sup> See Resnik 1997 p. 107.

<sup>42</sup> Where 'A' denotes abstract, 'M' denotes mathematical, 'ST' denotes spacetime, 'C' denotes concrete, and 'P' denotes physical. I do not endorse this, but Max Tegmark does. See Tegmark 2007.

<sup>43</sup> As David Baker notes, "The notion that quantum field theory (QFT) can be understood as describing systems of point particles has been all but refuted by recent work in the philosophy of physics. Rigorous

**CA** is supported (yet remains unjustified) by certain contemporary platonist assumptions that I have already shown to rest on certain dubious theses, at least with respect to mathematical platonism. I will return to an alternative conception of relations below, one that does away with a difference between things that universals/properties are instantiated in and universals/properties themselves. Thus, the proponent of EOSR has both the grounds and means to outright reject **CA** while nevertheless retaining the remaining theses characterizing the categories 'abstract' and 'concrete' (though such an outright denial of **CA** cuts out the meat of the abstract/concrete distinction).

One objection that might be raised here goes along the lines that rejecting the abstract/concrete distinction eliminates the possibility of distinguishing between the patterns studied in say pure mathematics and those studied in physics. A possible move here is to take the structures studied in mathematics as representational devices enabled upon selecting a particular graphico-syntactic medium. By graphico-syntactic medium, I mean language, symbolic system, or kind of graphical representation (the arrows of category theory, the curly braces of set theory, representational bundles to be articulated below, etc.). The structures studied by mathematicians are those that (a) are represented through a particular graphico-syntactic medium and (b) are constrained by axioms or rules determining how those graphico-syntactic items are concatenated. This sketch of a response is not to be mistaken as a kind of Formalism which entails that the truths of mathematical statements are merely the result of symbolic manipulation. The idea above goes beyond Formalism in that it does not merely focus on logical graphico-syntactic systems and because it does not commit one to the view that mathematical statements are merely the result of symbolic manipulation. However, the idea presented above does give a principled point of demarcation. Scientific structures are those graphico-syntactic structures that find application to empirical phenomena following acceptance by the scientific community. Here, once again, scientific structures are taken to be a subset of mathematical structures which again diverges from the view that mathematical entities are distinct and separate from scientific entities.

### 1.3 Infinitism

By far the biggest criticism of EOSR has been that one cannot have relations without *relata*.<sup>44</sup> If we take EOSR at face-value, we must dispense with the ontological category of individuals. But it seems that in dispensing with the ontological category of individuals we can no longer furnish relations with *relata*.

Proponents of EOSR have responded to this objection by pointing out that relations do have *relata* but that these *relata*, under closer inspection, turn out to be structures after all<sup>45</sup> – relations can take other relations as *relata*. The problem with that reply is that it leads the advocates of EOSR into an infinite regress. The contemporary wisdom has it that chains of relations must bottom out in non-relational entities - i.e., that relations supervene on intrinsic properties (a thesis that derives from physicalism). Insofar as relational structure is all that exists, it must be that chains of relations go on *ad infinitum*. It seems that proponents of EOSR are committed to an infinitary ontology, which is ontologically profligate, and to the denial of there being a fundamental level to nature, which is deeply counter-intuitive.<sup>46</sup>

There are a number of replies to this objection. First, we already have good reason to reject physicalism from whence such intuitions derive. Second, Ladyman and Ross hold that "we have good inductive grounds for denying that there is a fundamental level to reality."<sup>47</sup> Every time we think we've hit the bottom, so to speak, we find yet another fundamental level.

However, fundamental physicists may very well develop a complete microphysical, empirically adequate, theory that posits a fundamental level to nature. In such an event, the proponent of EOSR would be out of luck -

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forms of the interacting theory cannot sustain a "quanta" interpretation in which the fundamental entities are countable. And even the free theory is provably unable to describe particles localized at or around points" though he argues the field interpretation view. See Baker 2008 pp. 585-586.

<sup>44</sup> See Ladyman and Ross 2007 pp. 154-156 and Frigg 2011 p. 262.

<sup>45</sup> See Frigg 2011 p. 262.

<sup>46</sup> Schaffer 2003 p. 498 notes that talk of a "fundamental level of reality" pervades contemporary metaphysics.

<sup>47</sup> See Ladyman and Ross 2007 p. 178.

they would be committed to an extravagant metaphysical hypothesis. Fundamentalism remains a strong epistemic possibility and one that is seriously entertained by many physicists. Advocates of EOSR would be better served by an ontology that can also accommodate the epistemic possibility of there being a fundamental level to nature rather than merely one that is wed to an infinitary picture of the world. All things being equal, it is better to have an ontology that is open to more epistemic possibilities than one that is open to fewer.

