

# Macro Class Notes - EC4010

Rory McStay  
14323268  
mcstayr@tcd.ie

May 1, 2018

## Contents

<b>1</b>	<b>Consumption and Labour</b>	<b>4</b>
1.1	A Static Model . . . . .	4
1.1.1	Income and Substitution Effects . . . . .	4
1.1.2	First Order Conditions . . . . .	5
1.2	Extending to an Intertemporal Model . . . . .	6
1.2.1	Permanent Income Hypothesis . . . . .	7
1.2.2	Euler's Equation . . . . .	8
1.3	Uncertainty . . . . .	9
1.3.1	Precautionary Savings . . . . .	9
1.4	Taxation and Labour Supply . . . . .	10
1.4.1	Brief Review on Labour Supply and Taxation . . . . .	13
1.4.2	Optimal Taxation Policy . . . . .	14
<b>2</b>	<b>Stimulus</b>	<b>14</b>
2.1	Ricardian Equivalence . . . . .	15
2.2	Empirical Testing . . . . .	16
2.3	Tax Smoothing . . . . .	17
<b>3</b>	<b>The Risk Free Rate of Interest</b>	<b>17</b>
3.0.1	Sensitivity to the Risk Free Rate . . . . .	20
3.1	Long Run Interest Rate . . . . .	21
3.1.1	Quantitative Easing . . . . .	21
3.1.2	Liquidity Theory of Long Run Rates . . . . .	22
3.2	TIPS . . . . .	22
<b>4</b>	<b>Monetary Policy</b>	<b>22</b>
4.0.1	Sticky Prices . . . . .	23
4.1	Money Non-Neutrality . . . . .	23
4.1.1	State of the Economy and the Rates . . . . .	24
4.2	Expansionary Channels . . . . .	24

4.3	Government Budget, Inflation and Rates . . . . .	25
4.3.1	Government Budget . . . . .	26
4.4	Issues of Monetary Policy . . . . .	26
4.4.1	Inflation Targets . . . . .	26
4.4.2	Monetary Base . . . . .	26
4.4.3	Pre-empting and Inflationary Environment . . . . .	27
4.4.4	Zero Lower Bound on the Nominal Interest Rate . . . . .	27
<b>5</b>	<b>New Keynesian Model</b>	<b>28</b>
5.0.1	Consumption and Savings . . . . .	29
5.0.2	Firms Pricing Behaviour . . . . .	30
5.0.3	The New Keynes Philips Curve . . . . .	30
5.0.4	Money Market Equilibrium . . . . .	33
5.1	The Three Equation Model . . . . .	33
<b>6</b>	<b>Real Business Cycle Theory</b>	<b>34</b>
6.1	Dynamics . . . . .	34
6.2	The Model . . . . .	36
6.2.1	Consumption and Labour Supply . . . . .	37
6.3	Exogenous Shocks . . . . .	38
6.4	The Solow Residual . . . . .	38
6.4.1	Neoclassical Models and the Great Depression . . . . .	39
6.5	Dynamic Inconsistency . . . . .	40
6.5.1	Capital Taxation . . . . .	40
6.5.2	Monetary Policy . . . . .	40
<b>7</b>	<b>Ramsey Model</b>	<b>41</b>
7.1	Government Expenditure . . . . .	43
7.2	Capital Taxation . . . . .	44
7.3	Implications of the Ramsey Growth Model . . . . .	44
7.3.1	Dynamics of Consumption and Capital . . . . .	45
7.4	The AK Model . . . . .	45
<b>8</b>	<b>Growth and Development</b>	<b>45</b>
8.1	Endogenous Growth Model . . . . .	45
8.2	Malthusian Model . . . . .	46
8.3	Kremer Model . . . . .	47
8.4	Geographical Features and Growth Implications . . . . .	47
<b>9</b>	<b>Issues Related to the Labour Market</b>	<b>48</b>
9.1	Search theory and the Beveridge Curve . . . . .	48
9.1.1	Recent Movements in the Beveridge Curve . . . . .	49
9.2	Stagnant Real Compensation in Lower Skill Labour markets . . . . .	50
9.2.1	Evolution of Household Income . . . . .	50

<b>A</b>	<b>Appendix</b>	<b>52</b>
A.1	Tutorial 1 . . . . .	52
A.2	Tutorial 2 . . . . .	53
A.3	Tutorial 3 . . . . .	55
A.4	Tutorial 4 . . . . .	56
A.5	Tutorial 5: Exam Corrections . . . . .	58
	A.5.1 MCQs . . . . .	59
A.6	Tutorial 6 . . . . .	60
A.7	Tutorial 7 . . . . .	61

# 1 Consumption and Labour

We begin our macroeconomic analysis through modelling an economic agent through consumption and labour. An agent receives an income in a period from supplying labour and uses this income for his own consumption. All payments happen at the beginning of each period. First we examine a static model and extend it to incorporate multiple time periods and uncertainty.

## 1.1 A Static Model

Economic agents face a trade off between working and leisure. We begin our analysis of this with a static model where by we seek to maximise some utility function  $u(c) - v(l)$ .  $v$  is said to measure disutility of labour. The utility function is concave down to ensure diminishing marginal returns to consumption. The dis utility of labour is concave up, so that additional labour input has a greater and greater cost. Intuitively, this is why overtime is paid at a premium.

$$\max_{c,l} \{u(c) - u(l)\} \quad (1)$$

subject to  $c = wl + d$ .

The typical functional form of equation (1) is as follows

$$u(c) - u(l) = \frac{c^{1-\theta}}{1-\theta} - \gamma \frac{l^{1+\sigma}}{1+\sigma} \quad (2)$$

In this way, we can capture the level of convexity of  $u$  and  $v$  by varying the level of  $\theta$  and  $\sigma$  respectively. Relatively high levels of  $\theta$ , increase the convexity, meaning that an economic agents marginal utility of consumption decreases at a faster rate. They become **satiated more rapidly**. As consumption is derived from labour input. A high value for  $\theta$  implies a lesser reward from supplying a given level of labour.

A higher value for  $\sigma$  implies that an agent's marginal cost to supplying one more unit of labour will increase at a faster rate. The more work they do, the marginal increase in dis utility increases. Intuitively we can interpret this as that agent **finds work more difficult**. The agent's dislike of work is modelled by  $\gamma$ . In this way we can model an individual who finds work easy with a low  $\sigma$  but dislikes work with a high  $\gamma$ .

The assumptions imposed on  $u$  and  $v$  can be expressed more formally as

$$\underbrace{u'' < 0}_{\text{Concave}}, \underbrace{v'' > 0}_{\text{Convex}} \quad (3)$$

Non labour income can be included in the model as  $d$ .

### 1.1.1 Income and Substitution Effects

**Example 1.1.** Consider the following formulation of an economic agent's utility from consuming  $\omega l$  and working  $l$ .

$$\max_l \left\{ \frac{\omega l^{1-\theta}}{1-\theta} - \frac{1}{5} l^2 \right\} \quad (4)$$

The first order conditions as follows

$$\frac{\omega}{\omega l^\theta} = l \quad (5)$$

$$\Rightarrow l = \omega^{\frac{1-\theta}{1+\theta}} \quad (6)$$

From equation 6, we can examine two different types of effects. The substitution and income effect. When an agent supplies  $l$ . They receive  $\omega$ . An agent can choose to work more if the wage increases as they will desire a larger income. We call this the substitution effect. Then there is the income effect, if the wage increases the agents income increases. As such, they will choose to work less.

- $\theta = 1$ : The substitution effect cancels out the income effect and  $l = 1$  as the amount of labour supplied is independent of the wage set.
- $\theta > 1$ : The income effect will dominate as the labour supply will decrease following an increase in the wage.
- $\theta < 1$ : The substitution effect will dominate as the labour supplied will increase following an increase in the wage.

There is empirical evidence to suggest that between 1750 - 2000 the income effect dominated. This is also true in the cross sectional variation dimension. Since the 1960s, labour supply has remained relatively stable.

#### Response to a changes in the wage

- **Substitution Effect:** Where an individual increases labour supply following an increase in the wage. Leisure is more expensive and the reward to work is higher making them work more.
- **Income Effect:** Where an individual decreases their labour supply following an increase in the wage. They feel richer and do not feel the need to work as much.

There are associated empirical issues with testing the relation between labour hours and the wage level. If we were to specify labour hours as  $l = \alpha + \beta\omega$  on a cross section, we would experience omitted variable bias due to heterogeneity issues with our specification. Different jobs give different levels of fulfilment or differing levels of human capital accumulation.

### 1.1.2 First Order Conditions

The maximisation condition for the functional form in (1) can be shown simply to be

$$u'(c) \frac{dc}{dl} - v'(l) = 0 \quad (7)$$

$$\Rightarrow \underbrace{u'(c)\omega}_{\text{Marginal Benefit}} = \underbrace{v'(l)}_{\text{Marginal Cost}} \quad (8)$$

The amount of labour supplied by an economic agent is up until a point where the cost on utility of supplying one more unit of labour is equal to the benefit of the additional

consumption. We can examine the **extensive margin** of whether or not someone will decide to join the labour force. If  $\omega u'(c) < v'(0)$ , that agent will not join the workforce. This implies that the cost of joining is greater than the benefit of additional utility.

As a result of people joining the workforce (extensive margin), it may appear that the substitution effect dominates.

#### Example

Embedded within this is the income and substitution effects. Suppose that you are offered overtime for one night so that  $\omega \uparrow$ . Where as a result of this  $u'$  will fall infinitesimally small so the right hand side must increase and as such  $l$  must increase. This suggests that for a temporary increase in wages, the labour supply will increase. With a permanent change in income,  $u'$  will decrease. As a result the change in the labour supply is uncertain. However as previously mentioned, the data says that a permanent increase in wages results in a decrease in the labour supply.

## 1.2 Extending to an Intertemporal Model

We will arrive at our first major implication of adding additional time periods in which an agent will consume through the following example of a simple two period case. This model includes two of the basic parameters  $\beta$  and  $r$ , denoting rate of time preference, how much someone discounts future utility and the rate of interest respectively.

**Example 1.2.** Assume the rate of time preference,  $\beta = 1$  and a rate of interest  $r = 0$ . Noting that  $u'' > 0$ , maximise

$$U(c_1, c_2) = u(c_1) + \beta u(c_2)$$

Subject to the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

By using Lagrangian multipliers, one can arrive at the F.O.C

$$\mathcal{L} = U(c_1, c_2) - \lambda[c_1 + c_2 - y_1 - y_2] \tag{9}$$

$$\Rightarrow c_1 = c_2 \tag{10}$$

From the first order condition of example 1.2. We can see that faced with two periods to consume and a lifetime budget constraint of income in two periods. An agent will smooth consumption over time periods. The consumption in period one or two is independent of income in respective periods. This result can be generalised to an economic agent facing an infinitely lived amount of periods in which to consume.

However if we examine empirically what happens over a 12 month period, one will find that December has a higher level of consumption in say January. We can model this behaviour rationally by specifying the utility functions in each period.

### 1.2.1 Permanent Income Hypothesis

We can show that consumption in any period  $T = \{1, 2, \dots, n\}$  is

$$c_1 = \frac{\sum_{t \in T} y_t}{n} = \dots = c_n \quad (11)$$

$$\Rightarrow \frac{dc_i}{dy_j} = \frac{1}{n} \forall i, j \in T \quad (12)$$

This implies that an increase in  $y$  today will only increase consumption by a fraction  $1/n$ . Considering conventional Keynesian school of thought, a major implication was that  $c = \alpha y$  where  $\alpha$  is the multiplier on an increase in incomes effect on the level of consumption. The implications of equation 12 can reduce the magnitude of  $\alpha$ .

By the Euler equation formulation, an individual smooths marginal utility over time.

This has implications that policies that increase the disposable income of agents would be more effective in countries such as Japan, as on average an individual is older than the average age of Ireland for example. Therefore agents in Japan have fewer periods to plan for and as such, the change in consumption will be smoothed over less time periods.<sup>1</sup>

The major implication of the PIH is that consumption should not react to anticipated income changes.<sup>2</sup>

Suppose we were to receive an unexpected, large one off payment (the lotto). The discounted value of lifetime income now changes and the agent will increase consumption in every period. This could only be buy a small amount in the instance of a one off payment.

Suppose that instead the agent experienced a **persistent** change in income. Then the lifetime value of income increases so the agent will increase consumption today and in all periods. If all future income streams double, then consumption will double.<sup>3</sup>

The theory is consistent with what is found empirically in the case of a persistent increase in income. Where as in the case of say a Christmas bonus, it is not.

In order to arrive at the permanent income hypothesis, it is necessary to assume that agents have complete availability of credit. The excess variation found in the data could be explained by liquidity constraints present in reality. This is rationalised in the following example.

**Example 1.3.** Suppose an economic agent is faced with two periods and has a liquidity constraint. Suppose  $y_1 = 0$  and  $y_2 = 20$ . The simplest case would predict that the agent would consume 10 in each period. However the agent faces a liquidity constraint and cannot borrow from future earnings if his income is zero. Therefore, if his income in period one increase by 10, his consumption will increase by 10 and not 15 if there was not liquidity constraint.

---

<sup>1</sup>See Souleles (1999) where they find significant response of consumption to predictable income tax refunds.

<sup>2</sup>Chang-Tai Hsieh (2003) - Used Alaskan oil fund as a natural experiment to determine the effects of anticipated exogenous payments in the form of dividends to residents. - Found evidence in favour of the PIH. Consistent with many other studies of PIH when income change is large and regular.

<sup>3</sup>See Evan & Moore, a study on substance abuse related mortalities and an increase in income. This is evidence against the PIH - There is excess sensitivity of consumption on income

The PIH implies that changes in income have little effect on consumption. Consumption is determined by total lifetime income. The alternative case is that by taking income in the next period as uncertain, consumption will follow a random walk. Campbell and Mankiw test whether or not consumption is a random walk tested on aggregated data. They used a heterogeneity condition that a certain proportion of individuals will abide by the PIH and the others consumption will depend on their disposable income.

They rejected the null hypothesis that consumption is a random walk and conclude that the PIH is central to understanding consumption.

The strongest evidence for the PIH was found in an exploratory study of a *natural experiment* on the Alaskan Oil fund. Dividends are paid out to the population of Alaska who had registered for the payment. This was an anticipated and predictable once of payment. Evidence was found that Alaskan households smoothed their dividend end payments consistent with PIH.

### 1.2.2 Euler's Equation

We will now consider the intertemporal case without explicitly specifying  $\beta$  and  $r$  with the following example, in order to arrive at Euler's Equation.

**Example 1.4.** Suppose an economic agent is faced with two periods in which to consume with utility function  $U(c_1, c_2) = u(c_1) + \beta u(c_2)$  and faces the following budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \quad (13)$$

where  $\beta \neq 0, r > 0$  The F.O.C of this model yields Euler's equation

$$\underbrace{u'(c_1)}_{MC} = \underbrace{\beta(1+r)u'(c_2)}_{MR} \quad (14)$$

$$\Rightarrow \frac{u'(c_1)}{u'(c_2)} = \beta(1+r) \quad (15)$$

From this we can see how our relative consumption in periods 1 and 2 changes given the rate of interest and the rate of time preference. If  $\beta$  is high,  $u'(c_2)$  will be small. As  $u'' < 0$ ,  $c_2 > c_1$ . If  $\beta$  is low, we will consume a lot more today.

We may add to the Euler Equation by considering  $U(c_1, c_2) = E[u(c_1) + \beta u(c_2)]$ , so all the uncertainty is coming from future income. So all one could do is maximise their expected utility.

**Example 1.5.** Suppose the income stream is  $Y_1 = 10, EY_2 = 10, EY_3 = 10$ . And as such, the expected utility and consumption will also all be equal. We will assume that  $\beta(1+r) = 1$ . We will assume certainty equivalence. Then  $u'(c_1) = Eu'(c_2)$ . Suppose in period 2, we get a positive shock to income and  $y_2 \uparrow 50$ . Now at this point, the aim is to have  $u'(c_2) = Eu'(c_3)$ . Our lifetime income is now 60. So we should expect to see  $c_2 = Ec_3$ . So  $c_2 = 30 = Ec_3$

In a deterministic world, this adjustment would not occur. However, in the case of uncertainty, Euler's equation is more like a set of contingency plans.



