Imagine a car-like agent wishes to exit a maze like the one shown below:

The agent is directional and at all times faces some direction $d \in (N, S, E, W)$. With a single action, the agent can either move forward at an adjustable velocity $v$ or turn. The turning actions are left and right, which change the agent’s direction by 90 degrees. Turning is only permitted when the velocity is zero (and leaves it at zero). The moving actions are fast and slow. Fast increments the velocity by 1 and slow decrements the velocity by 1; in both cases the agent then moves a number of squares equal to its NEW adjusted velocity. Any action that would result in a collision with a wall crashes the agent and is illegal. Any action that would reduce $v$ below 0 or above a maximum speed $V_{\text{max}}$ is also illegal. The agent’s goal is to find a plan which parks it (stationary) on the exit square using as few actions (time steps) as possible.

As an example: if the agent shown were initially stationary, it might first turn to the east using (right), then move one square east using fast, then two more squares east using fast again. The agent will of course have to slow to turn.

1. If the grid is $M$ by $N$, what is the size of the state space? Justify your answer. You should assume that all configurations are reachable from the start state.

2. Is the Manhattan distance from the agent’s location to the exit’s location admissible? Why or why not?

3. State and justify a non-trivial admissible heuristic for this problem which is not the Manhattan distance to the exit.
2 Admissible and Consistent Heuristics

In this question we’ll explore the relationship between admissible and consistent heuristics.

Recall the definitions for admissible and consistent heuristics. Let \( s' \) denote the successor of \( s \), and \( \text{FutureCost}(s) \) denote the minimum cost from \( s \) to the goal.

- **Admissible**: \( h(s) \leq \text{FutureCost}(s) \).
- **Consistent**: \( h(s) - h(s') \leq \text{Cost}(s, s') \) and \( h(\text{Goal}) = 0 \).

1. If we used an inadmissible heuristic in A* graph search, would the search be complete? Would it be optimal?

2. If we used an *admissible* heuristic in A* graph search, is it guaranteed to return an optimal solution?

3. Give a general advantage that an inadmissible heuristic might have over an admissible one.

4. Show that A* search always find the optimal solution when using a consistent heuristic.

5. Consistency is a stronger condition than admissibility. Show that every consistent heuristic is admissible. (*Note that this is not true the other way around: there are admissible heuristics that are not consistent*)