1 Neural Network Representations

You are given a number of functions which are graphed below, from a to h. For the Neural networks indicated on the following pages, indicate which of these functions they can represent.

(a) $2x$

(b) $4x - 5$

(c) $\begin{cases} 2x - 5 & x \geq 2.5 \\ 0 & x < 2.5 \end{cases}$

(d) $\begin{cases} -2x - 5 & x \leq -2.5 \\ 0 & x > -2.5 \end{cases}$

(e) $\begin{cases} -x + 3 & x \geq 2 \\ 1 & x < 2 \end{cases}$

(f) $\begin{cases} 3 & x \leq 0 \\ 3 - x & 0 < x \leq 3 \\ 0 & x > 3 \end{cases}$

(g) $\log(x)$

(h) $\begin{cases} -1.5x & x \leq 0 \\ 0 & 0 < x \leq 3 \\ 3x - 9 & x > 3 \end{cases}$
1. Consider this neural network, with input being a real number $x$, weight $w$ and output $o$, such that $o = wx$.
Which of the functions can be represented by this neural network where For the options which can, write out the appropriate value of $w$.

\[
f(x,w) = wx
\]

(i) a
(ii) b
(iii) c
(iv) d
(v) e
(vi) f
(vii) g
(viii) h

2. Now we introduce a bias term $b$ into the same neural network, such that $o = wx + b$. Which of the functions can be represented by this network? For the options which can, write out an appropriate value of $w, b$

\[
f(x,w,b) = wx + b
\]

(i) a
(ii) b
(iii) c
(iv) d
(v) e
(vi) f
(vii) g
(viii) h
3. We can introduce a non linearity into this neural network as indicated below. We use the ReLu nonlinearity, which has the form ReLu(x) = max(0, x). Now which of the functions can be represented by this neural network with weight w and bias b? For the options which can, write out an appropriate value of w, b.

\[
\text{ReLu}(x) = \max(0, x)
\]

(i) a
(ii) b
(iii) c
(iv) d
(v) e
(vi) f
(vii) g
(viii) h

4. Now we consider neural networks with a hidden layer as indicated below. We now have input still a real number x, but two sets of weights and biases \(w_1, b_1\) and \(w_2, b_2\). We define the hidden unit h such that \(h = w_1 x + b_1\), and \(o = w_2 h + b_2\). Which of the functions can be represented by this network? For the options which can, write out an appropriate value of \(w_1, w_2, b_1, b_2\).

\[
f(x, w, b) = wx + b
\]

(i) a
(ii) b
(iii) c
(iv) d
(v) e
(vi) f
(vii) g
(viii) h

3
5. Next we consider adding ReLu non-linearities to the network above as indicated below. Which of the functions can be represented by this network? For the options which can, write out an appropriate value of $w_1, w_2, b_1, b_2$

(i) $a$
(ii) $b$
(iii) $c$
(iv) $d$
(v) $e$
(vi) $f$
(vii) $g$
(viii) $h$

6. Next we consider adding another hidden layer to the network along with a ReLu non-linearity as indicated below. Which of the functions can be represented by this network?

(i) $a$
(ii) $b$
(iii) $c$
(iv) $d$
(v) $e$
(vi) $f$
(vii) $g$
(viii) $h$
7. We next consider a neural network as shown below with two units in the hidden layer. Which of the functions can be represented by this network?

(i) \( a \)
(ii) \( b \)
(iii) \( c \)
(iv) \( d \)
(v) \( e \)
(vi) \( f \)
(vii) \( g \)
(viii) \( h \)
Next we consider adding ReLu non-linearities to this network as indicated below. Which of the functions can be represented by this network?

(i) a
(ii) b
(iii) c
(iv) d
(v) e
(vi) f
(vii) g
(viii) h