To extend the examples made through out the text this far to include multiple periods. One can use either hyperbolic or exponential discounting. With hyperbolic discounting we can model a present bias through the functional form

$$U(c_1, \dots, c_n) = u(c_0) + 1/2(u(c_1) + \dots + u(c_n)) \quad (16)$$

In this way we can see how an individual perhaps maybe indifferent to different periods in the future, but weight the present much more than other periods. In this way, the economic agent would not be able to stick to there plan over successive periods.

We can see how once period one is over, agents greatly prefer the next period to following periods. However, before this the consumer is indifferent to between the second and third period. This is known as **dynamic inconsistency**. A consumer continually changes plans and consistently puts a greater weight on the present in their total utility.

### 1.3 Uncertainty

In all our previous examples, we have taken future periods of income as known. In reality it is not certain what future incomes will be. We will now assume that  $y_t$  is a random variable. How will uncertainty effect our results?

**Example 1.6.** Suppose that  $y_1 = 10$  but the next periods income  $y_2$  is a random variable which is either 0 or 20 in each period. The most intuitive thing to do is find the expected value of  $y_2$  and proceed as normal. Then one would expect  $c_1 = 10$ . What will happen in the case  $y_2 = 0$ ? There will be nothing to consume in the next period.

This introduces what is know as **certainty equivalence**. This is a natural starting point to consider precautionary savings. In the event of uncertainty, agents will reserve a portion of their income in case of unforeseen events. In the last example, we did not consider  $d$ , the non labour income. If this related to welfare payments, then we would expect to see better welfare schemes reduce precautionary savings.

By considering the expectation of future income at time  $t$  as  $\mathbb{E}_t[Y_t]$ , we can consider how changes to a representative agents expectations can alter the level of consumption today. By considering the

#### 1.3.1 Precautionary Savings

In the case of certainty equivelance and consumpotion smooting, we base our decisions purely on the mean/expected value. However, in reality if their is a possibility of having an income of zero we may decide to hold back some savings in

**Example 1.7.** Suppose in period 1, we will recieve either 30 or 10. In the next period we relieve 40 or 0. In this instance  $\mu_{1,2} = 20$ . This is known as a **mean preserving spread**.

As we previously discusses, the existence of liquitdity constraints adds sensitivity to the data. In the liquidity constrained world, we have a more keynesian type response as an individual may consume all of an income increase. In the data, we have the existence of bequests, what is left over after all consumption occurs. Detrmining the source (motivations)

of savings is very important to determining growth. We expect agents to spend all their income. Are savings intended for bequesting reasons or is it precautionary<sup>4</sup>?

One part of it is that they are completely accidental in the case of unexpected death. It also may be altruistic where by the welfare of your children would be accounted for in your own utility function. Alternatively it could be strategic motive to guarantee better care as one becomes older. Some believe that bequests are the main form of savings<sup>5</sup>.

One puzzle in the data is that annuities are not utilised more by old people. Annuities are where you give an insurance company your assets at death in return for an income. This would be desirable in the case the PIH holds. This points to altruism or a strong precautionary motive.

## 1.4 Taxation and Labour Supply

We will now investigate what effects taxation policy has on the labour supply of a representative agent.

### Example

Suppose we have a lump sum tax and the following formulation

$$u(c_1) - v(l_1) + \beta(u(c_2) - v(l_2)) \quad (17)$$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - T_1 - \frac{T_2}{1+r} \quad (18)$$

In the case of taxes, consumption will decrease. And as a result,  $u'(c)$  will increase and as wages have remained the same, we get a pure income effect. The individual will feel poorer so will choose to work more. The marginal reward to work and hence leisure remain the same.<sup>a</sup> To see this more clearly,  $u'(c)\omega = v'(l) \Rightarrow c \downarrow u'(c) \uparrow \Rightarrow v'(l) \uparrow \Rightarrow l \uparrow$  as  $\omega$  stays constant.

In the case of a proportionate taxation, the resultant change in labour supply will be ambiguous as there is both a substitution effect and income effect. Empirically however, the income effect tends to dominate. That is, following an increase in taxation, there tends to be an increase in labour supplied.

<sup>a</sup> Consider the paper by Imbens et al (2001), Holtz-Eakin et al (1993): Effects of Unearned Income. Survey on those who receive large bequests and lottery winners substantially reduce their labour supply.

<sup>4</sup>Summers/Bernheim/Shleifer (1987) compare number of visits to rich parents by family members with one child and those with more than one child. They find that those with more offspring receive more visits. Suggesting that bequests have a strategic motive.

<sup>5</sup>Kotlikoff and Summers (1981, 1987) find that bequests are a large part of wealth. Furthermore, Kotlikoff finds that intergenerational tax transfers are a large part of US aggregate capital formation.

## Prescott's Redistribution Story

Prescott has a paper asking *why do americans work so much more than europeans*. He put down the main findings to the differences in taxation. Considering the reward to working as  $\omega/p$ , the real wage. Then after taxes we have

$$\frac{(w - t)l}{(1 + \tau)p} \quad (19)$$

Both types of tax lower the spending power of the reward to work. As such, for this study the two types of taxes were aggregated. In essence, aggregated taxes are lower in the US and as such the real reward to work is higher and the substitution effect then dominates. Where as in the EU, real aggregate taxes are higher so there is not as big a substitution effect.

One question not adressed is what the government do with the tax revenue? This  $tw$  is sent back to the public or generally redistributed. Recall when the wage goes down there is a negative substitution effect. With the income effect, an individual feels poorer and as such will work more. However, in Prescott's study, the income effect was killed when the aspect of redistribution was considered. So we are left with an unambiguous decrease in the labour supply.

The results of this paper depend crucially on the cancellation of the income effect. Due to the inequality of redistribution where by poorer people get the bulk of it, it does not necessarily always hold.

Following on with the redistribution implications, if the top 1% of earners experience an increase in taxation there will only be a very small income effect due to the small proportion of the tax to their overall wealth. As a result, their labour supply will increase a small amount. Relative to the income effect of the high earners, our labour supply will go down a lot as the tax revenue is redistibtruted to everyone else and as such the aggregate labour supply goes down.

### Tax Bracket Example

Consider now there is differing tax brackets with the following example.

$A : (\leq 10000, 0\%), B : (\leq 20000, 10\%), C : (\leq 30000, 30\%), D : (\geq 30000, 50\%)$

- Suppose there is one individual at the cusp of the 20000 bracket. If the tax rate of the next bracket goes from 30 to 20 percent. There will be no income effect as they have not earned anymore. But there is a substitution effect as the person is incentivised to work more. The marginal rate of taxation falls for an individual in the instance.
- On the other side of the bracket, there is no substitution effect but there is an income effect incentivising the individual to work less. An individual on this side of the bracket will experience an increase in income.
- Overall, the change in labour supply is ambiguous. It depends on how much people are in each bracket.

If the rate for someone between B and C, the marginal rate would fall if they reduced the rate of C. We can consider the effects of this type of tax plan that clustering may occur at tax bands. In practice people do not have that flexibility to arrange their income in that way.

Suppose the highest rate would fall to zero. There would be a pure substitution effect for the individual in the highest band of tax. Their marginal rate of taxation falls to zero and could choose to work more. Suppose that the individual was the only person in that tax bracket. Then everyone would be better off. It would be a Pareto improvement.

As a result of imposing a distribution of tax bands, the true distribution of earnings in an economy can become kinked at the upper end of each band as their marginal rate of taxation will be much higher.

There has been extensive research as to the effects of taxation on labour supply. However, there is a lot of noise in the data which could be attributed to the following. In regards to the changes in labour supply following changes in taxation, prime aged workers are not affected on the intensive margin (already participating). This is likely due to inflexible work hours<sup>6</sup>. Furthermore, working may increase the human capital for an individual so they will accumulate more experience rather than respond with a decrease in labour supply.

- Tournaments: Presence of CEO's massive pay, incentivises people to work hard to maybe one day to become CEO.
- Top end exception: Empirically their taxable income is very sensitive to higher marginal

---

<sup>6</sup>Chetty et al; Adjustment costs, firm responses and micro vs macro labour supply elasticities: Evidence from Danish Tax records. to show that Danish unions accommodate workers' desire to work less in the face of higher taxes. This shows that many workers indeed want to work less but can't. A common trend is that long run elasticities are higher than short run.

rates. However, there is flexibility on the top end to chose the method in which to be compensated<sup>7</sup>.

- The effective marginal tax rate: As they work more they begin to lose benefits. Eg loses housing allowance if work too much. One way to go about this is a negative income tax.

### 1.4.1 Brief Review on Labour Supply and Taxation

Iceland's natural experiment refers to the year of 1987 where tax collection timing changed from paying in full at the end of the year to pay as you go, leaving a year where no tax was collected. Whilst in that year, individuals still had to pay last years tax. The short duration of the zero tax rate, therefore minimised the income effect whilst the substitution effect remained intact. Marco Bianchi studied this phenomenon in an attempt to find an upper bound as to the effects of tax rates on labour supply. He found that men increased their labour supply by 14.3 % (self-employed by 24.1%) and women by 10.3%. Bianchi considered the difference between men and women to be down to the home responsibilities of a women (self-employed female: -12.6%). Increases in labour supply occurred on the intensive and extensive margin.

### Common Empirical Issues in Labour Supply

If we use a regression to parse our the nature of

$$l = \alpha + \beta w \quad (20)$$

- Short run inelasticity of hours implies an underestimation of  $\beta$ .
- Unobserved heterogeneity. Some index of taste or ambition for work should be included. Taste for work will appear in the error term and is almost surely correlated with the wage.
- High hours lead to high earnings so in a progressive tax system will reduce the after tax wage. So there can be a reveres causation issue here.

The way we fix this regression is to find some exogenous variable. Such as a tax change. We can then do a difference in differences regression. This is analogous to the treatment effect. There are many famous examples<sup>89</sup>.

Nada Eissa conducted a study using difference in differences. This looked at the tax reform in the 1980s. The top rate of tax went down by 30%. This particular study looks at the behavioural response to wives of top income earners. As a result of this tax reform, the

---

<sup>7</sup>A natural experiment Kleven et al 2014: Taxation and international migration from the foreigner tax scheme in Denmark. Here the government introduced a 30% tax rate for 3 years to encourage high income earners come into Denmark. There was a large increase in immigration of such workers to Denmark.

<sup>8</sup>Oettinger (1999); An empirical analysis of the daily labour supply of baseball vendors. Vendors work more on days of big baseball games on an intensive and extensive margin.

<sup>9</sup>A study of New York Taxi cab drivers on rainy days where the wage is higher found that they tended to work less hours.

marginal tax rate of the wife fell dramatically. The question asked was what the behavioural response to this change was. The 75th and 90th percentile were the controls, the HIGH was the group of interest.

The resultant change in labour supply was a 12.7% increase in High labour supply. and 3.6% and 6.8% respectively. This represents the intensive margin. The extensive margin we saw a 19.5% increase in people participating in the work force.

### 1.4.2 Optimal Taxation Policy

When we consider the optimal tax policy. We hope that the tax schedule is as least distortive as possible. The taxed world should look as similar as possible to an untaxed world. **Ramsey's Rule** suggests that this can be achieved by taxing inelastic goods. The tax rate should be inversely related with the elasticity. Alternatively the optimal policy may be found by using lump sum taxation as it does not effect prices. Consider a lump sum tax on land. It will not effect the motivations of owning or investing in the property as much as a value tax on property.

In light of this, there has been a strong fall in the marginal tax rate for corporations. This shows the increased mobility of corporations in the modern economy. Recall that Ramsey's Rule suggests to tax inelastic goods more than elastic goods. Akin to what is happening to corporation marginal rates.

There is evidence of increased income and lower tax rates for the very rich. Is this a result of rich people lobbying for lower tax rates or is it an increase in income as a result of lower tax rates? To determine this we look at a cross section. In contrary to the Ramsey rule, rich people have the most elastic labour supply and experience the highest marginal tax rates.

A study was conducted on the labour supply on taxi cab drivers during rainy days. Conflicting results, found that empirically labour supply goes down. However, a short term increase in the wage should absolutely increase labour supply by a dominant substitution effect.

Taxation through time should also be smoothed as to not distort peoples incentive to work in different time periods.

## 2 Stimulus

In this section we examine the effects of government spending on the levels of consumption, investment and labour supply. Government policy can be expansionary or contractionary. The former being increasing the deficit by increasing government expenditure and/or reducing taxes and vice versa. Consider a change in  $G \uparrow$  what happens to  $Y$ ? There are three main schools of thought.

### Keynes

The standard keynesian school of thought is that increasing government expenditure will increase output by a large amount. This works through the determination of output by aggregate demand. As such, we get a large *multiplier effect*.

- $Y = C + I + G \uparrow$ ,  $C + \alpha + \beta Y \Rightarrow Y \uparrow$ . A big effect

## Neoclassical

In neoclassical models later discussed, output is determined through supply and demand determined by a representative agent and firm maximising utility and profit. This encapsulates the PIH and Ricardian equivalence which results in a smaller effect of government expenditure.

- $Y = C + I + G \uparrow$  People internalise the increase in  $G$  and as a result know they will be taxed more. So to combat this they work more (Pure negative income effect), labour supply increases and as a result  $Y \uparrow$ . However  $C \downarrow$  due to saving for higher future taxes.

## Neoclassical with expectations

This extends the neoclassical theory to include fiscal expectations. The representative agents views on the future of government budgets can effect consumption today.

- $Y = C + I + G$ . You expect government spending to increase. As a result you know that your taxes will increase. You know you will be poorer in the future so will save for tomorrow's consumption.  $C \downarrow$  due to negative income effect. Thus  $Y \uparrow$

## 2.1 Ricardian Equivalence

Ricardian Equivalence suggests that if the government increases government spending to stimulate the economy (expansionary fiscal policy), as a result of forward looking consumers, they know they will be taxed more in the future so will save for this event. As a result there will be no change in demand/consumption. A government faces the following budget constraint

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} \quad (21)$$

A rational agent internalises the government budget constraint above. If government expenditure goes down in both periods today, the resultant tax bill is lower and consumption increases. This is known as an **expansionary fiscal contraction**. The consumer's budget constraint becomes

$$C_1 + \frac{C_2}{1+r} + G_1 + \frac{G_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad (22)$$

If total government expenditure falls, income held fixed, consumption must increase. As future taxes are generally uncertain, this may only work where individuals truly believe that the government will reduce its borrowing requirement in the future (**fiscal consolidation**). The strongest evidence of an expansionary fiscal contraction was from Denmark and Ireland in the 1980's. This is believed to work through the upwards altering of government budget surplus expectations in the period of consolidation which initiates a wealth effect.

However in light of this, if the fiscal consolidation does not seem credible then negative Keynesian effects may prevail. A fiscal consolidation reduces the risk premia of debt issuance

encouraging private investment, as well as increasing savings available, the real interest rate will fall encouraging consumption. If the former case prevails, then the private investment channel will not occur and agents will not expect lower government expenses and there will be no induced wealth effect.

Note that this extends to how a government finances its expenses. As a government ultimately finish with a zero net budget, if they finance expenses by borrowing, they are essentially borrowing on behalf of the people. This means people will foresee higher taxes in the future to repay the debt.

When looking to the data, the ricardian equivalence implication of the fact debt and tax finance are equivalent does not appear. This is due to overlapping generations. If debt is issued, an individual may not have to pay it back in their lifetime. However, this objection to ricardian equivalence can be mitigated by considering intergenerational links, causing a household of multi generations to act as an infinitely lived agent who will experience any burden of government far into the future.

Note that RE and PIH are closely linked, especially if we make consider smoothing after tax lifetime income and substitute expected government expenses in place of taxes.

## 2.2 Empirical Testing

Suppose we wanted to test the strength of a change in government expenditure we are often faced with a strong **Reverse Causal Bias**. Does a cut in spending lead to recession or does recession cause a cut in spending.

In the most simple case  $Y = \alpha + \beta G + \epsilon$ , we will have omitted variable bias and an endogeneity problem. We may find an instrumental variable in order to solve the endogeneity problem. Military expenses is often used for this. The GDP multiplier was found to be approx 0.55. This suggests that consumption will go down as a result of government expenditure. An important question; "By how much does GDP rise if the government buys more output?".

Ultimately the problem lies with the motivations for tax cuts. They could be in response to the current economic environment. This leads to the changes in output and economic activity to be down to another underlying reason. This makes analysing the effects of tax cuts difficult. By analysing tax cuts which were down to ideological reasons (to reduce an inherited budget deficit per say), we may analyse the effects of a tax cut which is independent of the state of the economy. In effect, exogenous.