## 2.0 Interlude

In order to overcome the objection that relations require relata, the eliminative ontic structural realist may counter that relational structures serve as the relata for relations.

In making such a move, the eliminative ontic structural realist appears committed to both an infinitary ontology (infinite relations) and picture of the world (infinite levels).

In the following sections, I will propose a way for eliminative ontic structural realists to preserve the strong intuition that there is a fundamental level in nature. The proposal that I have in mind is that structures can come as bundles of relations. I will first survey two mathematical theories in which such structures are already accepted in order to motivate my proposal in sections 3.1 and 3.2. I will lay out the specific details of the proposal through section four.

## 3.0 Category and Graph-Theoretic Structure

Many ontic structuralists have rebelled against the hegemony of set theory and first-order classical logic in determining an ontology and representation of structures.<sup>48</sup> Model-theoretic construals of structure take their inspiration from first-order predicate logic which privileges an objectual ontology. By contrast, category theory and graph theory lend themselves to an ontology of relations.

Ladyman and Ross note that:

"[The] structuralist faces a challenge in articulating her views to contemporary philosophers schooled in modern logic and set theory, which retains the classical framework of individual objects represented by variables subject to predication or membership respectively. In lieu of a more appropriate framework for structuralist metaphysics, one has to resort to treating the logical variables and constants as mere placeholders which are used for the definition and description of the relevant relations even though it is the latter that bear all the ontological weight."<sup>49</sup>

In the two subsequent sections, I will turn to category theory and graph theory to motivate the possibility of relational bundles.

## 3.1 Category Theory

Category theory is ubiquitous in its application throughout mathematics and is presently employed in many of the natural sciences. Participants to the mathematical foundations debate have recently proposed category theory as an alternate foundation for mathematics,<sup>50</sup> as a framework for a mathematical structuralism of *in rebus* structures,<sup>51</sup> and as a framework for physical ontology.<sup>52</sup> Category theory is becoming an increasingly important tool in theoretical computer science particularly in programming language semantics and in its application to domain theory.<sup>53</sup> Category theory is also finding increased relevance in fundamental physics where categories are given a physical interpretation.<sup>54</sup> Many participants to the structuralism debate have

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<sup>48</sup> See Ladyman and Ross 2007 p. 155. Bain (forthcoming) unites EOSR with category theory.

<sup>49</sup> See Ladyman and Ross 2007 p. 155.

<sup>50</sup> See McLarty 2004 p. 41-44.

<sup>51</sup> See Landry 2006.

<sup>52</sup> See Bain (forthcoming).

<sup>53</sup> See Pierce 1991.

<sup>54</sup> See Bain (forthcoming) pp. 10-11 and Coecke 2006.

argued that category theory deserves a larger role in formulating an ontology of structures<sup>55</sup> and in representing structure.<sup>56</sup>

A *category* is typically presented as a collection of *objects* and a collection of *morphisms*<sup>57</sup> such that:

- (1) For each morphism  $f$ , one object serves as the *domain* of  $f$  and one object serves as the *codomain* of  $f$ .
- (2) For each object  $A$ , there is an *identity morphism*,  $1_A$ , which has domain  $A$  and codomain  $A$ .
- (3) For each pair of morphisms  $f : A \rightarrow B$  and  $g : B \rightarrow C$ , there is a *composite morphism*  $g \circ f : A \rightarrow C$ .
- (4) For each morphism  $f : A \rightarrow B$ ,  $f \circ 1_A = f$  and  $1_B \circ f = f$ .
- (5) For each set of morphisms  $f : A \rightarrow B, g : B \rightarrow C, h : C \rightarrow D$ ,  $(h \circ g) \circ f = h \circ (g \circ f)$ .

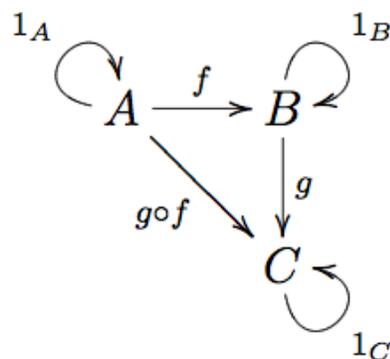


Fig. 2

Figure one reveals a sample category.

