

10

$$\begin{array}{l} 10 = 5 + 5 \quad \checkmark \\ 10 = 0 + 10 \quad \checkmark \\ 10 = 10 + 0 \quad \checkmark \\ 10 = 4 + 6 \quad \checkmark \\ 10 = 6 + 4 \quad \checkmark \\ 10 = 7 + 3 \quad \checkmark \\ 10 = 3 + 7 \quad \checkmark \\ 10 = 11 - 1 \quad \checkmark \\ 10 = 20 - 10 \quad \checkmark \\ 10 = \end{array}$$

Heidi: "This is the work of one student. What I want you to do is to look at this work...and see what you notice. You're going to have to look at it really closely to see what you notice. I'm going to use the sticks [popsicle sticks with student names for random calling], so you don't have to raise your hand. Everyone should be looking, and everybody should be thinking."

"Just so you know, this is Alvin's work. See what you notice about his work...What do you notice?"

*Heidi waits for more than **two minutes** while students thought.*

Heidi: "Alicia, what did you notice?"

Alicia: "I noticed that you put checks."

Heidi: "No, *I* didn't put checks! Why do you think *he* put checks, Alicia?"

Alicia: "To make sure he did it correct."

Heidi: "You think he double-checked them to make sure he thought they were right? Do you want to tell us why you put checks, Alvin?"

Alvin: "I checked them."

Heidi: "So you went back and checked them? Malcolm, what do you notice?"

Malcolm: "That he didn't make them too messy."

Heidi: "OK, I want you to think about the math. What did you notice about the math? You're right; he kind of organized his paper."

(Wait time 30 seconds)

Malcolm: "Um. He um. He changed the numbers um, like he changed it left to right and right to left."

Heidi: "What do you mean by that?"

Malcolm: "Like $7 + 3$ and $3 + 7$."

Heidi: "I'm going to ask someone if they can repeat what you said. Octavia, can you repeat what Malcolm just said?"

Octavia: (silent)

Heidi: "Malcolm, say it again, 'cause what you said is *really* important. Everybody listen because I think Malcolm is peeking into Alvin's brain, and he's making a guess about what was going on in Alvin's brain, and Alvin will tell us if Malcolm was right." Malcolm: "That he changed the numbers right to left and left to right."

Heidi: "So give us an example."

Malcolm: "So, see like $7 + 3$ and $3 + 7$."

Heidi: " $7 + 3$ and then $3 + 7$. So you're saying that this was left to right and then here he did it right to left?"

Malcolm: "Yes."

Heidi: "So he turned it around. Alvin, were you thinking about numbers like that when you did this? Were you thinking about putting them together? Putting the ones that go together together?"

Alvin: (shrugs)

Heidi: "No? When you wrote $7 + 3$, did you then think, 'Oh, I know, if $7 + 3$ equals 10 then $3 + 7$ equals 10?'"

Alvin: "I already knew it."

Heidi: "You already knew it, so it's kind of a habit? Does anybody know a name for this? Mabel?"

Mabel: "Turn-around facts."

Heidi: "These are called turn-around facts. Can I write that here, Alvin?"

Alvin: "Yeah."

Heidi: "Does anybody see another turn-around fact?" (8 seconds wait time.) "Jaleesa?"

Jaleesa: "10, 0, 10, and then 0, 10, 10."

Heidi: "Is this what you're looking at? $10 = 0 + 10$ and $10 = 10 + 0$? Is that what you said?"

Jaleesa: "Uh huh."

Heidi: "Does anybody see another turn-around fact? Naomi?"

Naomi: " $4 + 6$ and $6 + 4$."

Heidi: "Mm hmm. Alvin was being really organized when he did this! Does anybody notice anything else about Alvin's work? I'm going to take two more observations. Deandre? What do you notice, Deandre?"

(28 seconds wait time.)