- **Ramey Study:** If government expenditure is expected to increase. People absorb this into their own budget constraint, and as a result feel poorer. They as a result work more. As labour supply increases, wage decreases. This was studied through mentionings of war in the paper. If a country is expected to go to war, then future government spending will be high. It was found that increasing government expenditure had negative effects on consumption. The multiplier was no more than the direct effect.
- **Chodorlw Reich et al;** "Does state fiscal relief during recessions increase employment?" Reverse causality problem here, they could be getting money due to the state in the economy. Found states during the financial crisis, that recieved lots of money



from the state did better than others. These expenditures were exogenous as it was based on previously agreed deals. Employment benefits were also better in the states who received more money. This suggests a Keynesian response.

- **Romer/Romer:** A narrative study whereby documents recording the motives of tax changes were studied to find exogenous changes in tax. The study took the following specification  $\Delta Y = \alpha + \beta \Delta T + \epsilon$ , where  $\Delta T$  incorporated two components, exogenous and endogenous changes. The macroeconomic effect of tax changes. They examine what they call changes in taxation. In practice they examine changes in taxes which were not induced because of the state of the economy but down to ideological reasons (eg. Reagan tax cuts). Increases in taxation result in large negative effects on the economy. Furthermore, tax cuts intending to *starve the beast* reducing government expenses found to be unsuccessful
- **Shoag (2010):** By using state pension fund returns as an instrument for  $G$ . Namely, when state pensions were doing well they spent more. Crucially the performance of the pension funds was unrelated to the economic situation in the state economy. In a sense, therefore the rises in expenditure were made exogenous. Shoag found evidence of a Keynesian response to government expenditure.

## 2.3 Tax Smoothing

Suppose in year one there is a marginal tax rate of 50%, another year the rate is 0%. Firstly, in practice to minimise the pain to working we spread the work in both periods. This is the socially optimum response to working. In designing an optimum tax schedule, we hope to minimise distortion.

In this proposed tax schedule, people would opt to work in the second period. So to be a close to the optimal schedule is to have 25% in each period. Now suppose that the government needed to go to war. They could increase taxes to finance the war but as a result we would distort the true labour supply in either period. Ideally they should run up debt and raise taxes a small amount. Then the small increases in taxes overtime would pay off the debt.

By this rational, it can be efficient for a government to have a deficit. Furthermore, the increasing cost of taxation means running a deficit may be optimal as to not distort the motivations<sup>10</sup>.

## 3 The Risk Free Rate of Interest

We discussed before the risk free rate of interest in our simple intertemporal model in section 1. Consider the usual constraints. We seek to find  $s_1 = y_1 - c_1$ .

$$C_1 = \frac{1}{1 + \beta} \left( y_1 + \frac{y_2}{1 + r} \right) \quad (23)$$

---

<sup>10</sup>There are other reasons as to why there may be a deficit for example ideological or strategic reasons. These are not necessarily effective and have been researched extensively. There tends to empirically be a deficit bias.

As an aside, consider the effect of  $r$ . An increase in  $r$  will decrease the consumption today. The present value of all your labour earnings go down and as a result reduces consumption. That is independent of the income/substitution effect. Savings then becomes

$$s_1 = y_1 - \frac{1}{1+\beta} \left( y_1 + \frac{y_2}{1+r} \right) \quad (24)$$

The way this is set up, an increase in the interest rate would raise the level of savings. In the context of thi representative agent model, it is difficult to consider the savings as there is only one agent. So we will introduce heterogeneity (a second agent).

We will introduce our second agen to have  $y_1 = 0, \beta = 1$ . As a result of this they will be disaving in period 1.  $y_2$  is the income of the borrower in period 2. So savings for agent A in period 1 is

$$s_1^A = -\frac{1}{1+\beta} \frac{y_2}{1+r} \quad (25)$$

$$s_1^B = y_1 \left( 1 - \frac{1}{1+\beta} \right) \quad (26)$$

In this simple capital market, the sum of all savings will be equal to zero. In this market, the interest rate will adjust for this condition to hold. As a result,  $r$  becomes

$$r = \frac{y_2}{y_1} - 1 \quad (27)$$

If  $y_2$  is very large, agent A will want to do alot of borrowing so the interest rate will be high. This also suggests that low interest rates give a bad prognosis of the state of the econonmy. In this example, we did not allow for any production.

We now present an extension to this model through production. Assume their is a firm with a concave production function.

$$AF(K) = AK^\alpha \quad (28)$$

Firms invest in period 1 and produce in period 2. The firm borrows  $I$  in period one to finance investment, which will yield profits next period. They must pay back  $(1+r)K$ . The firm starts off with no capital in period 1 and sells off all the capital in period 2. The present dicounted value of capital is

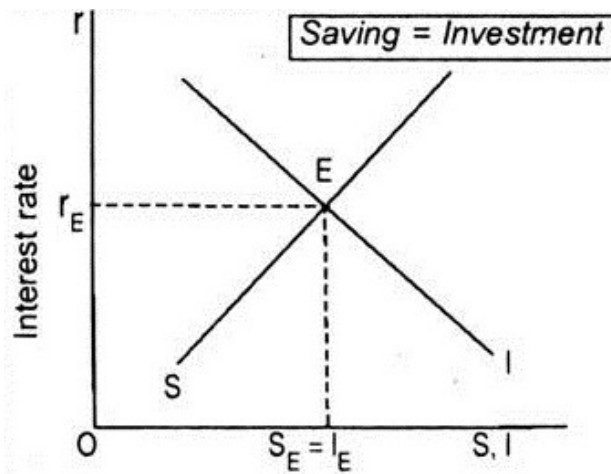
$$\frac{AF(k) - (1+r)K + K}{1+r} \quad (29)$$

The firm chooses  $K$  to maximise profits. Finding the first order conditions we can show

$$AF'(K) = r \quad (30)$$

The marginal productivity of capital. If  $r$  is high, then the marginal rate of capital is high. This implies that capital is scarce conditional on the assumptions  $AF' > 0, AF'' < 0$ . This means that high levels of interest rates, discourage investment.

From this we can derive two intersecting curves  $s(r), I(r)$  in  $(I, r)$  space. There is a negative slope on  $I$  and positive slope of  $s$ . The slope of the savings curve is very much dependent on  $\theta$ .



A high  $\theta$  results in a higher slope of  $s$ . We can then rationalise an increase in global savings as a shift in the  $s$  curve. Broadly speaking, savings consists of private government and international savings.

We may use this to examine what happens to interest rates following changes to the economy. The savings curve is derived from equation (27) and the investment from (30). Note that total savings includes public and private savings. Public savings depends on the budget deficit.

In the 80s, there were large budget deficits. This causes savings to fall. This would be an inward shift in the savings curve. Then  $r$  will increase. Then there is more capital inflows. If everyone is demanding dollars to save at high us interest rates, the dollar would appreciate. So the current account would worsen.

- Budget Deficit

- $\Rightarrow r \uparrow$

- More capital inflows

- $\$ \uparrow$

- $CA \downarrow$

This was very prominent in the 80s, since then there is a lot more going on so there is a lot more noise. Clinton decided that he needed to bring down the deficit.

Suppose  $\bar{y} = c(r) + I(r)$ . Before Keynes, we thought that if  $c$  falls, the interest rate would go down. The interest rate would fall sufficiently to stimulate more investment. Thus the economy was always be at potential. In this sort of world however, the great depression would be impossible. In the long run this is not a bad model.

In the Keynesian world, we denote  $S = sy$  as aggregate savings, people save a certain proportion of GDP. Keynes says, as a result of a fall in consumption, there is an increase in savings which would cause a recession. Then the level of savings  $s$  increases but  $y$  falls. Overall savings will return to where it was and we will remain in a recession. This is

known as the paradox of thrift. This rise in savings causes a recession. Where as in the other world a rise in savings caused an investment spree. The implication of this is that there is no natural adjustment mechanism to bring an economy out of a recession. If everyone increases their savings rate, consumption falls and people are saving a larger proportion of a smaller amount.

Where savings and investment intersect, we find the natural rate of interest. This corresponds to the natural rate of employment also. This rate is the rate at which the economy is at potential.

Given inflation, we can find the real nominal rate as  $i = \pi + r_n$ <sup>11</sup>.

### 3.0.1 Sensitivity to the Risk Free Rate

We may examine how sensitive an individual is to changes to the risk free rate depending on their level of risk aversion. Consider the representative agent living in  $T$  periods with a budget constraint. Consider the following lifetime utility

$$U = \sum_{t=1}^T \frac{1}{(1+\rho)^t} \frac{c_t^{1-\theta}}{1-\theta} \quad (31)$$

Subject to

$$\sum_{t=1}^T \frac{1}{1+r_t} c_t \leq A_0 + \sum_{t=1}^T \frac{1}{(1+r_t)^t} y_t \quad (32)$$

We may show that the Euler equation is of the following form

$$\frac{c_{t+1}}{c_t} = \left( \frac{1+r}{1+\rho} \right)^{\frac{1}{\theta}} \quad (33)$$

Now consider how an individuals consumption today will change following an increase in the interest rate. For  $\theta = 1$ , An increase in the interest rate, decreases consumption. The relation between relative consumption and the interest rate is one to one. For  $\theta > 1$ , the relative consumption between two periods also increases but by less than one to one. That is, for higher levels of  $\theta$ , an individual is less sensitive to the interest rate.

This analysis also portrays the relationship of the intertemporal elasticity of substitution (IES). When  $\theta$  is high, the IES is low (in fact the  $IES=1/\theta$ ). There is also an income and substitution effect with a change in the interest rate. The income effect acts to decrease savings following an increase in  $r$ , where as the substitution effect increases savings as a result of an increase in  $r$ . Saving becomes relatively more lucrative and the cost of consumption increases as that money could have been saved at higher rates. For the income effect, the individual feels richer following an increase in savings so has to save a smaller proportion of their wealth to maintain the same future consumption. This is easy to examine in the two period case through indifference curves. where the slope of the curve is  $-(1+r)$ . If the slope increases (in magnitude), period 1 consumption decreases.

---

<sup>11</sup>Prescription for Japanese economy - drive up expected inflation to make a negative natural interest rate.

### 3.1 Long Run Interest Rate

Consider two different ways to invest. Invest in a one year bond in each period for two years. Or invest in a two year bond. We only care about the monetary return. Then we can find the arbitrage condition

$$(1 + i_1)(1 + Ei_2) = (1 + y_2)(1 + y_2) \quad (34)$$

solving for  $y_2$  by taking logs and equating  $\log(1 + x) \approx x$

$$y_2 = \frac{i_1 + Ei_2}{2} \quad (35)$$

So how does monetary policy effect the long run rates of return. By communicating expected short term interest rates they can guide the long run rates (open mouth operations).

#### 3.1.1 Quantitative Easing

When a central bank buys long term bonds. Suppose the previous arbitrage condition holds. We will examine how QE destabilises this condition.

$$(1 + i_1)(1 + Ei_2) = (1 + y_2)(1 + y_2) \quad (36)$$

QE  $\Rightarrow$  price of two year bond goes up (QE means CB buying long run bonds). Given nothing else has changes,  $y_2$  goes down. The condition becomes

$$(1 + i_1)(1 + Ei_2) > (1 + y_2)(1 + y_2) \quad (37)$$

Holders of the two year bonds will now sell them to avail of the higher returns of the one year bond. Thus, the price will fall and we return to the equality condition. This private sector response means that QE will not work in this world.

One school of thought is that the mechanical aspect of QE has no effect on the interest rates. Where it might effect rates is the long rates and the future trajectory of short rates.

What is crucial in this theory is that all bonds are perfect substitutes. There is nothing that makes an investor prefer a long run bond over a short run bond.

#### QE Explanation

Suppose we have 50% AAA bonds and 50% Treasury bonds. Now we replace 30% of the Treasury bonds with more AAA bonds. What happens to the yield? If bonds are perfect substitutes there will be no difference in yields.

Now imagine that the Treasury bond has other characteristics such as collateral or liquidity advantages. Then if we reduce the amount of this bond available, the price of the treasury bond will go up and resulting yields will decrease

By reducing the supply of a particular type of bond, under market segmentation theory, the price of that bond will increase, reducing the yields of those particular type of bond. In this way, QE can reduce the yields of certain segments of the market.

The previous example was a way in which we can think about how QE works. In essence, QE works by purchasing treasury bonds, which reduces the yields and as such the interest rate.

There is also concern that this methodology is counter productive because of the alternative uses to holding treasury bonds.

Next we can consider a risk story in how QE works. Suppose we have many risky bonds in the private sector. If the Central Bank now buys these risky bonds, they are removing the risk from the market. This then allows the market to buy more risk. This results in a fall in the yields of risky bonds. The aim of QE is to reduce the interest rate on all risky assets. This is known as **risk absorption**.

### 3.1.2 Liquidity Theory of Long Run Rates

Combine pure expectations theory and segmented market theories to get the liquidity premium

$$i_{nl} = \frac{\sum_{t=1}^n \mathbb{E}[r_t + \pi_t]}{n} + \rho_n \quad (38)$$

Where  $\rho$  refers to the term premium, which is related to supply and demand for bonds of maturity  $n$ .

## 3.2 TIPS

A TIP bond is a bond with no inflation risk. The treasury will compensate the investors for the level of inflation of that period. Suppose that  $r = 2\%$  is a guaranteed return of a TIP bond. We have then nominal rates  $i = r + \pi^e$  paid out of the tip. In the case where  $i = 6, r = 2 \Rightarrow \pi^e = 4$  The market determines the rate of return of a tip. So we can infer expected rates of inflation. However, there is an issue associated liquidity risk which is accounted for in the spread of a normal treasury and TIPS. The spread of a Treasury and TIPS is known as the break even inflation.

## 4 Monetary Policy

Recall the taylor rule

$$i^* = r_n + \pi + b(\pi - \pi^*) + h(y - y^*) \quad (39)$$

Inflation is the increase in the average price of goods and services in terms of money. It is standard to think that demand for nominal money is increasing in income and decreasing in the nominal rate of interest. that is  $L_i < 0, L_y > 0$ . The money market is then equilibrium at

$$\frac{M}{P} = L(i, y) \quad (40)$$

Where  $i$  is the nominal rate. Assuming perfectly flexible prices in the long run. It is not unreasonable to assume that inflation is caused only by increases in the money growth. As it is unlikely that there will be sustained large fall in output and the interest rate has shown

little variance. Empirically the income elasticity of money demand has been shown to be close to 1. If we consider a permanent increase in money growth, noting that  $P = M/L$ , this causes an increase in the price level. In the long run we assume that  $\bar{Y}, r_n$  are constant. Then the real interest rate does not change and the interest rate will rise. This is known as the **fisher effect**. Recall the fisher identity  $i = r + \pi_e$ . The nominal rate encapsulates expected inflation.

We have shown that in a long run setting, an increase in money growth increases nominal rates. However the immediate effect of money expansion is to lower short term nominal rates. This is known as the **liquidity effect**. If prices are not completely flexible, then an increase in the money growth will cause a fall in  $r_n$  and an increase in output. In the long run,  $r_n$  will return to its original level. Thus, if the real effect outweighs expected inflation effect, then nominal rates will fall in the short run and increase in the long run.

#### 4.0.1 Sticky Prices

Now consider  $r_n$  as the natural interest rate, and  $\pi^*$  is target inflation. In the theory, we suppose that the CB sets  $i^*$  in accordance to the Taylor rule. When an economy is at potential and inflation is at target, then the rate of interest will equal the natural rate. Now recall the condition of money demand

$$\frac{M^d}{P} = L(i, y) \Leftrightarrow M^D = PL(i, y) \quad (41)$$

Noting that  $i = r + \pi^e$ . If the money supply is  $\bar{M}$ , which is exogenously determined.

$$\frac{\bar{M}}{P} = L(r + \pi^e, y) \quad (42)$$

If  $\bar{M}$  doubles. Instantaneously, the price level will double in a world with frictionless prices. However, if we did not have frictionless prices, then  $r$  would have to go down and  $y$  would increase in order for the money market to be in equilibrium. Now we have a model where monetary policy/cash has real effects and is **non-neutral**.

In the long run model, we consider the natural level of output and rate of interest  $\bar{Y}, r_n$  as constant.

### 4.1 Money Non-Neutrality

There is a lot of empirical evidence ,for money-output relationship this suggests causation from  $M$  to  $y$ .