Note that objects here play a minor role. They merely "receive" morphisms and "send them out". They have no identity or essence over and above the morphisms in which they stand. In fact, it is possible to present a category without objects by simply replacing all talk of objects with identity morphisms.<sup>58</sup> Category theory may therefore be presented in a way amenable to the EOSR line. The category displayed in figure one may be presented as a collection of morphisms  $\{1_A, 1_B, 1_C, g, f, g \circ f\}$  satisfying suitably revised rules (1)-(5). Note that each relata of each morphism is itself a morphism. Category-theoretic structure thereby lends *prima facie* support for the possibility of relational bundles but it is important to note that there is some disagreement on that point.

Vincent Lam and Christian Wüthrich argue that category theory does not offer any support for eliminative ontic structural realism.<sup>59</sup> Their objection is two-fold. First, a category such as that offered above does not provide an interpretation of physical systems directly. Rather, what a category does do is provide a framework for the *models* of a particular physical system. If one were to adopt category theory to do the work of eliminative ontic structural realism, one would need for the models of the system to dispense with objects. Unfortunately, the objects of those models appear to be sets. While some have attempted to reformulate the concept of a set to fit an object-less ontology so as to provide a direct categorical interpretation of physical phenomena, there is some suspicion that the concept of an object (though perhaps much deprecated) is not truly eliminated at least insofar as the concept of an 'element' is retained (whether those elements are objects or something else appears undetermined). Furthermore, *n*-ary relations are typically defined as Cartesian products (which are sets of sets) which can be formulated in some categories and not others. Thus, Lam and Wüthrich

<sup>55</sup> See Bain (forthcoming) for such a view.

<sup>56</sup> See Landry 2007.

<sup>57</sup> Sometimes called 'arrows'. By 'morphism' one usually means 'a structure-preserving mapping' - a kind of function or binary relation.

<sup>58</sup> See Bartles 2010 though such a conception of categories has been known for much longer in the literature.

<sup>59</sup> See Lam and Wüthrich 2013.

conclude our best theory may deliver a category in which either elements of objects, or relations attributable to objects, or both, do not exist.<sup>60</sup>

I agree that category theory may very well, and presently appears to be, unsuitable for a full formulation of EOSR. However, the claim made in this paper is merely that category theory provides conceptual space for a formal system without objects. Unlike set theory which is built from the bottom-up by an infinite universe of objects, category theory can be built from the top-down from an infinite categories of categories. Insofar as categories can be represented without recourse to objects, so category theory makes room for the view that there can be relations without objects which in turns lends support to the conceptual possibility of EOSR (which I have noted is one of the main objections to it). Whether category theory is fully capable of fleshing out EOSR or appropriate to the task of modern fundamental physics remains an open question (though perhaps we may have good reason to doubt both). This adds additionally urgency to the task of this paper which is to provide a formal system and hence a conceptual framework capable of attending to the demands of EOSR.

### 3.2 Graph Theory

Graph theory is another ubiquitous theory that has found a home in the biological sciences<sup>61</sup>, computer science, chemistry, and throughout mathematics. Briefly, a *graph* is a collection of *vertices* joined by a collection of *edges*. Each edge models pairwise relations between vertices. More formally, an ordered tuple  $\langle V, E \rangle$  is called a graph whenever  $V$  is a set of vertices,  $E$  a set of edges, and  $E \subseteq \{\{a, b\} \mid a, b \in V, a \neq b\}$ . Graphs come either *labeled* or *unlabeled*. A labeled graph is just a graph where each element in  $V$  is indexed to a set. An unlabeled graph is a graph in which individual nodes have no distinct identifications except through the edges that connect them.



Fig. 3

The vertices in an unlabeled graph thereby have no identity over and above the edges between them. Figure two demonstrates an example unlabeled graph.

Jason Taylor notes that we can identify a graph with recourse only to the edges it is composed of.<sup>62</sup> The top-most node in figure two may be identified by the nodes it is connected to:



Fig. 4

Here the top-most node is left black and the connecting nodes are colored green. The left-most connecting node bears two relations and can therefore be identified as  $\{2\}$ .<sup>63</sup> The right-most connecting node bears three relations and can be identified as  $\{3\}$  accordingly. The top-most node can then be identified as  $\{2, 3\}$  and the remaining node can be identified as  $\{3\}$ . The graph thus consists in the following relational defined collection

<sup>60</sup> See Lam and Wüthrich 2013 pp. 10.

<sup>61</sup> See Mason and Verwoerd 2007.

<sup>62</sup> See Taylor 2011 pp. 111-115.

