

# Processing DGS AT1M marine gravity data

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## 1 AT1M Introduction sensor

The AT1M sensor is a scalar gravity /specific force sensor mounted in a 2-axis gyro stable platform that keeps the sensitive axis of the gravity meter aligned with the local gravity vector. Only the magnitude of this vector is recorded on the data file.

The main sensor basic principle is the suspension of a “Gravitational sensing mass” by a very stable and low temperature sensitivity metal spring. The small variation of local gravity is measured by measuring the current of a Lorentz force actuator which together with position detector keeps the sensing and spring at a constant length.

The instrument measures specific force acting on the sensitive axis of the instrument. During post processing all the kinematic effects need to be removed from the measurement. In the most Marine surveys, the short vertical accelerations can be neglected because the average vertical position tends to be constant after gravity data is filtered with relatively long filter times, since the sea level tends to be constant.

Therefore, the only kinematic correction applied during the processing is the Eotvos correction. The instrument has an arbitrary offset with respect to the full field gravity or the value that we will measure with an absolute gravity meter in the same location. This offset can be easily determined by occupation of location with known absolute gravity values.

The sensor has a relatively low drift rate, in the order of 2 mGal a month and below of 0.2 mGal / month after aging. This drift is very lineal and can be removed by re occupation of gravity locations during the survey.

The gravity meter is positioned perfectly aligned to local level to guaranty that the sensor measures the local full gravity vector. This continuous alignment is performed by a 2 axis the gyro stabilized platform. The platform uses only the accelerometers outputs. The mechanical gimbal together with gyros, torque motor and accelerometers behave as if the gravity meter were hanging from a pendulum with long period. This long period platform attenuates the tilt response of the gimbal and gravity meter when the ship changes velocity or heading. If the ship is traveling at constant speed in straight line or turning with a big radius, the gravity meter will be aligned with the local gravity level vector and no level correction will be necessary. Only when the ship is experiencing long period velocity or heading variations, the gyro stable platform will experience meaningful deviations from the local vertical, and in this situation the measurement axis of the gravity meter will be miss aligned with the local gravity vector. A number of corrections can be used to compensate for the unmeasured fraction of the local gravity vector.

## 2 Gravity processing equations

To obtain a useful gravity measurement that can be used by generate gravity maps and useful gravity data, the output of the sensor must be corrected for dynamical effects in the sensing mass. The sensor measures specific forces acting on the sensing mass, and any dynamic Inertial effects need to be removed from the measurement.

### **Full field gravity calculation**

The full field gravity is the gravity that we would measure if we were using an absolute gravity meter performing a static for each data point.

$$g_{FF} = g_M + gOffset + \delta Eotvos + \partial Tilt + CC + gDrift + Tidal \quad [1]$$

$g_{FF}$  = Full field gravity

$g_M$  = Output of the gravity meter. [Data file \[2\] Gravity](#)

$gOffset$  = Gravity meter offset to the Absolute gravity in the Tie.

Tie= Physical place or location with a known gravity absolute value.

$\delta Eotvos$  = Correction for the vertical components of Coriolis and centripetal accelerations

$\partial Tilt$  = Correction for platform verticality.

$CC$  = Dynamical cross coupling

$gDrift$  = Gravity meter drift compensation

$Tidal$  = Tidal gravity compensation.

### **gOffset determination**

When possible, the gravity meter is located on a location with known absolute gravity value (Tie), and then we can find the gOffset as the quantity that we have to add to the gravity meter to agree in the measurement with Tie gravity.

$$gOffset = Absolute\ gravity_{tie} - Meter\ graviti_{tie} \quad [2]$$

### **$\delta Eotvos$ correction**

During marine gravity surveys, the gravity data are acquired on a moving platform. Thus, the centrifugal force acting on the gravity sensor is different from when the mere is still. An eastward component of the velocity adds to the velocity of the rotation of the earth thus decreases the gravity. A westward component does the opposite. In addition to these components, the northward component creates a new component of centrifugal force. All these effects add together is called Eotvos correction. The formulae were given in many geophysical textbooks.

$$\delta Eotvos = 4.04 \cdot |V| \cdot \cos \phi \cdot \sin \alpha + 0.001211V^2 \quad (7) \quad V=Km/h \quad [3]$$

$$\delta Eotvos = 7.503 \cdot |V| \cdot \cos \phi \cdot \sin \alpha + 0.004154V^2 \quad (8) \quad V=Knots/h \quad [4]$$

where:  $\alpha$  and  $\phi$  are the course heading relative to north and the latitudes. In Equations. (3) and (4), the units if V is Km/h and Knots/h, respectively. The final unit of the correction for both equations is mGals.