St Louis regressions took the form

$$\Delta Y = \alpha\beta\Delta M + \epsilon \quad (43)$$

This formulation has endogeneity issues due to the nature of the control of money supply. We need to examine increases in money supply that are independent of the state of the economy. There is one case where the FED chair died. He was very contractual (like a German) and replaced with some one much more expansionary (like an Italian).

A more prominent example of how this was investigated with when countries moved from the gold standard<sup>1213</sup>. They were essentially gaining the ability to use monetary policy. During the great depression, many countries left the gold standard and saw increases in output. This was the treatment effect in the analysis.

#### 4.1.1 State of the Economy and the Rates

Fed fund rate was too low for too long coming up to the financial crisis. Meant people sought higher yields/risky stuff. However there was also a savings glut coming into the US from Asia. As the FED can only effect short run rates, they could not overpower the influx of savings. Teaser rates are the short run rates set by the fed. For a long time, consumers were offered rates on mortgage equal to these teaser rates.

Irish rates were very low during the time we were booming. Looking at the suggested taylor rule rate, it was much higher during the boom times. Ireland's rate was much lower than what was required during the boom time.

Consider if 1 year rates are higher than 3 year rates, mechanically, short run rates will be lower in year 2 and year 3. This likely means that a recession is imminent.

## 4.2 Expansionary Channels

We can rationalise an expansionary monetary policy to work through some of the following channels.

1.  $r \downarrow \Rightarrow I \uparrow, c \uparrow$ <sup>14</sup>
2.  $r \downarrow \Rightarrow$  Cashflow increase for variable rate mortgage holder/businesses.
3. By altering the risk free rate, monetary policy can effect a variety of asset prices as other asset prices are strongly linked to the risk free rate. For example, if the FED lower rates, the necessary return for investors on an asset is now less. Lower necessary returns means higher asset prices.  $P = \frac{D}{1+r} \Rightarrow$  **Wealth effect**<sup>15</sup>. This also means that the cost of servicing debt decreases and the profits likely increase.
4. Expansionary monetary policy weakens the exchange rate. Exports will increase, causing an increase in output.
5. Credit (**Balance sheet effects**)- As the price of assets increases, the value of ones collateral increases so someone can obtain more credit of a bank. This also applies a

---

<sup>12</sup>A natural experiment; Friedman and Schwartz (1963) examine the recovery of economies after the great depression. They find economies which abandoned the gold standard recovered quicker and in the order of which they left.

<sup>13</sup>Romer and Romer (1989) pick dates of exogenous falls in the money supply and find large real effects. Such as when Britain left the gold standard

<sup>14</sup>There is actually remarkably little evidence of this.

<sup>15</sup>Explanation for recent market downturn, we know that rates are set to increase and there will be more aggressive contractionary policy especially with expected inflation



lot to banks balance sheet. In this case they may also be willing to offer more credit. This credit channel works from both sides.<sup>16</sup>

Furthermore, at lower interest rates, firms who would normally not be able to service high interest loans, are now more likely to obtain capital as the cost of debt is cheaper. This makes banks more likely to lend and the default rate less, increasing output and the monetary base. This is known as the supply side credit channel.

6. Tobin's Q - When interest rates go down, asset prices tends to rise. Firms in the stock market, find it easier to find investment. For the same reason, when house prices rise, housing production also rises.
7. Portfolio Rebalancing - If an investor has a portfolio of different assets in an optimal proportion. The fed comes along and buys a load of risky stuff of them. Now the portfolio is no longer balanced in the way it was before. Now they can re balance and buy more risky stuff. It *induces a portfolio rebalancing*. This means that the economy has now a higher capacity for risk. The act of rebalancing induces a search for yield. More risky assets are purchased (corporate debt and equities) and the yields of these assets will eventually fall. In the case of equities, the price increases and induces a wealth effect.

Asset pricing bubbles bursting tends to induce a recession. Greenspan rationalises QE as doing a mop up after it bursts. What if central bank just targeted asset prices? In reality they have no power over asset prices except by open mouth operations. However they do not do this in practice.

Consider a CPI target, where we always try keep a basket of goods at 100. If there was a lot of deflation, it could come down to 50. It would take a lot of expansionary policy to bring up the CPI to 100. Instead, we can search for an inflation target of 2%, then our CPI target would be 51 and we let "bygones be bygones".

A price level target would reduce the amount of uncertainty in the economy, as CPI could increase from 50 to 100, then a CPI of 102 would be sought. Inflation is desirable for a number of reasons such as having a difference between the real and nominal interest rate. Furthermore, it changes real wages with inflation during a recession which would be otherwise hard to do.

A price level target is also much harder to hit, so it is desirable to have an inflation target which they have a better chance of hitting which will build confidence in their abilities. A price level target calls for a lot more of an aggressive path. An inflationary target is for only the next period.

### 4.3 Government Budget, Inflation and Rates

When we think about rates, we can consider short and long term interest rates. A natural way to think about this is the yield curve. We will develop an intuition in order to consider effects to the yield curve.

---

<sup>16</sup>Clinton years - Kept a surplus balance sheet, interest rates went down, banks portfolio went up in value and offered more credit.

Investors demand and the supply of bonds determines the price at which they are sold for. This gives a yield to maturity which in essence the interest rate for that given term of bond. Investors consider the state of the economy further forward the longer the duration of the bond. This way, segmented bond market theory can have implications on long and short rates to move in different directions.

### 4.3.1 Government Budget

A government can choose to make a surplus or deficit budget for a given term. As a result of the natural interest rate being determined by the clearing of the investment/savings market, the reduced government savings causes an inward shift in the savings curve. As there is now a smaller pool of savings to invest from, the interest rate increases as a result of a surplus and *vica versa*.

Recall that the UIP condition implies investors will seek foreign economies with higher rates of interest. As a result that currency will appreciate as people demand that currency to invest.

This relationship has shown up multiple times in history. Most notably the Regan tax cuts caused an appreciation of the dollar and higher interest rates.

If a government surplus is expected, then the savings pool will be greater in the future *ceteris paribus* and the future natural interest rate will decrease. This causes the long end of the yield curve to come down.

## 4.4 Issues of Monetary Policy

Here we consider what can make monetary policy less or more effective.

### 4.4.1 Inflation Targets

It is in the central banks interest to set a positive inflation target. Due to the fisher effect, the nominal interest rate will be higher if  $\pi > 0$  as opposed to zero inflation. This means that the central bank will have more scope in which to reduce the interest rate and will avoid the liquidity trap for longer.

By raising the price level, the real wage to workers is less if the nominal wage is sticky. This increases real rigidity and as such makes monetary policy more effective.

Having a constant positive inflation reduces the likelihood of deflation. If there is deflation, this acts as an incentive to spend in the future as opposed to today as real rates will be higher. This reduces aggregate demand today.

High inflation means that the real value of government debt decreases, and taxation revenue increases naturally. This acts as a revenue stream for the government.

### 4.4.2 Monetary Base

The central bank only effects the money supply indirectly through the monetary base. Recall that the money supply has a multiplier effect  $M = \mu mb$ , where  $\mu$  is the multiplier. The size of the multiplier depends on banks willingness to lend. Due to the credit channel of the effects of QE, the multiplier is pro-cyclical

So the effectiveness of monetary policy depends on the state of the economy.

#### 4.4.3 Pre-empting and Inflationary Environment

Banks react immediately based on their expectations. This makes inflationary expectations hard to counter. Furthermore, inflationary environments cause monetary policy to be less effective as prices become more flexible. This reduces real rigidity and makes monetary policy less potent.

#### 4.4.4 Zero Lower Bound on the Nominal Interest Rate

If a central bank sets its nominal rate according to its rule, in the event that it suggest a nominal rate to less than zero, it cannot do so. As people would hold alternative assets promising a return at least zero. This is known as the zero lower bound on nominal interest rates. Thus a central bank may find it difficult to stimulate the economy when they reach this lower bound. One may consider using fiscal policy at this point but the economic response on aggregate demand will be uncertain as we have seen already.

Although the central bank cannot lower nominal rates below zero, they may be able to effect the real rate to become below zero. Expanding inflation expectations may have the result in reducing the real rate accordingly. Expansions of the money stock need to effect expected inflation in order to work. They must see a sustained increase in the money supply. If they believe the central bank will restore the money supply after any effect on aggregate demand, then it will not effect expected inflation and the policy will not work.

An example of open market operations in order to lower the real rate could be the central bank purchasing foreign currencies at favourable rates. As they print money in order to pay, they receive a large demand for home currency which devalues the home currency. This increases expected inflation and stimulates the economy. Alternatively, instead of buying short run government bonds, they can buy alternative assets such as long run government or corporate bonds in sufficient volume to effect the interest rate available on them. This increases expectations of inflation in the long run reducing real rates.

Economist generally fall into two camps. One believes that once the zero lower bound on nominal rates is met and monetary policy is powerless to stimulate the economy, known as **the liquidity trap**. The other stresses that the CB can print as much money as it wants and in the event of price rigidity, can effect real rates so the liquidity trap never occurs.

## 5 New Keynesian Model

This model provides a micro foundation and ultimately becomes the IS-LM model. It allows us to explain money non-neutrality and is motivated by empirical evidence on money. The Keynesian element is sticky prices (nominal rigidities). Micro founded refers to a representative agent who seeks to **optimise their utility function based on consumption and labour supply**.

We have  $N$  **monopolistic firms** (as such set prices) who are profit maximisers. This then dictates labour demand. Production always takes place in the form of labour. We assume that there is no capital and as such no investment. Wages are taken as given and we invoke **market clearing conditions**. This implies **no unemployment**. Labour hours vary with demand and as such gives us the idea of unemployment in this model.

We also have **demand determined output**. We have pricing decision and sticky prices. Menu costs make price inertia optimal. What is important is that  $p > MC$ . This way, if prices are fixed firms will respond to an increase in demand by increasing output. This allows for a degree of demand determined output. If prices are flexible, money is still neutral. The money supply is exogenous to the model.

As demand for goods increases, firms must increase their output. This results in an increase for labour demand. Consider now the role of  $\sigma$  in the consumer's dis utility of labour. If  $\sigma$  is high people find work hard and as such will require a higher reward to work if they are to work more (slope of labour supply). An increase in labour demand will then cause a greater increase in the wage.

The role of  $\theta$  and hence  $1/\theta$ , the IES mediates the consumers willingness to smooth consumption over time. If theta is high, then consumers will be less responsive to changes in the money supply. We require some responsiveness to the interest rate so we will assume that the IES is low. We will assume that the substitution effect outweighs the income effect. If the interest rate increases, consumers will be eager to save more.

The interest rate is set by a **Taylor rule** by the central bank who conduct interest rate smoothing to reach the rate set by the Taylor rule.

We maximise the utility function in terms of labour and consumption as we have seen already.

$$U = \sum \beta_t \left( \frac{C_t^{1-\theta}}{1-\theta} - \frac{L_t^{1+\sigma}}{1+\sigma} \right) \quad (44)$$

Subject to the budget constraint.

$$W_t L_t + (1+i)B_{t-1} + \Pi_t = P_t C_t + B_t + T \quad (45)$$

Our Euler Equation becomes

$$\frac{1}{P_t} u'(C_t) = \beta \frac{1+i_t}{P_{t+1}} u'(C_{t+1}) \quad (46)$$

Our  $N$  monopolistically competitive firms face a downward sloping demand curve. They maximise with respects to  $P_i$ , there only choice variable. Nominal wages are flexible. the firms demand is

$$Y_i = \left( \frac{P_i}{P} \right)^{-\eta} \frac{Y}{N} \quad (47)$$

Where  $\eta$  is the elasticity of demand. Subject to their production function

$$f(L) = L \quad (48)$$

And the maximisation condition is

$$P_i = \frac{\eta}{\eta - 1} W \quad (49)$$

$\eta$  gives us the elasticity of demand and from this, the mark up the proportion they charge over the cost of production. If  $P$  is fixed and  $MC$  increases, then the must fall. We can model marginal costs as  $W$  or as  $W/A = W/MPL$ <sup>17</sup>.

$$L_t = \left( \frac{W_t}{P_t C_t^\theta} \right)^{\frac{1}{\sigma}} \quad (50)$$

Recall that  $\sigma$  is how difficult we find work. A high  $\sigma$ , the more *painful* work becomes. Thus following an increase in demand, higher  $\sigma$  will mean that real wage will have to go up more in order to compensate.

### 5.0.1 Consumption and Savings

The following Euler equation

$$u'(c_t) = \beta(1 + r)u'(c_{t+1}) \quad (51)$$

gives us the relation between output and the interest rate. Note that the this is only one of the channels which monetary policy works. By subbing in  $u'(c) = \frac{1}{c^\theta}$  and taking logs we then show.

$$c_t = \frac{\rho - r_t}{\theta} + \mathbb{E}_t c_{t+1} \quad (52)$$

There is also a government in the background where  $\log(G_t) = g_t$ . So total aggregate demand is

$$d_t = \frac{\rho - r_t}{\theta} + \mathbb{E}_t c_{t+1} + g_t \quad (53)$$

We may iterate forward  $n$  times and find that

$$c_t = n \frac{\rho}{\theta} - \frac{n}{\theta} R_n + c_{t+n} \quad (54)$$

Where  $R_n$  is the long run interest rate by the PEH. Thus consumption today depends on the future path of short run interest rates. This gives us an indication as to how the central bank may stimulate the economy by effecting expectations of future short run interest rates. By committing to keep short run rates low for a while they can effect the long run rate thus increasing output.

---

<sup>17</sup>Marginal Productivity of Labour is equivalent to the level of technological progress. Higher  $A$ , each worker is more productive and the  $MC$  of a good is lower.

### 5.0.2 Firms Pricing Behaviour

Our  $N$  monopolistic firms choose prices to be some constant of their marginal cost of production, the wage. The following is the maximisation condition for real profits to the firm.  $\eta$  is the elasticity of demand.

$$P_i = \frac{\eta}{\eta - 1} \omega \quad (55)$$

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{\omega}{P} \quad (56)$$

$$\text{Log} P_i = \text{Log} P + \log\left(\frac{\eta}{\eta - 1}\right) + \log\left(\frac{\omega}{P}\right) \quad (57)$$

Thus the price depends on the price level, mark-up and the real wage (note that the last  $P$  term could drop out but we keep it to show real wage). The optimal price of any given period is the result we just arrived at. Suppose now that we cannot set the price every period. The natural thing to do is take

$$P_{1,2} = \frac{P_1^* + P_2^*}{2} \quad (58)$$

Exactly how this is set depends on the motives of the manager. This brings us to the main result that if we have to set prices today for the next number of periods, I take into account the level of inflation for successive periods. If there is high future expected inflation, then there will be higher inflation today as a result. In the fully fledged model, 20% (say).