<sup>63</sup> I note that this process is entirely arbitrary. For example, one might instead take the top node to be identified as  $\{2\}$ , the bottom-left node as  $\{2, 3\}$ , the middle-bottom node as  $\{3\}$ , and the right most node to be  $\{1\}$ . I thank Renee Jorgensen for providing an objection that lead to that conclusion. I note that Taylor does not address that point.

of entities  $\{\{2\}, \{3\}, \{2, 3\}, \{3\}\}$ . I note that while some graphs are categories (and vice-versa) many graphs are not categories. Graph theory therefore lends additional *prima facie* support for the possibility of relational bundles and hence, EOSR.<sup>64</sup>

#### 4.0 A Bundle Theory of Relations

Now to the task of articulating an ontological framework suitable for the proponent of EOSR.<sup>65</sup> I begin by conceiving of relation as having *legs* (I have selected this name due to the method of diagramming articulated in 4.1). By a leg I mean a place that a relatum may take (though relatum here is restricted to other legs/relations). It is natural to think of legs as "slots" or "places." An arity 2 relation thereby has two legs, an arity 3 relation has three legs, and so on.

I should like to note the multitude of relations which have relations as their relata. Such relations are ubiquitous in philosophical discourse. Consider the bundle theorist's compresence relation<sup>66</sup> which takes monadic relations as relata, the second-order monadic relation "is a property" that takes other monadic relations as its relata, and the dyadic relation "has more relata than" which takes  $n$ -ary relations as relata. The relational bundle theorist thus takes on the following seemingly uncontroversial thesis:

**(RR)** Legs may take relations.

An entire relation may take the spot provided by a leg. For example, the legs of the relation "has more relata than" take whole relations and not just their legs. The relational bundle theorist also accepts the postulate:

**(LL)** Legs may take other legs.

One may couple relations together by "coupling" legs to each other - each coupled relation serving as a relatum for the other - in the same way that the two ends of a key ring clasp fit together each taking the other to lock it into place. Each relation that is coupled with another relation per **LL** is compresent with the other relation it is bound to.<sup>67</sup> Here, relations are reconceived without recourse to a distinction between *positions*<sup>68</sup> and *things that positions take* which is really a distinction between *ersatz* objects<sup>69</sup> and normal objects. Here "positions" take "positions".

Relations may also take relational structures as relata. The objects of a category may be categories themselves and first-order model theory allows for morphisms<sup>70</sup> between structures. Relational structures may therefore be composed of other relational structures. To **LL** and **RR** the relational bundle theorist adds the axiom:

**(RS)** Legs may take relational structures.

A collection of relations or relational structures bound together in the manner of **RR**, **LL**, and **RS** I call a *relational structure* - a bundle of relations. I take it that there is one  $n$ -ary compresence relation per relational structure though the relational bundle theorist might take compresence to be a dyadic relation holding between pairs of relations or pairs of relational structures and relations without loss or addition to the overall theory.<sup>71</sup>

<sup>64</sup> A contention shared by Taylor. See Taylor 2011.

<sup>65</sup> Taylor defends the possibility of relational bundles but does not explicate a relational bundle theory in much detail. No other person in the literature has provided an explicit formulation of a relational bundle theory nor an explicit formulation of EOSR though category theory and sheaf topology have been suggested as a basis for such a view (such views may require a commitment to set theory, however, despite the fact that they do provide *prima facie* support for EOSR).

<sup>66</sup> The relation that binds properties, and here relations with an arity greater than one, together.

<sup>67</sup> Taylor discusses this point at length. See Taylor 2011 p. 120.

<sup>68</sup> A major criticism raised against Stuart Shapiro's *Ante Rem* Mathematical Structuralism.

<sup>69</sup> See Lam and Esfeld 2011.

<sup>70</sup> Such as isomorphism and homomorphism.

<sup>71</sup> One might worry that the compresence relation itself needs to stand in some relation with the other relations composing the bundle à la a Bradley-style regress. Such a worry has been attended to by trope bundle theorists

Alternatively, one might hold that no relation of compresence is required, that the mere case of a relation taking another relation requires no further relations to bind them. In fact, that seems to be the case with just about any second-order relation (a relation that takes another relation). Consider "is a second order relation to", "has the same relata as", "is a second order relation to".<sup>72</sup> There does not seem to be any reason to hold that an additional relation is required to relate the two former relations, by way of the latter, together. The view offered here is incompatible with the traditional conception of relations (namely that there is an abstract universe of universals in "platonian heaven" that are instantiated in concrete objects because there are no objects under EOSR and because the abstract/concrete distinction is itself suspect).<sup>73</sup> On this conception, relations are the only things that exist, and they exist in virtue of being related to other relations. One argument for this view is that the EOSR relational bundle view is less ontologically profligate (relations and their relational bundles, which are merely relations, versus universals and their instantiations). Unfortunately, save for trope or nominalist views, there is no view that escapes the argument just given as all such other views presuppose such a distinction of one sort or another though they may not use the term 'instantiate' or 'universal'. But then, one wonders why we should use the term nominalist anyway.

