Q1. Probabilities

(a) Fill in the circles of all expressions that are equal to 1, given no independence assumptions:

- $\sum_a P(A = a \mid B)$
- $\sum_b P(A \mid B = b)$
- $\sum_a \sum_b P(A = a, B = b)$

(b) Fill in the circles of all expressions that are equal to $P(A, B, C)$, given no independence assumptions:

- $P(A \mid B, C) P(B \mid C) P(C)$
- $P(C \mid A, B) P(A, B)$
- $P(A \mid B) P(B \mid C) P(C)$
- $P(A \mid B, C) P(B \mid A, C) P(C \mid A, B)$

(c) Fill in the circles of all expressions that are equal to $P(A \mid B, C)$, given no independence assumptions:

- $\frac{P(A, B, C)}{\sum_a P(A = a, B, C)}$
- $\frac{P(B, C \mid A) P(A)}{P(B, C)}$
- $\frac{P(A, C \mid B) P(A, C)}{P(B, C)}$

(d) Fill in the circles of all expressions that are equal to $P(A \mid B)$, given that $A \perp B \mid C$:

- $\frac{P(A) P(B \mid C)}{P(B)}$
- $\frac{P(A) P(B \mid C)}{P(A \mid C)}$
- $\frac{\sum_c P(B, A, C = c) P(A, C = c)}{\sum_{c'} P(A, B, C = c')}$

(e) Fill in the circles of all expressions that are equal to $P(A, B, C)$, given that $A \perp B \mid C$ and $A \perp C$:

- $P(A) P(B) P(C)$
- $P(A \mid C) P(B \mid C) P(A, B)$
- $P(A \mid C) P(B \mid C) P(C)$
- $P(A \mid C) P(B \mid C)$
Q2. Bayes Nets: Independence

Consider a Bayes Net with the following graph:

Which of the following are guaranteed to be true without making any additional conditional independence assumptions, other than those implied by the graph? (Mark all true statements)

- $P(A \mid C, E) = P(A \mid C)$
- $P(A, E \mid G) = P(A \mid G) \times P(E \mid G)$
- $P(A \mid B = b) = P(A)$
- $P(A \mid B, G) = P(A \mid G)$
- $P(E, G \mid D) = P(E \mid D) \times P(G \mid D)$
- $P(A, B \mid F) = P(A \mid F) \times P(B \mid F)$