By choosing a price, the firms are implicitly choosing an output. Subbing in the optimal price for each period into 58.

$$p^* = \frac{1}{2} \left( \text{Log} P_1 + \log\left(\frac{\eta}{\eta - 1}\right) + \log\left(\frac{\omega}{P_1}\right) \right) \quad (59)$$

$$+ \text{Log} P_2 + \log\left(\frac{\eta}{\eta - 1}\right) + \log\left(\frac{\omega}{P_2}\right) \quad (60)$$

### 5.0.3 The New Keynes Philips Curve

Suppose we have a firms optimal price setting relation

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{\omega}{P} \quad (61)$$

$$\log(P_i) - \log(P) = \log\left(\frac{\eta}{\eta - 1}\right) + \log\left(\frac{\omega}{P}\right) \quad (62)$$

At equilibrium, the  $P_i = P$

$$\Rightarrow 0 = \log\left(\frac{\eta}{\eta - 1}\right) + \log\left(\frac{\omega}{P}\right) \Rightarrow \log\left(\frac{\eta - 1}{\eta}\right) = \log\left(\frac{\omega}{P}\right) \quad (63)$$

Denote the equilibrium real marginal cost as  $\widehat{\log\left(\frac{\omega}{P}\right)}$  and we know that this is equal to the log mark-up. Substituting the log mark-up for negative equilibrium real marginal costs by

using general log rules.

$$\log(P_i) - \log(P) = \log\left(\frac{\omega}{P}\right) - \widehat{\log\left(\frac{\omega}{P}\right)} \quad (64)$$

$$p_{it} - p_t = \log\left(\frac{\omega}{P}\right) - \widehat{\log\left(\frac{\omega}{P}\right)} \quad (65)$$

$$p_{it} = p_t + \underbrace{\log\left(\frac{\omega}{P}\right) - \widehat{\log\left(\frac{\omega}{P}\right)}}_{\text{Proportional to output gap}} \quad (66)$$

$$\Rightarrow p_{it}^* = p_t + \alpha(y_t - y_n) \quad (67)$$

$$p_{it} - p_{t-1} = p_t + \alpha(y_t - y_n) - p_{t-1} \quad (68)$$

$$\pi_{it}^* = \pi_t + \alpha(y_t - y_n) \quad (69)$$

This is by how much a firm *would* change their price by, if they had the opportunity to. Some fraction of the output gap. If the output gap is negative, they will decrease their prices and vice versa. Now suppose that only a fraction of firms  $\delta$ , get the opportunity to do so. This is the level of *nominal rigidity* in the economy. Recall that *real rigidity* is the extent to which firms want to change their price. So the price level in period  $t$  is

$$p_t = \delta p_t^* + (1 - \delta)p_{t-1} \quad (70)$$

$$\Rightarrow \pi_t = \delta \pi_t^* \quad (71)$$

Firms know that they may not get the opportunity to change their price again for a number of periods. So they take into account each optimal price for the next number of periods. They discount each future optimal price by some  $\phi$ . This says how much a firm cares about future profits. So the optimal price change today is

$$\pi_t^* = \sum_{i=1}^{\infty} (1 - \delta)^i \phi^i \mathbb{E}_t[\pi_{t+i} + \alpha(y_{t+i} - y_n)] \quad (72)$$

Subbing this into our equation for inflation today (71).

$$\pi_t = \delta \sum_{i=0}^{\infty} (1 - \delta)^i \phi^i \mathbb{E}_t[\pi_{t+i} + \alpha(y_{t+i} - y_n)] \quad (73)$$

From this, isolate  $\mathbb{E}_t[\pi_{t+1}]$

$$\phi \delta (1 - \delta) \mathbb{E}_t[\pi_{t+1}] = \delta \pi_t + \delta \sum_{i=2}^{\infty} (1 - \delta)^i \phi^i \mathbb{E}_t[\pi_{t+i} + \alpha(y_{t+i} - y_n)] \quad (74)$$

subtract (73) from (74)

$$\pi_t - \phi \delta (1 - \delta) \mathbb{E}_t[\pi_{t+1}] = \delta \pi_t + \delta \alpha (y_t - y_n) \quad (75)$$

$$(1 - \delta) \pi_t = \phi \delta (1 - \delta) \mathbb{E}_t[\pi_{t+1}] + \delta (y_t - y_n) \quad (76)$$

$$\pi_t = \phi \mathbb{E}_t \pi_{t+1} + \frac{\delta \alpha}{1 - \delta} (y_t - y_n) \quad (77)$$

$$\pi_t = \phi \mathbb{E}_t \pi_{t+1} + \zeta (y_t - y_n) \quad (78)$$

This yields the new Keynesian Philips curve. Note how  $\zeta$  depends on  $\alpha$  and  $\delta$ .  $\alpha$  dictates how much flexible marginal costs change as a result of changes in the output gap. This captures the level of nominal and real rigidity in the economy.  $\zeta$  is small for high levels of rigidity.

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} \frac{Y}{N} \quad (79)$$

Where aggregate demand here  $Y$  is equivalent to  $d_t$  found earlier.

Earlier we assumed that output is "demand determined" If there is an increase in aggregate demand, then firms will meet this demand. If prices rise, this reduces demand so people have to hold a lot more cash. As a result, there is less money put into banks and as such interest rates will rise.

#### Example

Suppose we have a helicopter drop of money and all prices are fixed for a year but wages are flexible. Aggregate demand will go up for every firm in the market.

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} \frac{Y}{n} \quad (80)$$

Where  $Y$  is aggregate demand. In order for firms to meet the increase in aggregate demand they must increase labour supply, causing an increase in the marginal cost to the firm. Now the Price level is greater than  $MC$ .  $L^D \uparrow \Rightarrow W \uparrow \Rightarrow W/P \uparrow$  As wages have increased and prices cannot change, the mark up must fall.

In the next period, firms will want to have prices set at there markup so as a result, prices will increase and aggregate demand will fall back down. If wages were not as flexible, this would have lasted for longer. The more real rigidity in the economy, the longer this will last for. Suppose that instead of hiring more of the same workers, the firms could hire in cheaper labour from China. This would be a degree of real rigidity. The firm will be happy to increase output as long as  $P > MC$ . The increase in AD will last as long as  $MC \not\geq P$

The previous example was dependent on prices being fixed for a year. This stems from **menu prices** theory or **nominal rigidity**. It is akin to an implicit contract between firms and consumers. For sure however these menu costs have gone down substantially in recent years.

There is also **real rigidity**, which is some other force which reduces the incentive to increase prices. It is something real which relieves the upward pressure on marginal costs. In a boom time, it is cheaper for firms to borrow money, this incentives firms to keep prices fixed. Both real and nominal rigidity are captured in the new Keynesian Philips curve..

The state of labour supply has effects on the level of real rigidity within an economy. As more people join the labour force, the real wage must decrease. This reduces the upward



pressure on marginal costs and acts as real rigidity.

What else may occur is if competition increases in a boom, then  $\eta$  will increase in a boom, making mark ups fall which could mean prices remain fixed. Empirically, (Gali How well does IS LM fit US Data) inflationary pressures remain after a shock to the economy.

#### 5.0.4 Money Market Equilibrium

Consider the money market equilibrium  $\frac{M}{P} = L(r_n^-, Y_n^+)$ . Suppose that prices are fixed, and the central bank increases the money supply

We may extend the New Keynesian model to include a probability  $\delta$  that you adjust each period. For a higher,  $\delta$  people are more sensitive to inflation. As such, if  $\delta$  is high, there will be a higher level of inflation. Furthermore, firms discount the future and care more about today's profits. The optimal price becomes

$$p_{it}^* = p_t + \alpha(y_t - y_n) \quad (81)$$

Where  $\alpha$  measure the sensitivity of marginal costs to the output gap. Recall, a high sigma means that real wages will go up more following a change in labour demanded, this is analogous to  $\alpha$  here. If we take away  $p_{t-1}$  from both sides,

$$\pi_t = \phi \mathbb{E}_t[\pi_{t+1}] + \zeta(y_t - y_n) \quad (82)$$

This is the new Keynesian Philips curve. As opposed to a philips curve, people in this model are forward looking. In contrast, the standard Philips curve are backwards looking, inflation depended on inflation last year.  $\zeta$  will depend on the number of people changing prices  $\delta$  and  $\alpha$  the sensitivity to the output gap.

- If there is very little real rigidity,  $\zeta$  will be high. A lot of Real rigidity means MC are very stable.

To reduce inflation here, the central bank could announce that they are bringing down inflation. This alters expectations of inflation in a hope to bring down inflation. Alternatively, you could drain upward pressure on prices by creating a recession. Note that bringing in cheap labour to lower marginal costs is not on the mandate of the central bank.

In the Volker disinflation both parts of this relation played a roll. By decreasing the money supply, the interest rate rose, causing a fall in the output gap, as such a recession. This caused the high inflationary pressure to fall from at its peak 12% to a reasonable level. In the Volcker disinflation episode, the coefficient of inflation in the Taylor rule rose markedly.

### 5.1 The Three Equation Model

We have derived the new Keynesian Philips curve, found an expression for demand/output and noted that central banks follow a Taylor rule. Our model then becomes

$$\begin{aligned} y_t &= \frac{\rho}{\theta} - \frac{1}{\theta}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t c_{t+1} + g_t + \nu_t \\ i_t &= r_n + \pi_t + \gamma(y_t - y_n) + \beta(\pi_t - \bar{\pi}) + \epsilon_t \\ \pi_t &= \phi \mathbb{E}_t \pi_{t+1} + \zeta(y_t - \bar{y}) + u_t \end{aligned}$$

Note here that we have substituted the real rate  $r_t$  as in the demand equation for the nominal rate minus inflation. When discussing economies such as Ireland in the Euro-zone, fluctuations in demand in the Irish economy does not result in a change in the Taylor rule. Thus we consider what happens to changes in the real rate with  $i$  remaining constant.

## 6 Real Business Cycle Theory

Waves of creative destruction. Not an isolated theory historically other theories put emphasis to *real* shocks

### No Money

Real shock to the economy. No role for money. QE is reverse causal in this model. In a boom money demand increases. CB increases money supply which brings down the interest rate. This is a controversial aspect of the money. Think about the Volker disinflation. This is contradicting. Exogenous changes in  $M$  change  $Y$ . Where as this theory denies it. It considers the reverse causal aspect.

### Frictionless

No information asymmetries and prices can adjust instantly to make markets clear. As such each period is an equilibrium and as such efficient.

### Long Run

#### 6.1 Dynamics

We have a production function

$$Y = AK^\alpha L^{1-\alpha} \quad (83)$$

In contrast to previous theories we have included capital, which implies that investment is fundamental to this model. We can decompose the output to

$$Y = C + I \quad (84)$$

Suppose we have an impulse response curve starting off at  $A$ . At time zero, there is a jump up in  $A$ . Overtime, the shock fizzles out and  $A$  returns to its old level. The key thing here it is **temporary but persistent**. To fit the data, we need both of these features. Here  $A$  is pretty broad. It could include having to employ lawyers or when the weather is bad. For example, we could model the snow last week in this manor.

$A$  follows an autoregressive structure as such, shocks to  $A$  are persistent. If  $A$  goes up today, it will be high tomorrow all things equal. As a result of  $A$  going up, the marginal product of labour also goes up. As a result, the demand for labour goes up so  $Y$  gets an additional *kick*. Real wages will also go up as a result.

If  $A$  was not persistent, then there would be not be as large an increase in consumption (just as the PIH predicts of course). Implicit in the model is a strong substitution effect and

weak income effect. In order to have this, we must have a high intertemporal elasticity of substitution. This induces a rise in savings and labour supply today as a result of the shock.

### Propagation Mechanisms

The models internal way of amplifying a technology shock are caused by:

- **Intertemporal Elasticity of Substitution:** Causes an increase in savings/investment and labour supply<sup>a</sup>. Work is better rewarded today then tomorrow so labour supply will increase. The return to savings is higher today so labour is encouraged to save the extra income.
- **Capital Accumulation:** As productivity of capital increase, consumers will want to supply investment today as the returns next period will be high. More investment today, more capital tomorrow and hence more output tomorrow.<sup>b</sup>

<sup>a</sup>Beast From the East: No one could work during the snow, reduced labour demand and hence the wage, so people shifted work to next period.

<sup>b</sup>Beast From the East: Savings and Capital did not change, just what people could do with them during this period.

$$\uparrow A \Rightarrow Y \uparrow \quad (85)$$

We now are interested in what resultantly happen to the decomposition to  $C + I$ . Output always equals potential and hence the rate of interest always equals the natural rate. Prices are flexible so they move such that the markets always clear, in stark contrast to the new Keynes model.

There is an increase in consumption by the PEH but only by a little bit. Most of the increase in output is saved in the form of investment. As a result, the capital stock will be higher next period.

There is also an increase the interest rate. This has the resultant effect of incentivising saving capital for future use. If  $Y$  goes up alot and  $C$  only goes up a little bit, then  $I$  must go up a lot.

In this model, the wage is equal to the marginal productivity of labour. This results in a pure substitution effect. So from an shock to  $A$ , the productivity of labour goes up, thus the wage goes up and so does  $L$ . This is a propagation mechanism of the shock. This is also known as the **intertemporal substitution** of labour. If the wage goes up today, I will work more today.

$$\frac{\partial Y}{\partial L} = (1 - \alpha)AK^\alpha L^{-\alpha} = w \quad (86)$$

The other propagation mechanism is capital accumulation. We say in any given period,  $K$  is fixed. Investment today, builds up capital stock for tomorrow. That is

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (87)$$

And

$$Y_{t+1} = A_{t+1}K_{t+1}^\alpha L^{1-\alpha} \quad (88)$$

This describes the propagation of a technology shock. All the extra capital that we have built up will eventually depreciate.  $L_{t+1}$  reverts back also (as capital is in the wage). We will converge back to some level of steady state capital.

## 6.2 The Model

We are now going to move away from the aggregate and look at the firms perspective and motivations as well as the consumer. The profit function for the **perfectly competitive** firm (as opposed to the monopolistic firms in the new Keynesian model).

$$\Pi = AK^\alpha L^{1-\alpha} - wL - rK \quad (89)$$

We assume that sticky prices and the nature of the firms is not quantitatively important. We maximise this with respects to  $L$  and  $K$ . We rationalise this as the firm pays the households for use of the capital.

$$\frac{\partial \Pi}{\partial L} = (1 - \alpha)AK^\alpha L^{-\alpha} - w = 0 \quad (90)$$

$$\frac{\partial \Pi}{\partial K} \alpha Ak^{\alpha-1} L^{1-\alpha} - r = 0 \quad (91)$$

Wages and labour are pro-cyclical Interest is pro cyclical and so it the rate of technology. The rate of interest is nothing to do with the FED.  $L$  in this model is hours per worker. We are on the intensive margin here. This is somewhat problematic as most of the movement in  $L$  during the business cycle is on the extensive margin. In the New Keynesian model, we could patch this up pretty easily. ( $I, C, r, w, Y, A$ ) all pro-cyclical This model predicts that  $I$  is *very* pro-cyclical

As a percent of the variance of GDP - ( $c = .75, I = 4.7, P = .57, L = .6, labour.prod = 62.7$ ). In this model, all the fluctuations are **pareto** optimal. This is a walrasian equilibrium? This implies that there is no need for stabilisation policy. For example, the government couldn't force people to go out and work in the snow. The depression is often known as the great vacation in this world. Thus there is times where this model does fit the data quite well but its limitations should always be kept in mind. If there was an increase in  $G$ , there would be a negative wealth effect which would cause labour supply to go up. It does so by making people poor and miserable.

There is another fact which accentuates the intertemporal substitution of labour. That is the pro-cyclical interest rate. If  $r$  goes up and I invest my wages, I get a greater return on my work. Thus the reward to work is higher and increases labour. This is akin to an increase in the opportunity cost of labour.

In all the impulse diagrams, if we look at  $Y$  at its trend level, the increase in  $A$  pushes us up above trend. We look at the percentage deviation from trend. (Recall taking the logarithms in last years macro). From our analysis, investment should have the greatest deviation from trend.

### 6.2.1 Consumption and Labour Supply

Consumers have lifetime utility

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left( \log(C_t) - \frac{l_t^{1+\sigma}}{1+\sigma} \right) \quad (92)$$

And the following budget constraint

$$w_t l_t + k_t(1 + r_t - \delta) = c_t + k_{t+1} \quad (93)$$

Setting up the lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \log(C_t) - \frac{l_t^{1+\sigma}}{1+\sigma} \right) - \sum_{i=1}^{\infty} (w_t l_t + (1 + r_t - \delta) k_t c_t - k_{t+1}) \quad (94)$$

We maximise with respects to  $c_{t+1}$ ,  $l_{t+1}$  and  $k_{t+1}$  which yields

$$\beta^{t+1} \frac{1}{c_{t+1}} - \lambda_{t+1} = 0 \quad (95)$$

$$\Rightarrow \beta^{t+1} \frac{1}{c_{t+1}} = \lambda_{t+1} \quad (96)$$

$$-\beta^{t+1} l_{t+1}^{\sigma} + \lambda_{t+1} w_{t+1} = 0 \quad (97)$$

$$\Rightarrow \beta^{t+1} l_{t+1}^{\sigma} = \lambda_{t+1} w_{t+1} \quad (98)$$

$$\lambda_t + \lambda_{t+1}(1 + r_{t+1}) = 0 \quad (99)$$

Setting  $\delta = 0$  and considering  $r_{t+1}$ , net of depreciation we arrive at an Euler equation

$$\frac{1}{c_t} = \mathbb{E}_t \beta (1 + r_{t+1}) \frac{1}{c_{t+1}} \quad (100)$$

This determines consumption each period and as such investment. From this we can see temporary increases in the interest rate will cause consumption to fall and savings to rise. This is the rate they get next period. They respond today by decreasing consumption for investment and renting it out next period.