What about change? Various relations between relational bundles between different times or events account for change. The persistence conditions of relational bundles can be specified by something like isomorphism<sup>74</sup> of essential relations or a weaker kind of morphism which require but subtle reformulations of existing theories of persistence (i.e. endurantism and perdurantism). Consider that a relational bundle at time  $t$  may remain identical to a relational bundle at time  $t+1$  or that a series of relational bundles may be related by weaker morphisms such that they can be conceived as a temporal worm. The relational bundle theorist now has all the ingredients for relational bundles and accepts that:

**(BTR)** Legs do not take anything other than relations, other legs, or relational structures.

One may therefore conceive of a bundle of relations as a collection of relations, legs, or relational structures or as the same standing in a relation of compresence satisfying **RR**, **LL**, **RS**, and **BTR**. As stated before, one may hold that there is no need for a relation of compresence. Are such relations tropes? No. They are not strictly tropes because compresence is essential to the concept of trope bundle whereas compresence is not essential to the concept of relational bundle. However, these relations exhibit certain features that tropes exhibit. But they also exhibit some features that repeatable entities exhibit.<sup>75</sup> On a trope theory view, a particular predicate is defined, perhaps arbitrarily or by convention, as a set of numerically distinct properties. Thus,  $\text{Red} =_{\text{df}} \{\text{red}_1, \text{red}_2, \text{red}_3, \dots, \text{red}_n\}$ . There is a sense in which certain relational, dyadic or higher-arity relations, may be clustered together and taken to define a particular relational predicate. However, because isomorphism is defined per footnote 73, a single relational structure may be understood as a repeatable entity. In some circumstances, a single relational bundle and hence, individual relations are isomorphic to other relational bundles. Thus, the naming of a particular individual relation is made purely by convention. For ease, I will take to treating the following relational bundles as separate and distinct from the relations in which they

and the eliminative ontic structural realist may help themselves to the standard replies should they insist on compresence. See Maurin 2002 pp. 139-166 and 2010.

<sup>72</sup> It is important to note that our conception of language is one framed by first-order logic. To some extent, first-order logic is a result of natural language at least insofar as natural language is often taken to be a meta-language within which the object language (FOL) is constructed. Thus, there are few ordinary examples to be given which is consistent with much of the mathematics utilized in contemporary theoretical physics. Consider the classical problem of linking quantum phenomena with Newtonian mechanics. Decoherence remains a controversial proposal and to this author's knowledge, the only one that has been offered. So, the problem of finding ordinary examples to frame a fundamentally different theory is not unique to the position articulated here.

<sup>73</sup> What I shall "contemporary platonism."

<sup>74</sup> Two relational bundles  $A$ ,  $B$  are isomorphic *if and only if* each relation in relational bundle  $A$  stands in one-to-one correspondance to each relation in relational bundle  $B$ . A consequence of this first approach to identity is that one relational bundle may be located in many places (distinct instances of which may be distinguished by the relations in which those instances stand). However, these are not universals, for there is no instantiation relation. I have given an argument above as to why the whole distinction between things in which so-called universals are instantiated and universals should not be endorsed.

<sup>75</sup> More on this below.

stand.<sup>76</sup> Consider the following: “is an expression”, “is represented by an English predicate”, “whose predicate shares a letter with the predicate representing”, “is a metalinguistic feature” - a particular relational bundle (which may be conceived by way of the diagrams articulated below). This is isomorphic to the relational bundle: “whose representation contains a verb”, “expresses a linguistic concept along with”, “is expressible in the same language as”, “is blue”.<sup>77</sup> Such relations are undirected, though the possibility remains for directed relations.<sup>78</sup> Now, both relational bundles are actually the same bundle when understood as separate and distinct from the other relations that they stand in (or even when they stand in particular relations as isomorphism can hold between substructures). Both relational bundles are actually the same relational bundle though they may be distinguished by the relations in which they stand. Hence, we might name a particular predicate or relational bundle class as a list of distinct relational bundles (individuated by the various relations in which they stand) or as a repeatable entity - as an entity that is present in many distinct and non-local, non-contiguous, relational bundles. Thus, the relational bundle view collapses a long-held distinction between nominalist and realist theories of propertyhood.