Heidi: "Can I say one thing that I noticed? Alvin, tell me if you think I'm right. I thought probably $5 + 5$ was something you knew pretty quickly, right? And then he did $10 + 0$ and $0 + 10$ and you probably knew those in a snap, right? And then he did $4 + 6$ and $7 + 3$, those turn-around facts—I bet he knows those pretty well. And *then*, I thought, he'd try to do some harder ones. I thought, oh, he decided he'd do some subtraction, which is harder. And so, after he wrote a bunch that he knew, he decided to *challenge* himself and put some equations that were harder to figure out. Did I get it right, Alvin?"

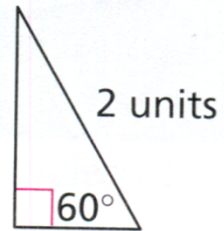
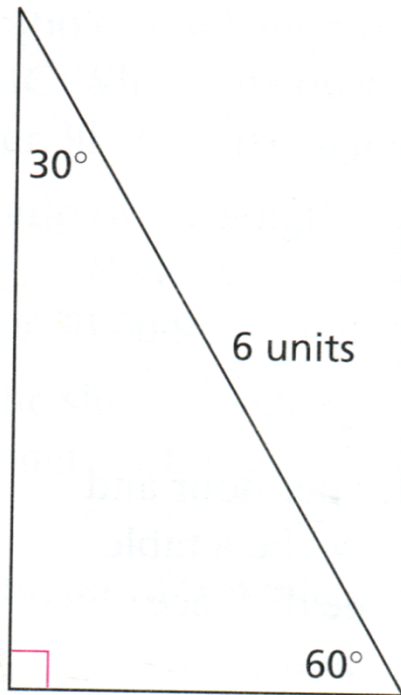
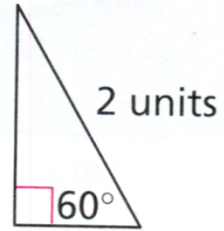
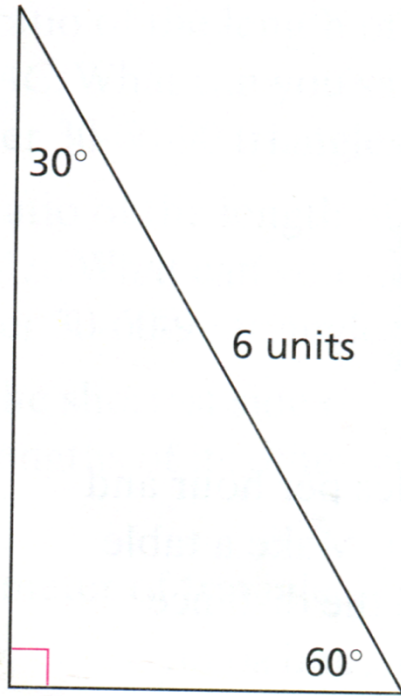
Alvin: "Yeah."

Written comments for students who are playing it safe:

- *It's time to bump it up a notch.*
- *This is a lot of problems, but is it challenging to you?*
- *I bet you can do much more complicated problems now!*
- *Can you challenge yourself even more? Try starting with really big numbers and subtracting to reach the number.*
- *I see you know turn-around facts! For a challenge, try starting at 50 and subtracting back to the number of the day.*
- *Can you use larger numbers? This is a wonderful start!*
- *Is $+20 - 20$ challenging to you? I am wondering if you could add steps that challenge your brain where you have to think about the numbers...*
- *You have a lot of good ideas and I know you could use a bigger challenge.*
- *I know you like a good challenge, so try starting with a number that doesn't end with 0.*
- *You got this pattern down! Try some things out of your comfort zone.*
- *I can see that you really like playing around with numbers! For your challenge, I would like you to try getting up to those high numbers and then get back to the number of the day without using \times zero. Good luck!*

Written responses to Students Taking Risks:

- *I see you are really bumping it up today! Woohoo!*
- *Wow! This is so much more challenging than your previous entries! I had to get out paper to keep track of your numbers! Keep it up!*
- *I see that you have really taken on the challenge and are trying harder problems! Good for you!*
- *Way to push your thinking!*
- *I circled the 2 that were the most challenging.*
- *I see you really like to challenge yourself!*
- *I see you are stretching your brain a little! Do you know any multiplication you can use?*
- *I think you might have meant 10,000. Nice job trying something really difficult! It looks like you could use a challenge!*
- *You are doing such an amazing job challenging yourself with these math problems.*
- *You're really on the right track of trying some harder problems!*
- *This is more like it! Remember that you are a second grader who likes a challenge!*
- *You have really stepped it up a notch!*
- *Good try! You are trying such challenging numbers. I think they are hard to keep track of. What can you do to help keep track of your totals?*



Clip 1:

Melissa: "You have a very, very, very, very, very, very small probability of winning."

