Search Continued

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[These slides adapted from Dan Klein and Pieter Abbeel; ai.berkeley.edu]
Recap: Search
Search problem:

- States (abstraction of the world)
- Actions (and costs)
- Successor function (world dynamics): \( s' \mid s, a \Rightarrow s' \)
- Start state and goal test
Example: Tree Search
Depth-First Search
Depth-First Search

Strategy: expand a deepest node first
Implementation: Fringe is a LIFO stack
Search Algorithm Properties
Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

Cartoon of search tree:
- b is the branching factor
- m is the maximum depth
- Solutions at various depths

Number of nodes in entire tree?
- $1 + b + b^2 + \ldots + b^m = O(b^m)$
Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If $m$ is finite, takes time $O(b^m)$

- How much space does the fringe take?
  - Only has siblings on path to root, so $O(bm)$

- Is it complete?
  - $m$ could be infinite, so only if we prevent cycles (more later)

- Is it optimal?
  - No, it finds the “leftmost” solution, regardless of depth or cost
Breadth-First Search
Breadth-First Search

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue
Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be $s$
  - Search takes time $O(b^s)$

- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^s)$

- Is it complete?
  - $s$ must be finite if a solution exists, so yes!

- Is it optimal?
  - Only if costs are all 1 (more on costs later)

- Diagram:
  - $s$ tiers
  - 1 node
  - $b$ nodes
  - $b^2$ nodes
  - $b^s$ nodes
  - $b^n$ nodes
Quiz: DFS vs BFS
DFS vs BFS

- When will BFS outperform DFS?
- When will DFS outperform BFS?
Video of Demo Maze Water DFS/BFS (part 1)
Video of Demo Maze Water DFS/BFS (part 2)
Iterative Deepening

- Idea: get DFS’s space advantage with BFS’s time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..

- Isn’t that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!
BFS finds the shortest path in terms of number of actions. It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.
Uniform Cost Search
Uniform Cost Search

Strategy: expand a cheapest node first:
Fringe is a priority queue (priority: cumulative cost)
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs $C^*$ and arcs cost at least $\varepsilon$, then the “effective depth” is roughly $C^*/\varepsilon$
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)

- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$

- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!

- Is it optimal?
  - Yes! (Proof next lecture via A*)
Uniform Cost Issues

- Remember: UCS explores increasing cost contours

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location
Video of Demo Empty UCS
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)
Up next: Informed Search

- Uninformed Search
  - DFS
  - BFS
  - UCS

- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search
  - Graph Search
Search Heuristics

- **A heuristic is:**
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Pathing?
  - Examples: Manhattan distance, Euclidean distance for pathing
Example: Heuristic Function

$h(x)$
Greedy Search
Greedy Search

- Expand the node that seems closest...

- Is it optimal?
  - No. Resulting path to Bucharest is not the shortest!
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state

- A common case:
  - Best-first takes you straight to the (wrong) goal

- Worst-case: like a badly-guided DFS
Video of Demo Contours Greedy (Empty)
Video of Demo Contours Greedy (Pacman Small Maze)
A* Search
A* Search
Combining UCS and Greedy

- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Greedy** orders by goal proximity, or *forward cost* $h(n)$

- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

Example: Teg Grenager
When should A* terminate?

- Should we stop when we enqueue a goal?
  - No: only stop when we dequeue a goal
Is A* Optimal?

What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!
Admissible Heuristics
Idea: Admissibility

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe.

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs.
Admissible Heuristics

- A heuristic $h$ is *admissible* (optimistic) if:
  \[ 0 \leq h(n) \leq h^*(n) \]

  where $h^*(n)$ is the true cost to a nearest goal

- Examples:

- Coming up with admissible heuristics is most of what’s involved in using A* in practice.
Optimality of A* Tree Search
Assume:
- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:
- A will exit the fringe before B
Optimality of A* Tree Search: Blocking

Proof:

0 Imagine B is on the fringe
0 Some ancestor n of A is on the fringe, too (maybe A!)
0 Claim: n will be expanded before B
   1. \( f(n) \) is less or equal to \( f(A) \)

\[
\begin{align*}
f(n) &= g(n) + h(n) & \text{Definition of f-cost} \\
f(n) &\leq g(A) & \text{Admissibility of h} \\
g(A) &= f(A) & h = 0 \text{ at a goal}
\end{align*}
\]
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
  1. \( f(n) \) is less or equal to \( f(A) \)
  2. \( f(A) \) is less than \( f(B) \)

\[
\begin{align*}
g(A) &< g(B) \quad \text{B is suboptimal} \\
f(A) &< f(B) \quad h = 0 \text{ at a goal}
\end{align*}
\]
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor \( n \) of A is on the fringe, too (maybe A!)
- Claim: \( n \) will be expanded before B
  1. \( f(n) \) is less or equal to \( f(A) \)
  2. \( f(A) \) is less than \( f(B) \)
  3. \( n \) expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal
Properties of A*
Properties of A*