More on tropes. Crassly, trope bundle theory requires *compresence* binding *tropes*. Thus, we might say that on the trope bundle view, compresence (an  $n$ -ary relation) is required in addition to the myriad unary relations (tropes) it is taken to bind together. So, the relational bundle theory is similar to the trope bundle theory in that we might say that both take relations as being bound to other relations. But we see that the relational bundle theory differs from the trope bundle theory in that it dispenses with objects entirely (more below) and is far more permissive than the trope theory in the sense that relational bundles need not conform to the “one compresence relation plus  $n$  unary relations” schema that describes the structure of trope bundles at all.

Relational bundles come into existence when a set of relations or relational bundles come to be bound together by a compresence relation in the manner of **RR**, **LL**, **RS**, and **BTR** and go out of existence whenever their relations and/or relational structures cease to exist or cease to be compresent with each other. Relational bundles are individuated by the relations and relational structures which compose them.

One might be tempted to draw a distinction between ‘thin’ and ‘thick’ objects. The former type of entity requires only that it be a referent of a singular term. The latter kind of entity satisfies not only that but at least the Principle of the Identity of Indiscernibles as well. One might be then tempted to argue that relational bundles are just thin objects after all. Yes, thick objects have been eliminated but not all objects. The obvious reply to that objection is simply that the view not only collapses the abstract/concrete distinction and the distinction between things that are instantiated/things that instantiate but it also collapses the distinction between objects and relations entirely. The intuition, so the replay goes, that relational bundles are really thin objects gets turned on its head, thin objects are merely bundles of relations – they *reduce* to them. There are no objects over and above relations period.

#### 4.1 Diagramming Relational Bundles

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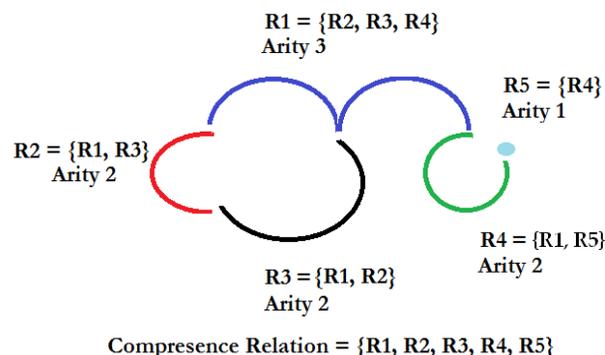
<sup>76</sup> In fact, such a condition may be taken to constitute a distinction between what I shall *inner relational bundles* and *outer relational bundles*. The former entities are merely distinguished by the relations that constitute it. The latter entities are distinguished by both the relations that constitute it and the relations in which it stands. As I see it, these are two different ways to conceive of relational bundles and present two ways to understand the identity conditions of them.

<sup>77</sup> As I observe later on, we note that “thin” objects (those that merely satisfy the condition for being the referent of a singular term) collapse into and reduce to relations on this view further collapsing the distinction between even this weaker conception of an object (and perhaps the weakest), relations, and properties. Thus, our understanding about semantics for natural language statements need not be changed save for the subtle reformulation of our metaphysical concepts. Taking a step back, natural language could be said to be truth-apt though not particularly metaphysically illuminating if ontology is read superficially from the *prima facie* ontology it seems to presuppose. That is to say, though natural language may seem to require commitment to thick objects (as many have thought) on first blush, it doesn’t though we may retain the same semantic machinery (i.e. - model theory and the formal semantics derived from it in linguistics) while changing the underlying ontological commitments of the model-theoretic machinery (switching set theory for relational bundle theory).

<sup>78</sup> The examples that follow below are undirected. See footnote 77 for further details.

Taylor outlines a way to represent relational bundles composed solely of binary relations but does not supply the proponent of EOSR with a suitable way to represent relational structures that are composed of ternary or higher-arity relations.<sup>79</sup> It is the purpose of this section to improve on Taylor's account and to give an explicit pictorial representation of the relational bundles laid out in the previous section. This is not a frivolous activity. One of the great criticisms against the EOSR view is that it is simply incoherent. I will below present a novel way to conceive of relations, relational bundles, and structures to not only demonstrate the absolute coherence of the EOSR position but to also equip its proponents with a means by which to dispense with object-centric mathematical frameworks like model theory and set-theory as a way of framing the entire structure discussion (because the proponent of EOSR has already lost half the battle by using object-centric talk to describe object-less structures). Why not use category theory or graph theory to diagram relational bundles? For ease and because such systems are not necessary. I'm of the opinion that having a variety of means by which to present and discuss a particular topic is preferable to having fewer. While category theory and graph theory can adequately represent relational structure with ternary or higher-arity relations they only do so in a somewhat complicated fashion. For example, a ternary relation on objects  $A, B, C$  is represented in category theory as the subobject of a ternary product  $R \rightarrow A \times B \times C$ .