Combining our maximisation conditions for labour and consumption we find the static labour/leisure optimality condition

$$w_t u(c_t) = v'(l_t) \quad (101)$$

This gives us labour supply for the consumers. Doing the same for leisure next period and dividing conditions for labour tomorrow by labour today we find

$$\frac{l_{t+1}^{\sigma}}{l_t^{\sigma}} = \frac{w_{t+1} c_t}{w_t c_{t+1}} \quad (102)$$

Setting  $\beta(1 + r_{t+1}) = 1$  for now, we can then set  $c_{t+1}/c_t = 1$  and hence

$$\frac{l_{t+1}}{l_t} = \left( \frac{w_{t+1}}{w_t} \right)^{\frac{1}{\sigma}} \quad (103)$$

Because of the increasing marginal disutility of labour, consumers will smooth labour over time. More generally

$$\frac{l_{t+1}}{l_t} = \left( \frac{w_{t+1}}{w_t \beta (1 + r_{t+1})} \right)^{\frac{1}{\sigma}} \quad (104)$$

If wages rise today relative to tomorrow, labour supply will rise today. If we note that  $w_t/c_t = l_t^\sigma \Rightarrow l_t = (w_t/c_t)^{\frac{1}{\sigma}}$  and taking next period, tomorrows labour supply is

$$\Rightarrow l_t = \left( \frac{w_{t+1}}{c_{t+1}} \right)^{\frac{1}{\sigma}} \quad (105)$$

By the PIH,  $c_{t+1}$  will rise a little bit so labour supply tomorrow will fall a little bit. This is due to the intertemporal substitution of labour. People will prefer to work when wages are high, and because of the income effect and PIH, labour supply will fall a little bit tomorrow following an increase in the wage today relative to tomorrow. However, given consumers are infinitely lived, we assume that  $l_{t+1}$  stays the same. Given a permanent increase in the wage,  $w_t/c_t$  will remain the same so labour supply will remain unchanged.

### 6.3 Exogenous Shocks

Suppose that capital will be injected into the economy independent of the state of the economy,.  $A$  remains constant.  $Y$  will increase, and labour will increase. As a result of the increase in  $K$  and **diminishing marginal returns to capital**. This brings down the interest rate. So  $K \uparrow \Rightarrow r \downarrow$ . With the increase in  $K$ , the marginal productivity of capital goes up. The other effect however, is like an income effect. If  $r$  goes down then as we already discussed,  $L$  goes down a small bit as there is two effects at play. Investment goes down as the interest rate goes down. Consumption will go up as it is *like winning the lottery*.

Consider the difference between economies with rich natural resources such as Africa, have the worst performing economies. This could be attributed to the income effect of having such great resources (dump of capital into the economy).

### 6.4 The Solow Residual

When we estimate  $Y$ , we can parse out a solow residual  $A$ . This model predicts that the Solow residual will be highly correlated with output.

We have a representative agent who maximises

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(C_t) - \frac{l_t^{1+\sigma}}{1+\sigma}) \quad (106)$$

Subject to  $wl + rk = c$ . This yields the familiar Euler equation. Every period we have optimal labour supply conditions also seen  $wu'(c_t) = w'(l)$ . The key feature is that  $w$  is going to be endogenous. It will depend on  $A, K$  and  $L$ . Recall that capital and labour are **complements**. The households supply labour and the firms demand labour. They both meet at some equilibrium in every period. Based on the model outlined, we will have

$$\frac{1}{C_t} = \mathbb{E}_t \beta (1 + r_{t+1}) \frac{1}{C_{t+1}} \quad (107)$$

For simplicity, set  $\beta(1 + r) = 1$ , then

$$\frac{l_{t+1}^\sigma}{l_t^\sigma} = \frac{w_{t+1}}{w_t}. \quad (108)$$

One implication of this is how we see tax cuts being **phased in** into the economy. A tax cut would stimulate labour supply. If it is not right away, the return to labour is higher in the future and you will not want to work right now. With uncertainty and not assuming  $\beta(1 + r) \neq 1$

$$\frac{l_{t+1}^\sigma}{l_t^\sigma} = \frac{w_{t+1}}{w_t} \frac{1}{\beta(1 + r_{t+1})} = 1. \quad (109)$$

Capital and labour demand are both increasing in  $A$ . One Keynesian counter argument about this is that an increase in  $A$ , maybe firms will demand fewer workers. Especially in the short-run In the long run, we have seen enormous increases in  $A$  and  $L$ . This is what drives up wages in the long run. In the short-run it is not clear.

A typical stochastic process for  $A$  can be

$$\log(A_t) = \rho \log(A_{t+1}) + \epsilon_t \quad (110)$$

If  $0 < \rho < 1$ , the shock will eventually die out. Otherwise it will be a unit root process and never die out.

#### 6.4.1 Neoclassical Models and the Great Depression

$A$  is exogenous in the RBC model.  $A$  is causing the business cycles. RBC theorists would then say this is evidence of this model. This could also be as a result during a boom, people work harder during a boom (learning by doing etc...). In a boom there is greater use of capital in labour. Then it is endogenous to the model. This is a neo Keynesian response to RBC arguments. This is essentially the Business Cycle causes fluctuations in  $A$ . As a result of greater labour utilisations.

Manufacturing hours falling dramatically 1930s. This suggests that the great depression was not lead by failing monetary policy/collapse of the banking system. The Keynesian story of great depression was a money supply/rigidity story. The RBC story for the great depression is s that wages were kept too high. Nominal wages couldn't fall committed to by the firm. Hoover permitted aloud a lot of monopoly power into the economy. Firms wouldn't drop wages but in return, the government wouldn't go after monopolies This kept wages artificially high. This combination of static nominal wages and deflationary environment, caused labour demand to fall dramatically. This also restricted output and competition.

### Conflicting Theories of Cole/Ohanian and Hoover

The standard theory of the cause of the great depression in the 30s is that a monetary contraction followed by a banking crisis, reduced the money supply causing the great recession. The US could not respond effectively because they were committed to the gold standard. This made it difficult to increase the supply of high powered money. However, observing the level of the money supply with manufacturing hours, it is clear that there was a significant fall in manufacturing hours, before the banking crisis and monetary contraction began.

Manufacturing hours fall substantially where as the money supply only fell between 1-4%. Ohanian' theory of the great depression, downplays the role of money and speculates that it was actually because manufacturing wages were well above the market clearing rates. This is from evidence of papers of people offering themselves for jobs with substantially lower wage demands than what unions suggested. Misguided policies were put in place which allowed and encouraged collusion amongst firms. Furthermore, wages were estimated to be 25% too high.

## 6.5 Dynamic Inconsistency

Dynamic inconsistency is one of the key policy ideas to emerge from RBC theory. This is concerned with the debate around rules and discretion. Should policy be bounded by a set of rules or should it be up to policy makers at the time.

### 6.5.1 Capital Taxation

Consider the economy today and next period. Suppose that the government can either tax labour and or capital. They announce today that they will not tax capital next period but tax labour instead. Labour is relatively inelastic in the short run as people will in general always need to work. As a result of this, it is optimal for people to invest income today to rent out tomorrow.

Capital investment from last period is brought forward into next period. As such it cannot be withdrawn by consumers. They have committed to supplying capital. In period 2 when the time comes to not tax capital, what is the optimal thing for the government to do? Should they hold true to their promise or renege? Of course, as the capital cannot be withdrawn it is optimal for the government to tax capital as it will be completely undistortinate. So modelling this in a game theory way, there would be no investment in period 1 as agents know they will be taxed despite the promises.

This is known as dynamic inconsistency. Formally, the initial policy is dynamically inconsistent. In period 1 it makes sense as there will be more investment but next period it is not optimal

### 6.5.2 Monetary Policy

In a more modern context, promising to banks that no more banks will be bailed out is in itself dynamically inconsistent.



Recall the money demand and supply equilibrium

$$\frac{M}{P} = L(i, Y) \quad (111)$$

In the classic long run, if we double  $M$  we also double  $P$  and nothing would change on the right. We would require at least some amount of rigidity to change the right. To get an effect on real economic activity, we need  $M$  to increase without  $P$  to increase.

Suppose that the central banks commit to not increasing  $M$  in the future. Suppose that firms believe the story of the central bank, that is the stability of  $M$ . The following year, when all firms have fixed their prices accordingly. What is the incentive to central banks to do with  $M$ ? It will be optimal to increase money supply as firms have set prices and there is rigidity. As a result, the government can induce a boom by increasing money supply.

The typical way to get around dynamic inconsistency, is to make the central banks independent institutions.

Milton Friedman postulated that to conduct good monetary policy, you need to increase the money supply at a constant rate over time. By setting rules, we remove discretion and dynamic inconsistency is minimised (rules vs. discretion argument). Consider the ECB for example, they have a very strict mandate of inflation targets of 2%. This is known as a nominal anchor. These arguments were very prominent in the 1970s, where monetary policy was seen to be very poorly conducted.

## 7 Ramsey Model

The Ramsey Model is closely related to the RBC model whereby both models are of a frictionless environment with flexible prices and perfectly competitive firms. It is a model of the *long run*. It is often referred to as the neo-classical growth model. Basically, it is a RBC model with fixed labour supply and constant productivity. The capital stock plays the central role.

Because it is a long run model, output always equals potential output. We let labour supply and productivity be constant. Thus, people choose between saving and consuming in each period.

The main dynamic in play in the Ramsey Model is the tension between consuming today and savings. Mediating this tension is the Euler equation.

Assume labour supply is constant and that there is no depreciation on capital. We can begin at the Euler equation

$$u'(c_t) = \beta(1 + r_{t+1})u'(c_{t+1}) \quad (112)$$

If we allow for log utility (more generally we use CRRA utility), this becomes

$$\frac{1}{c_t} = \beta(1 + r_{t+1})\frac{1}{c_{t+1}} \quad (113)$$

$$\frac{c_{t+1}}{c_t} = \beta(1 + r_{t+1}) \quad (114)$$

$$\frac{c_{t+1}}{c_t} = \frac{1 + r_{t+1}}{1 + \rho} \quad (115)$$

Subject to the budget constraint

$$l_t w_t + r_t k_t = c_t + \underbrace{k_{t+1} - k_t}_{\text{Increase in } k} \quad (116)$$

That is, labour income plus the return to capital must be equal to consumption today plus the increase in capital stock. Recall that firms are perfectly competitive so there are no profits and labour is constant ( $l_t = 1$ ). The left hand side is output ( $y_t$ ).

$$y_t = c_t + k_{t+1} - k_t \quad (117)$$

This is the aggregate resource constraint of the economy, given a representative agent. We may find tomorrows level of capital stock as

$$k_{t+1} = k_t + \underbrace{y_t - c_t}_{\text{Investment last period}} \quad (118)$$

Tomorrows level of capital stock is yesterdays level of capital stock, and what is left over from last periods of output after consumption  $c_t$ . Underlying the dynamics of this model is the tension between savings and consumption. This is governed by the Euler equation. If the interest rate is greater than the rate of time preference. Then consumption will grow over time. The interest rate grows with the level of capital stock.

The interest rate at any point in time is the marginal productivity of capital. It is the return consumers may rent their savings to firms at. Given perfect markets, they receive the MPK of capital. (Here we assume that  $l = 1$ )

$$f(k) = Ak_t^\alpha \quad (119)$$

$$\Rightarrow f'(k) = \alpha Ak_t^{\alpha-1} \quad (120)$$

Subbing this into the Euler equation

$$\frac{c_{t+1}}{c_t} = \frac{1 + \alpha Ak_t^{\alpha-1}}{1 + \rho} \quad (121)$$

At the steady state, consumption is constant so  $c_{t+1}/c_t = 1$ .

$$1 = \frac{1 + \alpha Ak_t^{\alpha-1}}{1 + \rho} \quad (122)$$

$$\rho = \alpha Ak_t^{\alpha-1} \quad (123)$$

$$\Rightarrow k^* = \left( \frac{\alpha A}{\rho} \right)^{\frac{1}{1-\alpha}} \quad (124)$$

If the interest rate is high, consumers will want to save more. As they save more, the MPK of capital will fall, up until a point where their rate of time preference is equal to the return of capital.

If we were to introduce labour into this model also, think about what results we would find. In steady state,  $f(k^*) = Ak_t^\alpha = k_{t+1} - k_t + c = c_t$ . In the steady state output is equal to consumption.

## 7.1 Government Expenditure

Government expenditure in the Ramsey model may be included in the total resource constraint as

$$y_t = k_{t+1} - k_t + c_t + g_t \quad (125)$$

Consider a temporary rise in government expenditure, first outside of the model with standard economic theory. If the government increase expenditure today, they must borrow from somewhere. This crowds out the market for loanable funds and as such the interest rate must rise. The Ramsey model also predicts a positive correlation amongst temporary fluctuations in government expenditure.

First let's examine a permanent rise in government expenditure. In the steady state,  $\dot{k} = 0$ . So the resource constraint in the steady state becomes

$$y_t = c_t + g_t \quad (126)$$

If the government increase taxation permanently today, consumption will fall one to one and the steady state level of capital will remain unchanged. This is because of the steady state condition that  $1 + f'(k^*) = \rho$ . The Euler equation also remains unchanged. So an increase in government expenditure permanently does not change the level of output.

Now consider that the government increases government expenditure temporarily today for one period and we are initially at the steady state.

$$y_t = c_t^L + g_t^H \quad (127)$$

Consumption falls one to one. Examining the Euler equation

$$\frac{c_{t+1}}{c_t^L} = \frac{1 + f'(k_t)}{1 + \rho} \quad (128)$$

The rate of consumption growth increases. Holding  $\rho$  constant, the numerator must fall. Thus, a temporary rise in government expenditure will cause the interest rate to increase, by a similar logic of a crowding out of the market for loanable funds.

### Permanent rise in government expenditure

Suppose now instead that the government increase expenses permanently with a lump sum taxation.  $g_t = T$ . Incorporating this into the budget constraint this becomes.

$$y_t = c_t + T + k_{t+1} - k_t \quad (129)$$

In the steady state,  $k_{t+1} - k_t = 0$ , so consumption must fall one to one. There is no change to the capital stock. The dynamic of savings/rate of time preference remains unchanged. It acts like a negative endowment and is a pure negative income effect. If there was labour in the model, consumers would work more.

## 7.2 Capital Taxation

If the government increases the tax on capital, what happens to investment, consumption and output? The steady state Euler equation condition becomes

$$(1 - \tau)\alpha A k_{t+1}^{\alpha-1} = \rho \quad (130)$$

Capital stock then becomes

$$k^* = \left( \frac{(1 - \tau)\alpha A}{\rho} \right)^{\frac{1}{1-\alpha}} \quad (131)$$

The return of capital will still be  $\rho$ . The capital stock as a result of the taxation has become smaller. However, as a result of the smaller capital stock, the return to capital has increased. The workers bear the instance of the tax. Wages are increasing as the level of capital stock increases. If it decreases, capitalists bear no loss of returns but workers experience a wage cut.

This is a very robust result in economics that labourers bear the burden of capitalist taxation. This however is not necessarily the motivations of trump's policy. However, his strategy will work against him. As he reduces corporate tax rate, there would be an inflow of foreign investment. This would result in an appreciation of the dollar and a worsening of the current account, of which he is trying to strengthen (trade war with China).

An issue with a policy to reduce corporate tax and the predicted outcomes of this model is that in the real world, firms are not perfectly competitive and as such profits are not competed away. Thus a rise in after tax profits from reducing taxation will go to foreign firms who are investing in the economy.

## 7.3 Implications of the Ramsey Growth Model

It is optimal to save in the outset of a development model. Capital accumulation is key when consumption growth is high. The savings when you are poor is very beneficial as you can grow rapidly/accumulate capital quickly. Take America for example, they will never experience rapid growth. China is a good example of massive amount of savings/investment and rapid growth.

If  $A$  goes up, this implies that the level of income and capital at steady state both go up. If  $\rho$  goes up, the steady state of  $y$  and  $k$  go down.

Growth rates relative to per capita to gdp of developed nations. When we examine independent of initial GDP/poor countries who are undeveloped, they will not necessarily have high growth rate.

In the case of a war, what will happen? The country will go to a lower level of capital, then it has the potential to grow fast. Consider the German miracle. Examining Germany, we can see a massive drop off at the war for per capital real GDP. Post war, they converge rapidly to their previous level of growth.

Note the difference in levels of per capita growth often times are down to **Conditional convergence**, if you are poor, you will converge rapidly to the steady state conditional that it is a good steady state. This is also referred to as the catch up effect.

If the level of productivity is fixed, then there is no long run growth. If we allow  $A$  to increase over time, we may model an economy where the level of capital stock at the steady state will increase over time.