Ann: "Absolutely. Anything else?"

Josh: "Overall, for all of the prizes you have about a 3% chance of winning so if you play 33 times you're probably going to win the small prizes if you play a few times. If you play 33 times you're going to get something."

Student: "Well, not necessarily..."

Student: "That's a theoretical probability."

Ann: "I like that conversation. I like that conversation. "

(A few additional thoughts on that idea.)

Student: "If everyone in the world played that way, what's it called, Powerball, that like just under 40 people would win."

Ann: "OK."

Student: "Jackpot."

Ann: "Assuming a world population of..."

Student: "Seven billion. Million. Yeah, billion."

(A few additional thoughts on that idea.)

Ann: "Ethan. This is cool."

Ethan: "There is a better chance of winning \$1,000,000 than winning \$10,000."

Ann: "Huh! Look at your numbers!"

Students: "What?" "Huh?" "Whoa!"

Ann: "What's going on there?"

Students: "Because the Powerball throws everything off!" "Stupid Powerball."

Ann: "Could that be right? I mean, why would they have done the prizes that way then?"

(Multiple student comments)

Ann: "Does anybody think that maybe the statisticians who figured it out for the state of New Hampshire might have done a better job than we did? [raises hand]. The people who figure out statistics for a living might have done a better job than we did? Maybe we made a mistake!"

Student: "Yeah."

Ann: "Maybe we made a mistake! Right, that doesn't seem to make sense that they would give a higher dollar amount for a more likely thing. Right?"

Clip 2:

Ann: "I'm not asking these questions because I want to say things to you. I'm asking these questions because I really have them. I have no idea what we were doing wrong yesterday. I don't get it yet! I haven't figured it out. I thought that solving an easier problem would help me figure it out. So that's what I created for you last night."

Clip 3:

Ann: "I created a problem that I thought would spark a discussion about something that I thought might help us figure out what's going on with the real Powerball problem. OK? Because how we're attacking the Powerball problem is different than the answer key I found online."

Students: "What?"

Ann: "Yeah, for figuring out the Powerball problem. I looked it up online. What do you guys do when you research stuff? You look it up online! No, I found a source that I trusted. I evaluated my source, all those things. But, it goes through and shows me how this math person solved the problem, right? This mathematician--I don't know if they're hired; if that's their job, right? But anybody who does good math, mathematicians--did this problem. So now I'm looking as another mathematician and you're looking as other mathematicians and coming up with a different way to solve the problem. We should get the same answer. But we didn't. So something must be up! And I've decided to doubt my own answer rather than their answer. I could have decided to doubt theirs, and try and figure out the mistake in theirs. Instead I've decided to doubt my own and try and figure out the mistake in my own. We might go through this and find that we didn't make a mistake at all; the one posted online made the mistake. But I think by examining ours and understanding ours, we'll come to determine which of those is correct, OK, and why.

Clip 4:

Ann: "What are we doing now?"

Class: "Thinking."

Ann: "Thinking!"

Josh: "At least some of us."

Ann: "All of you are. Every single one of you in here are. You totally were thinking, Josh. You have a formula up there that you figured out based on what you were doing. That is thinking, right? So that's what we're going to do. And when we figure it out, then we'll do our test. And you'll know more than you knew yesterday and today."

Liz: "OK, so I've been sitting here doing some thinking. I don't think it's right but I want to put it up just so people can say stuff about it."

Ann: "Please do it, please do it."

Liz: "I don't think this makes much sense at all, but it made sense when I did it."

Ann: "Good! OK. Let's talk about it."