I begin by diagramming an  $n$ -ary relation where  $n > 1$ . First, one represents each leg of a relation as a point connected by a curved line. A binary relation can then be diagrammed as a half-circle and a ternary relation as two half circles connected by the same middle point, etc. It is then possible to diagram legs that take other legs as their relatum by "connecting" legs to each other. Consider the following sample relational structure diagram:

Fig. 5<sup>80</sup>

The relational structure in figure three comes equipped with a set of relations  $R$ . For each relation  $r \in R : r \subseteq R$  and  $arity(r) = cardinality(r)$ .<sup>81</sup> An arity  $n$  relation is thereby compreant with  $n$  relation and can be individuated by the relation it is compreant with though, as I noted before, I do not think that such a relation is even necessary. It is included in order to provide a full range of the possible options available to the proponent of EOSR. For example,  $R_1$  is uniquely identified by the the set of relations  $\{R_2, R_3, R_4\}$ ,  $R_2$  by  $\{R_1, R_3\}$ , and so on. The relational structure,  $N$ , in figure three may then be defined as  $\{\{R_2, R_3, R_4\}, \{R_1, R_3\}, \{R_1, R_2\}, \{R_1, R_3\}, \{R_4\}, \{R_1, R_2, R_3, R_4, R_5\}\}$ .

<sup>79</sup> See Taylor p. 123.

<sup>80</sup> To simplify the diagram, the relation of compreance is left out. The unary relation  $R_5$  is merely included to illustrate a relation structure that is composed, in part, by a unary relation. A relational structure does not need to have any unary relations composing it.

<sup>81</sup> An ordered  $n$ -ary relation  $r$  can be identified as follows:  $r \subseteq R^n$ .

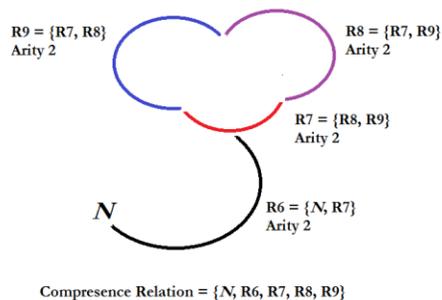


Fig. 6<sup>82</sup>

Per **RS**, relational structure  $N$  may stand in relation to some other relation. Figure four diagrams just such a case where, within a relational structure  $M$ , relational structure  $N$  stands in a relation to binary relation  $R_7$ . Note also that  $R_6$  takes the entire relation  $R_7$  as a relatum.

Diagrammatically, directed relations can be acquired by using a one-directional arrow to link legs. Consider the following diagrammatic example:

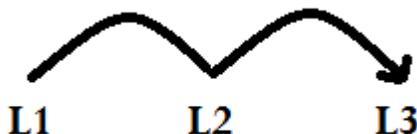


Fig. 7

Here each leg is denoted by an 'L'. Linguistically, a directed relation can be denoted using the following notation: ' $R_1 > R_2$ ' where '>' denotes a unidirectional relation between legs and/or relational bundles instead of the notation ' $R_1, R_2$ '.



Fig. 8

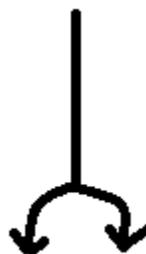


Fig. 9

Figure 8 displays a many-to-one relation and Figure 9 demonstrates a one-to-many relation.

I note that this formulation of relational bundles is neither a category nor a graph insofar as such bundles fail to satisfy the axiomatic constraints articulated earlier. That is to say, the relations need not be pairwise, not identity morphism is requisite on every non-identity morphism, the relations does not need to be transitivity,

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<sup>82</sup> Again, to simplify the diagram, the relation of compreance is left out.

etc. I stress that this is a new mathematical theory insofar as it neither a category nor a graph nor does it even need to rely on set theory (as I will show immediately below).<sup>83</sup>