### 7.3.1 Dynamics of Consumption and Capital

It is often useful to consider the dynamics of consumption and capital and its implications on the phase diagram when analysing the effects of taxation. First note

$$y_t = f(k_t) \quad (132)$$

$$\dot{k} = f(k_t) - c_t(1 + \tau) \quad (133)$$

$$\frac{\dot{c}}{c} = f'(k_t) - \rho\theta \quad (134)$$

An increase in consumption tax, causes the saddle path to shift downwards. This leaves the economy at a lower steady state level of consumption. An increase in taxation on capital stock decreases the level of capital thus consumption locus of points such that  $\dot{c} = 0$  must shift inwards. Therefore consumption and capital stock are lower.

A tax on consumption does not affect the agents preferences as these are governed by the Euler equations. Thus an increase in consumption makes consumption more expensive. Decreasing what is left over for investment. thus the  $\dot{k}$  locus shifts downwards. This reduces the level of consumption in the steady state and leaves the level of capital stock unchanged.

## 7.4 The AK Model

Suppose now instead of the traditional production function, we let  $\alpha = 1$ . Then the MPK of capital is constant and equal to  $A$ . The Euler equation becomes

$$\frac{c_{t+1}}{c_t} = \frac{1 + A}{1 + \rho} \quad (135)$$

This has the nice feature that it allows us to model perpetual growth as long as  $A > \rho$ .

## 8 Growth and Development

### 8.1 Endogenous Growth Model

The classical theory of economic growth may be described as  $f(k) = AK$ , where  $A$  is some constant representing the level of technological progress. This yields a constant returns to scale.

We may extend this by allowing  $A$  to vary, described by

$$\dot{A} = \gamma AL_A \quad (136)$$

In essence, we are endogenising  $A$ . By this we can attempt to explain how  $A$  grows over time. This is used to help the shortcomings of the Ramsey growth model. Suppose that

$L_A = sL$ , where  $s$  is some proportion of those involved in research development roles. Here  $A$  represents the number of new developments ideas or innovations within the economy. Ideas and new innovations are non-rivalrous. Everyone can benefit from them unlike capital.

$$\dot{A} = sAL \quad (137)$$

So the level of technological growth is dependent on the actual level of technology and the number of those employed in research segment.

Consider the nature of  $s$ , the larger the labour force, the greater the chance that an individual comes out with some *breakthrough* for  $A$ . Furthermore, growth in innovations are much more beneficial. Comparing innovation to capital per se, capital is for an individual's utilisation. Where as innovation can be shared.

$$Y = (1 - s)AL \quad (138)$$

We assume that  $(1 - s)$  and  $L$  is constant. In this setting, the growth rate of  $Y$ ,  $g_y = g_A$ . That is if the labour force and proportion of those in R&D is constant, then the growth of output will equal the level of output. Consider the trade-off, for increasing  $s$  and removing others from the labour force.

$$g_y = g_A = \gamma sL \quad (139)$$

$$\Rightarrow \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} \quad (140)$$

This implies that a higher population yields higher growth per capita. This is seen on the world level of macro data. More people has led to more output per person. Furthermore, the level of economic growth is increasing in  $L$ . Empirically this is a very strong result. Welfare is increasing in population size is known as the **scale effect**. Central to this idea is that ideas are non-rivalrous.

## 8.2 Malthusian Model

This model explains growth in the very long run. A model which explains an awful amount of world economic history. Consider the measure of human well being as GDP per capita,

$$\frac{Y}{L} = c \quad (141)$$

For most of history, changes in  $Y$  resulted in changes in  $L$ . Self inflicting constant per capita wealth. The agricultural revolution increased food production but as a result led to more births. Thus the standard of living remained constant. That is constant. Suppose that  $\uparrow Y$ . This results in an increase in  $L$ . **Demographic transition**. As people get richer they have fewer children, thus they are an inferior good. Broke free from the Malthusian trap, once  $Y/L$  becomes non constant.

The combination of the endogenous technology model and the Malthusian model, leads some economies to escape this malthusian trap. This is known as the Kremer Model

### 8.3 Kremer Model

Over the very long run, there is a relationship between population size and population growth rate. From our endogenous growth model,  $L \uparrow \Rightarrow g_L \uparrow$ . In the Malthusian model then implies that  $L$  must also go up in order for  $Y/L = c$ . We are able to have more resources, but they are expended by having to have a higher population growth rate.

### 8.4 Geographical Features and Growth Implications

Differences in growth, level of development. Reasons for the spread on the cross section today. First factor is geography - some areas are disadvantaged Second factor - institutions. Very much catch all variable - implies things such as property rights. GDP per capita by latitude Almost symmetrical across the equator. This is at least suggestive that geography is important. However it is hard to disentangle these two stories.

Major axis of the continents. Guns growth and steel. Geographically driven study. Provocative question, why did everything start in Europe? Geographical story. Europe has a number of geographical advantages. One of the main ones was the animals could easily be domesticated. In America you had grizzly bears, Australia had kangaroos and Europe had cows and horses. This allowed them to settle down and produce food. This allowed them to consider production of goods.

The fact that the Europeans lived closely to animals, meant that they developed immunisation to a lot of diseases. Furthermore, ideas could spread across Eurasia easily. This was a scale effect such that more people could develop technologies together and these things would be relevant across Eurasia (wide main axis). This may not have been possible for the Americas (long main axis). Economies with good institutions overcoming bad geography For example Las Vegas is one of Americas fastest growing states.

Institution examples of east and west German and north and south korea. Where countries have very similar geographies but vastly different institutions.

Distributions of power can be pluralistic (More diffused) or absolute (North Korea). We need some degree of centralisation in order to enforce property rights. Inclusive institutions are ones who encourage involvement to economic activity allowing people to get richer. (You can keep most fruits of your labour)

Institutions are very persistent. For example, a good institution allowing everyone to get richer, people then gain more power and it is hard to shut down. Extractive institutions (North Korea). Tension of how much to be a dictator. If you allow them to have too much wealth, they could overthrow the dictatorship.

Some degree of complimentary amongst geography and institutions. Europeans set up bad extractive institutions in areas they did not wish to settle. Tropical diseases disincentivised Europeans to settle good institutions in these locations. Bad institutions are also persistent. <sup>18</sup>

---

<sup>18</sup>Jared Diamond explains a long run theory of development up until around the 1600s. China and Europe on the same axis, good ideas spread laterally. Africa along the longitudinal axis. Good ideas didn't necessarily work nearer the equator. Europeans got guns from China through the diffusion of good ideas. Geography of Europe provided a good climate for an agricultural revolution and population density rose. They had the right animals as opposed to giraffes and tigers. They had physical resources and set up inclusive institutions

## 9 Issues Related to the Labour Market

Broad trends in the labour market. Rising college wage premium. There is an increased demand for those who have gone to college. Skill based economy, complementarity for technology change and skill. Arbitrage argument for supply of college students and the demand for college graduates has not held yet. Supply has failed to increase. Education system has not caught up in supply. High skilled manufacturing wages have fallen due to technology and globalisation. Theory - trade brings down non skilled labour. One theory, Low skill wages have not really risen that much. Why have wages at the very top up gone up more relative to the lower wage levels. Big inequality within high skilled people.

- **Superstar theory** - As the economy gets richer over time, people want *the best* doctor/lawyer etc... Thus the wages for the very top will go up a lot. Is the pay of CEO's justified by economic theory? Firms are much bigger and multinational, so the value of having such great CEOs is much bigger so they are more in demand.

### 9.1 Search theory and the Beveridge Curve

Brings in heterogeneity into the labour market. We normally assume that everyone is the same etc... This is obviously not the case. What implications does this have for the equilibrium?

Let the number of job matches, those unemployed who find jobs be  $M$

$$M = xU^\alpha V^{1-\alpha} = x(uL)^\alpha (vL)^{1-\alpha} \quad (142)$$

Where  $U$  is the number of unemployed and  $V$  is the number of vacancies.  $x$  is some parameter for the technology of the matching.  $v$  and  $u$  is the proportion of the total labour participation  $L$  vacancies and unemployed. Hence  $vL = V$  and  $uL = U$ . Note that the number employed is  $(1 - u)L$ .

- The **job finding rate** is  $M/U$ , the fraction of the the unemployed who found a job.

$$\frac{M}{U} = \frac{xLu^\alpha v^{1-\alpha}}{uL} = x\left(\frac{v}{u}\right)^{1-\alpha} = xa(\theta) \quad (143)$$

$$a'(\theta) = (1 - \alpha)x\theta^{-\alpha} > 0 \quad (144)$$

The inverse of the job finding rate is the time it takes to find a job.

- The **job filling rate** is  $M/V$ , the number of vacancies filled

$$\frac{M}{V} = \frac{xLu^\alpha v^{1-\alpha}}{vL} = x\left(\frac{u}{v}\right)^\alpha = x\left(\frac{v}{u}\right)^{-\alpha} = xq(\theta) \quad (145)$$

$$q'(\theta) = -\alpha x\theta^{-\alpha-1} < 0 \quad (146)$$

---

where they wanted to settle. This enabled property rights. These institutions were persistent and allowed the accumulation of wealth. Domestication of animals meant that they spread germs wherever they went (smallpox killing 90% of Native Americans). This caused a scale effect.



Where  $\theta = \frac{v}{u}$  is a measure of the *tightness* of the labour market, the number of vacancies relative to those unemployed. The higher is  $\theta$ , the less there are workers currently not in employment available for the vacancies. The job finding rate is increasing in  $\theta$ . The more there are vacancies available, then more unemployed will find jobs but the slower they are found.

This make sense, as not everyone is appropriate for a given job, so if there are relatively less people per vacancy, the slower matches will occur.

Jobs are destroyed at a rate  $\phi$ . So the change of unemployment is

$$\Delta U = \underbrace{\phi(1-u)L}_{\text{Lost Jobs}} - \underbrace{xa(\theta)uL}_{\text{Found Jobs}} \quad (147)$$

At equilibrium,  $\dot{U} = 0$

$$\Rightarrow \phi(1-u)L = xa(\theta)uL \Rightarrow u = \frac{\phi}{\phi + xa(\theta)} \quad (148)$$

From this, we see an increase in  $\theta$  (an increase in vacancies), implies an increase in the job finding rate and a decrease in unemployment. This implies a high vacancy rate is associated with a low unemployment rate. This traces out the Beveridge curve. A downward sloping curve with  $v$  on the  $y$ -axis and  $u$  on the  $x$ -axis, defined such that the number of unemployed is unchanging  $\dot{U} = 0$ . An increasing in matching efficiency  $x$  causes an inward shift in the Beveridge curve. For a given number of unemployed, there are less

### 9.1.1 Recent Movements in the Beveridge Curve

In recent years, the Beveridge curve has shifted outwards, reflecting some dysfunction in the labour market. This section proposes some reasons for this

- **Housing:** With households in negative equity, it is less easy for them to sell their homes and move to where jobs are. As a result, there is more unemployed for a given number of vacancies.
- **Uncertainty:** Faced with economic uncertainty, firms may post vacancies but be selective in who they decide to take on. Only hiring those who add the most value to the firm. This creates more vacancies for a given number of unemployed.
- **Welfare Benefits:** These raise workers reservation wages and reduce the intensity of the job search. This reduces workers incentive to accept jobs and as such for any given vacancy rate there is higher unemployment and the curve shifts outwards.
- **Skills Mismatch:** Construction workers losing their jobs meant that they did not have the skills necessary to accept or apply for job vacancies. This results in a higher level of unemployment for a given vacancy rate.

## 9.2 Stagnant Real Compensation in Lower Skill Labour markets

The following are our explanations of a fall in the participation rate of low skilled male workers over the past number of decades.

- Indirect effects of trade on the wages of low skilled workers. Same argument applies for high skilled worker imports. - **Stolper Samuelson Theorem**
- Skill Biased Technological change - Internet and technology increasing market for superstar workers. Technological advancements are displacing unskilled workers and benefiting skilled workers.
- Household Labour supply - Women joining the workforce causing an increase in household income leading to an income effect to men.
- Immigration - Globalisation is increasing the supply of cheap labour force. Furthermore, outsourcing work to places of cheap labour such as back end tasks in India and manufacturing in China.
- Disability benefits have increased markedly.

These forces have caused a decrease in the real wage and as such have made the non participation condition

$$wu'(c) < v'(0) \quad (149)$$

These combined have decreased  $L$  in the production function. Which is a primary reason as to why rates are so low currently. Many economists believe this to be a new norm in the economy. These real forces have policy implications. It is now less likely that Donald Trump can combat this with policies.

### 9.2.1 Evolution of Household Income

#### Examining The Change of Income at Different Tax Brackets

Log-normalised to zero of wage for different income brackets. Higher brackets increases much more than lower. Top 10% vs 1% and 10%, clearly there is a lot of upward pressure at the very top.

Examining the green line, it is a downward sloping line. A high vacancy rate is associated with low unemployment. Moving along this reflects changes to the business cycle. The north-east quadrant was best to be in. Shift outwards in the Beveridge curve during the financial crisis. Suggests a shift in something. Time series data on matching data

The fall in the participation rate of prime aged men on the lower end of the spectrum can be attributed to obvious reasons. There is a negative substitution effect from falling wages in the high skilled manufacturing segment and manual labour segment. In general, technological advancements such as increased automation are reducing wages.

Two sources of income in this segment. Disability payrolls have gone way up inducing an income effect reducing the labour supply. Disability payroll has increased from 1.5 million

Americans in the 1970s to 8.9 million in 2013. What is puzzling here is that the general health of Americans has gone way up through out this period<sup>19</sup>.

The increased inclusion of spouses in the labour supply has resulted in a household income effect. This reduces the labour supply.

Another item which may explain the large fall in the labour market is attributed to the large decline in dollar per unit of entertainment. This invokes an income effect. Leisure is now relatively cheaper so the individual will work less.

Fall in labour mobility, all else constant would increase series. One possible test for this Search Theory - Matching buyers and sellers. Good example ua

---

<sup>19</sup>Autor and Duggan (1993)

# A Appendix

## A.1 Tutorial 1

1. Total lifetime income:

$$\frac{NY}{T} = C \quad (150)$$

$$Y \frac{NY}{T} = Y(1 - N/T) \quad (151)$$

$$(152)$$

- 2.

$$u'(c_t) = E_t \beta (1 + r) u'(c_{t+1}) \quad (153)$$

$$u'(c_t) = E u'(c_t) \quad (154)$$

3. Specify two different utility functions, model naturally smooths utility
4. An inpersistent change in income will lead to only a small change in consumption. Temporary changes in income will cause a large increase in income inequality but will not be reflected as much in the consumption inequality.
5. The more wealth, the less precautionary savings. A fall in wealth will lead to an increase in precautionary savings.
6. FALSE, there is more of a need to precautionary savings in america due to the quality of the social welfare system in Europe. In china there is lots of savings. This could be because of the relatively underdeveloped welfare system.
7. Household income becomes more stable, less of a need for precautionary savings. Divorce will lead to a high unexpected cost, requiring precautionary savings.
8. Ambiguous result
9. Benefits  $\Rightarrow$  disincentive to work  $\Rightarrow$  exacerbated recession
- 10.

$$Y_{t+1} = y_t + \epsilon \quad (155)$$

$$y_1 = y_0 + \epsilon \quad (156)$$

$$y_2 = y_1 + \epsilon_t \quad (157)$$

$$(158)$$

We can see that a change in income will cause a permanent change in income. This will lead to consumption increasing a lot from a change in income.

11.  $\beta \uparrow, \gamma \downarrow, \sigma \downarrow, \theta \downarrow$

12. This can illustrate that people in fact do not obide by PIH. However, it is a result of a economig growth, leading to higher incomes if those who are earning. They smooth their consumption by saving for the future but the people who are old dont have much to spend. However they have smoothed consumption.
13. Prescott revenue given back. Here rev is not returend. Income and substitution effect  $\theta = 3 \Rightarrow$  Income effect dominates  $\Rightarrow$  Labour supply  $\uparrow$ .
14.
  - 20
  - i20
  - It does not change
  -

## A.2 Tutorial 2

1. Expectations are backwards looking. Taxes in each period will equal government expenditure. If government expenditure rises permanently in period 2 to 4. Then in that period, there will be a budget deficit of 2. Then in the next period,  $E_2t_3 = 6$ . If the movement was temporary, then taxes will ve 3 in each period. The optimal way to have the taxes is to have them all equal in each period as this will minimise distortion, this is why we would expect the deficit correction smoothed over remaining periods.

$$t_1 + E_1t_2 + E_1t_3 = G_1 + E_1g_2 + E_1g_3 \quad (159)$$

For a temporary change, there will be a budget deficit, in the third period there will be a surplus, the next balance will be balanced.