Uniform-Cost

A*
UCS vs A* Contours

0 Uniform-cost expands equally in all “directions”

0 A* expands mainly toward the goal, but does hedge its bets to ensure optimality

[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]
Video of Demo Contours (Empty) -- UCS
Video of Demo Contours (Empty) -- Greedy
Video of Demo Contours (Empty) – A*
Video of Demo Contours (Pacman Small Maze) – A*
Comparison

Greedy

Uniform Cost

A*
A* Applications
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...

[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]
Video of Demo Pacman (Tiny Maze) – UCS / A*
Video of Demo Empty Water Shallow/Deep – Guess Algorithm
Creating Heuristics

YOU GOT

HEURISTIC UPGRADE!
Creating Admissible Heuristics

0 Most of the work in solving hard search problems optimally is in coming up with admissible heuristics.

0 Often, admissible heuristics are solutions to relaxed problems, where new actions are available.

0 Inadmissible heuristics are often useful too.
Example: 8 Puzzle

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

Admissible heuristics?
8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h(\text{start}) = 8$
- This is a relaxed-problem heuristic

Statistics from Andrew Moore
What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

Total *Manhattan* distance

Why is it admissible?

$h(\text{start}) = 3 + 1 + 2 + ... = 18$

<table>
<thead>
<tr>
<th></th>
<th>Average nodes expanded when the optimal path has...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>...4 steps</td>
</tr>
<tr>
<td>TILES</td>
<td>13</td>
</tr>
<tr>
<td>MANHATTAN</td>
<td>12</td>
</tr>
</tbody>
</table>
How about using the *actual cost* as a heuristic?
- Would it be admissible?
- Would we save on nodes expanded?
- What’s wrong with it?

With A*: a trade-off between quality of estimate and work per node
- As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself
Graph Search
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.
In BFS, for example, we shouldn’t bother expanding the circled nodes (why?)
Graph Search

- Idea: never expand a state twice

- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but...
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

- Important: store the closed set as a set, not a list

- Can graph search wreck completeness? Why/why not?

- How about optimality?
A* Graph Search Gone Wrong?

State space graph

- S
- B (h=1)
- C (h=1)
- A (h=4)
- G (h=0)

Search tree

- S (0+2)
- A (1+4)
  - C (2+1)
    - G (5+0)
  - B (1+1)
    - C (3+1)
      - G (6+0)

Closed Set: S B C A
Consistency of Heuristics

- **Main idea:** estimated heuristic costs ≤ actual costs
  - **Admissibility:** heuristic cost ≤ actual cost to goal
    \[ h(A) \leq \text{actual cost from A to G} \]
  - **Consistency:** heuristic “arc” cost ≤ actual cost for each arc
    \[ h(A) - h(C) \leq \text{cost(A to C)} \]

- **Consequences of consistency:**
  - The f value along a path never decreases
    \[ h(A) \leq \text{cost(A to C)} + h(C) \]
  - A* graph search is optimal
Optimality of A* Search

- With a admissible heuristic, Tree A* is optimal.
- With a consistent heuristic, Graph A* is optimal.
  - See slides, also video lecture from past years for details.
- With h=0, the same proofs shows that UCS is optimal.
Search Gone Wrong?
A*: Summary
A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems
function Tree-Search(problem, fringe) return a solution, or failure
    fringe ← Insert(make-node(initial-state[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, state[node]) then return node
        for child-node in EXPAND(state[node], problem) do
            fringe ← Insert(child-node, fringe)
        end
    end
function Graph-Search(problem, fringe) return a solution, or failure

closed ← an empty set
fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)

loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
        add STATE[node] to closed
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
end
All these search algorithms are the same except for fringe strategies

- Conceptually, all fringes are priority queues (i.e., collections of nodes with attached priorities)
- Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
- Can even code one implementation that takes a variable queuing object
Search and Models

- Search operates over models of the world
  - The agent doesn’t actually try all the plans out in the real world!
  - Planning is all “in simulation”
- Your search is only as good as your models…
Search Gone Wrong?