The view here is grounded in set theory (which is object based) as a way to easily present the aforementioned method, though this need not be the sole way to present the view. First, we might utilize category theory as our underlying mathematical theory (as I have noted it might very well be that set theory reduces to category theory in the first place).<sup>84</sup> Second, one way to get around the set theoretic presentation is to replace all curly brackets with something else. In this way, no set-theoretic commitment through language is required. If it were, then we'd have good reason to think that set theory is in fact indispensable to this project and that objects are indeed presupposed by it contradicting the main aim of the paper. One way to do so is to utilize a relation of compressence and another without such that, for example,  $\{\{R_2, R_3, R_4\}, \{R_1, R_3\}, \{R_1, R_2\}, \{R_1, R_5\}, \{R_4\}, \{R_1, R_2, R_3, R_4, R_5\}\}$  in Fig. 5 may be presented as  $-R_2, R_3, R_4; R_1, R_3; R_1, R_2; R_1, R_5; R_4; R_1, R_2, R_3, R_4, R_5$  where '-' denotes a relation of compressence. Alternatively,  $R_2, R_3, R_4; R_1, R_3; R_1, R_2; R_1, R_5; R_4; R_1, R_2, R_3, R_4, R_5$  will suffice (wherein there is no relation of compressence at all). The former formulation, however, loses the credit of ontological parsimony against platonist conceptions of universals/properties (insofar as it requires a distinction between two kinds of relations) yet nevertheless remains equally parsimonious. Here, relational bundle theory employs the notion of a *collection* or *container* which is a more expansive concept than the more restrictive notion of a *set*.<sup>85</sup> In any event, I leave it to greater minds than I to more carefully develop the presentation of relational bundles elsewhere.

## 5.0 Conclusion

The proponent of EOSR may reply to the objection that relations require relata by taking the relata of relations to be relations themselves or relational structures. By endorsing a bundle theory of relations, advocates of EOSR may reject the thesis that relations supervene on intrinsic properties and take it that relations may come in relational bundles terminating regress chains in relational structures. The picture I sketch in sections 4.0 and 4.1 provides an alternative to category and graph-theoretic conceptions of structure and provide a way to represent relations and relational structures regardless of the arity.

Opposed to EOSR is the unholy alliance of physicalism, objecthood, contemporary Platonism, and the abstract-concrete distinction - four extremely entrenched positions found throughout the philosophy of science and the larger philosophical community as a whole.

Physicalism and objecthood seem to run up against the findings of contemporary physics. Either objects must be recast as entities that are not governed by the Principle of the Identity of Indiscernibles (which *prima facie* seems essential to the conception of an object or at least the conception of an object that is commonly presupposed throughout much of contemporary metaphysics) or we must dispense with objects entirely.

What I shall call contemporary Platonism requires a commitment to the abstract/concrete distinction which I have argued is unjustified or to a distinction between things that instantiate and things that are instantiated. On the former, loss of the abstract/concrete distinction may appear as a cost to many, but those are probably they who endorse contemporary platonism, as I have defined it, and therefore presuppose the truth of the abstract/concrete distinction in the first place. Commitment to the abstract/concrete distinction is also an additional theoretical commitment that is not present in nor required by a commitment to EOSR. By utilizing relational bundle theory, EOSR may legitimately shed the additional deadweight theoretical commitment that

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<sup>83</sup> The ordinals can be constructed in the following albeit somewhat inefficient way:  $R_a; R_b, R_c; R_d, R_e, R_f; \dots; R_1 > R_2 > R_3 > \dots > R_n$  where  $R_1 = R_a$  and  $R_2 = R_b, R_c$  and  $R_3 = R_d, R_e, R_f \dots$  (here, 1, 2, 3, ..., n are used for ease - we can very well replace those notational marks with something else aa, bb, cc, ..., n thus, this particular formulation of the ordinals is not parasitic on the number line).

<sup>84</sup> As noted in 3.1, though Landry argues that we shouldn't think in terms of foundational mathematical theories at all, an argument I find persuasive. See Landry 1999.

<sup>85</sup> The set  $\{2, 2\}$  is identical to the set  $\{2\}$  under the standard conception of a set. We might hold that those two sets are in fact not identical by employing an alternative conception of a set. In both cases we intuitively grasp that both conceptions of a set realize the more expansive notion of a collection or container - a thing in which other entities are contained in or clustered by. The mere fact that we can coherently consider alternative set theories also substantiates the claim made above.

there is a distinction between things instantiated and things that are instantiated.

I have also traded truth property monism for deflationism and have announced my sympathy for the view that set theory should be replaced by category theory as a foundation for mathematics though neither of these two positions are required in order to hold EOSR nor the relational bundle theory. For example, relational bundle theory may be taken as an independent mathematical theory itself and one may claim that it is improper to think of mathematics as making ontological posits in the first place.

EOSR may thereby come to address several major complaints raised against it by making recourse to a relational bundle theory and takes growing scientific consensus that the fundamental constituents of nature are not particles at face value. EOSR thus gains a leg up (no pun) against contemporary platonism and comes out ahead both in terms of theoretical parsimony and/or ontological parsimony against traditional objects and properties couched in terms of the abstract/concrete distinction all while earning the additional credit of being coherent with our best current scientific theories.

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