2. In the high  $\beta$  economy, people will value future consumption relatively more than current consumption. Therefore, they will be more likely to save for future consumption. Thus there will be a larger pool of savings in economy A. If there is a larger pool of savings, then the natural interest rate will be lower.

Note that the constraint of having closed economies, is necessary to remove the arbitrage potential of having differing interest rates internationally.

3. If the labour force participation  $L_A$  and rate of technical progress  $A_A$  of economy A is higher than economy B, we can note that  $A_AL_A^\alpha > A_BL_B^\alpha \Rightarrow Y_A > Y_B$ . There are potentially two dynamics at play here. If income levels are higher in economy A are higher than economy B, then the demand for money in A is higher. If the demand for money is higher, then the natural rate of interest is higher. If the rate of technical progress is higher in A, then expected incomes in economy A are high. People will want to borrow out of high expected incomes. Thus there will be more borrowing and a higher natural rate of interest.

In this question consider  $AF'(k) = r$ ,  $k$  must decrease in A increases. A greater labour force increase the marginal productivty of capital, thus causing a direct increase in the interest rate.

4. From the market clearing condition, we can sub in  $Y_i$  in place of  $C_i$ . Then we can rearrange the Euler equation to find

$$1 + r = \frac{u'(y_1)}{u'(y_2)} \quad (160)$$

In this world, there is no saving, so the interest rate will be set to make you happy. It will be sufficiently large such that they are just happy enough not to borrow.

This question relates to  $i_2^L = \frac{i_1 + E_2}{2}$ . In general we can say

$$i_n^L = \frac{i_1 + Ei_2 + \dots + Ei_n}{n} \quad (161)$$

Where  $i = r + \pi$  and  $I_n^L$  is the long run interest rate.

5. (a) If the government borrow more and save less, there will be an inward shift of the savings curve, thus interest rates for those time periods in the future will be higher. This will cause a steepening of the yield curve.
- (b) A permanent fall in money growth today, will cause less inflationary pressures. Lower inflation reduces the interest rate directly and will cause a shallowing of the yield curve.
- (c) Better growth prospects will result in higher future expected incomes today. This will cause an increase in the natural interest rate and a resultant steepening of the yield curve.
6. If the fed commit to keeping interest rates low for the next year and investment demand increases permanently, long run rates would increase and short run rates would remain low. This would steepen the yield curve.
7. Operation twist refers to a fed policy in 1961 seeking to raise short run rates and lower long run rates. To do so, the FED sold short run bonds and purchased long run treasuries. What happened based on a study by Modigliani in 1966 found that the term spread between long and short run treasuries narrowed moderately at best (implying did not work). If we consider that long run rates are the geometric average of all future expected short run rates, then this finding would support the pure expectations hypothesis.

This question relates to the expectation hypothesis. Consider now a literal twist in the yield curve around some point. They bought long run bonds driving up price. So  $(1 + y_2)(1 + y_2)$  falls, conversely, they sell short run bonds, the price will fall and the interest rate would go up.  $(1 + i)(1 + Ei)$  will increase. This condition is thrown out of equilibrium. Demand for long term bonds would fall. The private sector would reverse the mechanism (in theory QE doesn't work). "The problem is it works in practice but not in theory".

It is believed that the act of pursuing QE is merely a signalling mechanism.

8. This statement refers to a central bank's ability to alter the long run rates by implying future short term rates through its actions today. Not its ability to mechanically lower rates. If the Fed pursues a policy intending to lower rates by buying long run bonds, by the pure expectations hypothesis, this can be driven by falling expected future short rates. If investors expect rates to be lower in the future, then long run rates will also fall.
9. Suppose we have 100 bonds in a market, 50 treasuries and 50 AAA bonds. Both bonds are identical except for the issuer.
  - There would be no change to the interest rates if 25 more AAA replaced Treasury bonds.
  - If we consider Treasury bonds to be a collateral asset, and the same was done to the market as above. Then the supply of highly valued collateral bonds will increase. The return on the bonds would therefore be lower and the rate of interest would be lower.

### A.3 Tutorial 3

- The active potential output was thought to be 100, but in reality it is at 50 as a result of a slowdown. Suppose that GDP today is 80. What will the monetary policy do? They effectively think that they are in a recession. They as a result, reduce rates. So reducing interest rates during when output is above potential, there will be inflation.
- As a result of removing the illiquidity in the economy, there is more risk bearing capacity in the economy. This reduces the expected/required returns. Alternatively, more people buying illiquid assets, results in an increase in price and as such, lower yields.
- Long run rates will increase more in B. Recall the relationship between interest rates and bond prices. You know that they follow a Taylor rule, so a CB will keep inflation under control. So inflation risk premium will go up more in B.
- This essentially Ben Bernanke saying that the act of purchasing increases the price, reducing the yield. After markets priced this information, there was positive outlook on growth and as a result, prices of bonds went down and yields went up. This means if you think QE will work you expect booming economy down the road.
- Lower short rates in the future bring down the component of future expected short rates in equation (38). This is a sign of bad times ahead and the CB should pursue expansionary policies. Better Monetary policy should bring down the  $\rho$  term in this equation. Less economic risk would also bring down  $\rho$  in this economy. If long rates are going down because of reducing  $\rho$  (term premium), hence reducing the risk in the economy is in itself expansionary. So if  $\rho$  is falling, they should pursue a contractionary policy.

- No it is definitely not consistent with pure expectations hypothesis. With PEH, the difference in price would be arbitrated away. In this economy, the 10 year bonds are more attractive for collateral. This is a convention.
- If long run rates fall below short run rates, it indicates that expected short run rates are going to fall in the future. This was often a signal of impending recession. If QE takes place, it does not increase the term spread. QE would bring down the  $\rho$  component, reducing the term spread.
- Interest rate smoothing refers to central banks not jacking up interest rates to target in one go, but incrementally. This is because there is economic uncertainty and also not to shock the bond market and maintain stability.
- The difference in yields represents the breakeven inflation. Suppose that the TIP return is  $r^* + \rho$  and the treasury is  $i = r^* + \pi^e$ , now the difference is  $\pi^e - \rho$ , this underestimates expected inflation. Where  $\rho$  is some risk premium, generally liquidity premium. So the difference in yields is an inaccurate measure of inflation.

•

$$P_i = \frac{\eta}{1 - \eta} MC \quad (162)$$

- Recall from Micro that a monopolist has a downward sloping demand curve. There will be an optimal price ( $P^*, Q^*$ ). This will depend on the marginal productivity etc. At a point  $P > MC$ . If the firm choose to sell more, they must reduce the price on every product. If we had the opportunity to sell additional units at a constant price, they will be willing to do so.
- The real wage,  $w/p_i = \frac{\eta-1}{\eta} < 1$  Thus workers are paid less than their marginal costs. The real wage increases the wage, but the firm acts second, so they can act accordingly. So the change in  $w/p$  is zero. Infact the real wage stays at  $\frac{\eta-1}{\eta}$ .
- The mark up must fall.

## A.4 Tutorial 4

- A nominal anchor convinces people in the economy that inflation will be kept under control. An example is a fixed exchange rate. Thus if a central bank prints money they break the peg. Suppose the price level target is 100, then the price level falls to 50. With a price level target, the central banks mandate is to return the price level target. Therefore inflation expectations would increase. Noting that

$$r = i - \pi^e \quad (163)$$

Assuming that  $i$  does not go up, the ex-ante real interest rate would go down.

- Deflation is not conjusive to economic growth. If we take the stock of debt, and find the real value, following deflation the cost of servicing the debt will increase. In the case of creditors, they will gain so on aggregate it is ambiguous as to the net loss. So it is not clear as to why deflation is bad for the economy.



- Suppose the philips curve was

$$\pi_t = \pi_{t-1} + \beta(y_t - y) \quad (164)$$

In the forward looking variety of above, you can control/manipulate expectations. However in the case examined here, inflation today depends on inflation yesterday. So if it is high, the only channel would be through the output gap. Therefore it is much easier to control inflation through a forward looking philips curve. In the Volcker case, we did not see a massive output gap but he brought down inflation.

- The new keynesian philips curve

$$\pi_{t+1} = \pi_{t+1}^e + \beta(y_t - \bar{y}) \quad (165)$$

Inflationary expectations were not stable, despite the large negative output gap.

- The degree of nominal rigidity, can be encapsulated by nominal and real rigidity. The more rigidity there is, the lower  $\beta$  will be. Hence, the more rigidity there is, the less sensitive the economy is to deviations in the output gap.
  - A higher  $\sigma$  results in a steeper labour supply curve, so wages react more to increases in labour demand. This puts upward pressure on marginal costs so there will be more inflation which implies in a higher  $\beta$ .
  - There is more flexibility in prices as a result of lower rigidity. This means that money is more non neutral so monetary policy is less effective.
  - Higher sigma  $\rightarrow$  higher  $\beta$
- Nominal rigidity examines how much people change prices. If there is higher nominal rigidity, the  $\beta$  will be lower. In the new Keynesian model agents face a constant probability of changing prices ( $\delta$ , not structural).
- This formulation of output does not have constant returns to scale. In this case, costs will increase faster. So there is less nominal rigidity. Thus, the  $\beta$  will be larger.
- In a boom, asset prices go up so people have more collateral so the cost of credit goes down. This reduces marginal costs.
- Suppose that  $Y_i = A(Y)L$  Then in a boom,  $A$  increases. This reduces the upward pressure on marginal costs. This increases real rigidity and makes monetary policy more effective. In a boom, output per person goes down - productivity is countercyclical.
- - Volcker disinflation there was a big increase in  $i$ . Volcker essentially increases  $\gamma_\pi$ , suggesting a big dislike of inflation. As a result, the interest rate increases. Then as a result of this,  $y_t$  does down. Finally, as a result of the increase in the output gap (negative), the level of  $\pi$  goes down.
  - We have a falling  $i$ . This causes  $y_t$  to go up. This increases the output gap and causes increasing inflation. Then expectations of inflation in the output gap starts to go up. This

## A.5 Tutorial 5: Exam Corrections

### Section B

#### Question 1

1. Recall the optimal labour condition  $wu'(c) = v'(l)$ .

$$u(c) = n \log(c) \quad (166)$$

$$v(l) = l^2 \quad (167)$$

$$w \frac{n}{c} \quad (168)$$

$$l = \sqrt{\frac{n}{2}} \quad (169)$$

First thing to note, is that it is independent of the wage. It is increasing in the number of consumer goods available and the substitution effect and income effect cancels out.

2. The differential of labour supply,  $n$  has grown in the US. It is less obvious that there has been a fall in the labour supply in France and Germany. This could be explained in the difference of the non trade sector. With the big welfare state diminishes the amount of available consumer services in

#### Question 2

1. Recall

$$I_n = \frac{\sum_{i=1}^n r_1 + \pi_1}{n} + \rho = \frac{\sum_{i=1}^n r_1}{n} = \frac{\sum \pi}{n} + \rho \quad (170)$$

This equates to  $I_n = 1.5\% + 2.8\% + 0.4\%$ , the treasury yield. In the TIP yield part, in practice the  $\rho$  term is only really for the nominal yield term. 1.5 or 1.9 would be fine here. Also note there would be liquidity premium.

An estimate for the marginal productivity of capital is the real interest rate. Go back to the loanable funds model. This is a theoretical result.

2. Anything that causes a fall in the long rate. QE will bring down the long rates. A decline in inflation risk. Anything related to  $\rho$ .
3. Over time the real rate has trended downwards. A fall in  $A$  makes capital less useful which pushes down the real interest rate.
4. There are two points to note here. A fall in expected inflation reduces actual inflation. As inflation is trending downwards, people start adjusting their prices less. In the long run, expected inflation and the nominal rate will move together. Rest of marks for business cycle/rigidity story.

## Section C

1. Anything above 1000 is taxed at 50%. This rate goes down to 5%. This is a pure substitution effect. The reward to work has gone up and should increase labour supply. Government revenues have to go up. Societal welfare will go up. They are voluntarily working more. It is a pareto improvement.
2. There is a 10% increase in the wage of the poor. When  $\theta$  is between 0 and 1, there is a pure substitution effect. Overall labour supply goes up.
3. The implication of all this is that there is an increase in labour supply. This creates real rigidity.

### A.5.1 MCQs

1. F
2. B
3. C
4. E
5. C
6. C
7. E
8. A
9. A
10. E
11. D
12. E
13. A
14. D
15. D
16. D

## A.6 Tutorial 6

1. Marginal productivity of capital increases which raises  $r$ .
2. I takes the burden of adjustment as it is more volatile. If consumption goes up it is only due to the PIH that it will increase only a tiny little bit.
3. Recall the labour optimality condition

$$\frac{l_{t+1}}{l_t} = \left( \frac{w_{t+1}}{w_t} \right)^{\frac{1}{\sigma}} \quad (171)$$

As a result of the increase in taxes overtime, wages today are higher than in the future. So if wages increase today, labour tomorrow must fall or labour today must rise.

4. No it does not, it is often charactered in that way. It is a bad event but everyone is reacting optimally to the bad event. Each equilibrium is pareto optimal, hence there is no need for stabilisation policy.
5. Wages are higher in a boom so labour supply is higher during a boom. If the government was systematically lowering taxes in a boom and raising in a recession. This would amplify the effects of the business cycle.
6. Suppose that  $U(c) = \log(c)$ . Suppose you can work up to 12 hours a day. There is no disutility in the labour supply. Thus they will work 12 hours a day.  $Y$  will not be accentuated by movements in labour, the interest rate won't go up. In this mode the MPK goes up when  $L$  goes up. But here  $L$  is fixed so interest rates would be less cyclical. The amplifications in this model would be very small.  $Y$  will be essentially be tracking  $A$ .
7. i) Wages rise in a boom, they are arguably too procyclical. In the data they are only mildly procyclical. How do we rectify this? Can we throw something else in to the model to patch it up. If we assume government expenditure goes up, this would have a negative income effect. This would cause consumption to go down and labour to go down. The increase in labour supply would resultantly bring the wage down. So this feature would help us fit the model to the data.  
 ii) People who were initially unemployed will join the labour supply. In the RBC model (composition effect), there is no extensive margin. When people join on the extensive margin, they come in and bring down the empirical average wage in the economy. Both these points try to mitigate the problem of procyclical wages.
- 8.

$$U = \log(c_1) + \log(c_2) - \gamma(L_1 + L_2) \quad (172)$$

$$C_1 + C_2 = \underbrace{AL_1}_{\text{Output in P1}} - \underbrace{AL_2}_{\text{Output in P2}} \quad (173)$$

Labour supply is linear, you come up with some laboru suppply plan these represent output in period . IfWith the linearity, we are indifferent to supplying labour supply in either period. If we are more productive in period 1, then we will choose to work when we are more productive as wages are higher.

This is an extreme case of intertemporal substitution of labour. With Consumption, the log makes it so that we will want to smooth consumption.

9. The production function, ommitting capital  $Y = AL$ . What happens if  $A$  jumps up for one period. Then as there is no investment, consumption will go up one for one. Consider  $u'(c)w = v'(l)$ . The wage  $dy/dl = A$ , this implied  $A/AL = KL^\sigma$ . Thus labour supply is constant
10.  $(r_{t+1}, s_t, I_t, L_t^s) \uparrow$ .
11. A rise in uncertainty would cause savings to go up and consumptpion to go down. Referring back to the labour optimality condition, the labour supply would increase as a result in fall of consumption. Output would go up as a result of labour supply increase. Furthermore, investment would counterfactually go up.
12.  $\beta \downarrow$ , a preference shock. This means that we care less about the future. Consumption will go up as we prefer consuming today. As we care about our utility today more than tomorrow thus we will not work as much today. As a result of the fall in labour supply output will fall. The wage will go down as a result of the inward shift in the labour supply. Investment goes down as  $y$  went down and  $c$  went up. As a result of the increase in consumption, savings fall and as a result  $r$  *could* go up. however what is also at bay is the fall in  $L$  causing MPK to fall and as a result,  $r$  will fall.
13.  $Y = A(uK)^\alpha L^{1-\alpha}$ . If we go to the data, and we try parse out  $A$ , we will actually find  $Au^\alpha$ . This is distorted measure of  $A$ .  $s$

## A.7 Tutorial 7

- 1.