A more complete formula more suitable for Airborne applications (8), can be use for Marine corrections by setting the ellipsoidal H to zero.

$$\delta Eotvos = \frac{v_N^2}{a} \left[ 1 - \frac{H}{a} + f [2 - 3 \sin^2(\phi)] \right] + \frac{v_E^2}{a} \left[ 1 - \frac{H}{a} - f \sin^2(\phi) \right] + 2 w V_E \cos(\phi) \quad [5]$$

The correction can be calculated using Speed, Course and Latitude stored in the data file

V= velocity Knots/h. [Data file \[17\] Speed in knots](#)

$\phi$  = north heading [Data file \[18\] Course in degrees](#)

$\emptyset$  = Latitude [Data file \[15\] Latitude in decimal degrees](#)

### **$\partial Tilt$ correction**

The tilt correction is only necessary when the ship is experiencing significative changes in velocity or heading during the normal gravity acquisition.

Different methods can be used for correcting the platform Tilts generated by the speed or heading of the ship. It is important to know that this correction due to the quadratic terms, can generated bias in the correction when the reading of the acceleration contain noise. Specially care must be taken in using a good GPS positions, for instance using INS system or differential GPS. A more extended explanation of these out of level correction techniques can be found in [1]

For the processing of AT1M data we recommend the use of one of the next two methods to determine the platform tils and out of level correction

#### *A) Modeling and simulation of platform response by an IIR digital transfer function*

For the differential equation for a inertial damped AT1M platform for a single and uncoupled axis is;

$$\ddot{\emptyset} + 2f\omega_0(\dot{\emptyset} + \frac{\ddot{x}}{g}) + \omega_0^2(\emptyset + \frac{\ddot{x}}{g}) = 0 \quad [6]$$

$\omega_0 = 1/\text{Platform period (rad/sec)}$

$f = \text{Platform damping}$

Equation [6] can be discretized using bilinear transformation or any other method. The system can then be simulated as a IIR filter with input  $\frac{\ddot{x}}{g}(k)$  and with  $\emptyset(k)$  being the platform tilt.

The platform tilts can then be found  $\emptyset_y, \emptyset_x$  and then equations 9 through 11 can be used to calculate the gravity correction.

B) *Peters and Brozena, trigonometric [1] for more details*

$$\emptyset_y = -\frac{g + \sqrt{g^2 + 2a_y(a_y - am_y)}}{a_y} \quad [7]$$

$$\emptyset_x = -\frac{g + \sqrt{g^2 + 2a_x(a_x - am_x)}}{a_x} \quad [8]$$

$$\partial_y = \frac{g\emptyset_y^2}{2} - a_y\emptyset_y \quad [9]$$

$$\partial_x = \frac{g\emptyset_x^2}{2} - a_x\emptyset_x \quad [10]$$

$$\partial\text{Tilt} = \partial_y + \partial_x \quad [11]$$

$a_x = \text{cross accelerometer (mGal)}$ . [Data file \[3\] Long accelerometer\\*1000](#)

$a_y = \text{long accelerometer (mGal)}$  . [Data file \[4\] cross accelerometer\\*1000](#)

$am_x = \text{cross calculated acceleration from GPS (mGal)}$ . [Calculated from GPS section 2 and 3](#)

$am_y = \text{long calculated acceleration from GPS (mGal)}$ . [Calculated from GPS sections 2 and 3](#)

$A_E = \text{East acceleration (mGal)}$  [Calculated from GPS, sections 2 and 3](#)

$A_N = \text{North acceleration (mGal)}$  [Calculated from GPS sections 2 and 3](#)

$g = 980000 \text{ (mGal)}$

$\partial\text{Tilt} = \text{gravity correction (mGal)}$

## CC cross coupling correction

Most of accelerometers and gravity meter present variations of the output with the motions and acceleration in other than the sensitive axis, and sensitivity to vibrations.

The DGS AT1M sensor has much smaller cross coupling than other beam-based gravity meters since the beam keeps lock in position by the electronic feedback. In any case the final quality of the output of the gravity meter can be improved. The amount of improvement of the data depends in the sea conditions, and size of the ship.

Cross correlation method for finding the cross-coupling factors AT1M shipboard gravity data.

- 1) Filter Eotvos corrected gravity and all the cross-coupling channels with at least with a filter length in the range 100 to 300 seconds
- 2) Perform a double derivation of all the channels and Eotvos corrected gravity. Since we are only solving for the curvature.

*Important: Note that all the terms used on equation [12] must be double differenced by using a Tylor expansion derivative, or any other technique.*

*Also, when applying this cross-correlation solution, be sure to include lines in all directions at least, and better with multiple directions to minimize the correlation between the channels (monitors)*

Find the gains cross coupling correction gains G by solving the curvature equations:

$$-\ddot{g} = G_1 \ddot{v}e + G_2 \ddot{v}cc + G_3 \ddot{a}l + G_4 \ddot{a}x + G_5 \ddot{T}ilt \quad [12]$$

Where g=is Eotvos corrected gravity.

File data [2] Gravity-calculated Eotvos corrections

Ve= Data file [11] monitor Ve

Vcc=Data file [12] monitor Vcc

Al=Data file [13] monitor al

Ax= Data file [14] monitor ax

Tilt= Calculated tilt correction

Once the optimum cross coupling gains have been found  $G_1 \dots G_5$ , each gain is multiplied by the respective Data file recorded monitor to compute the cross coupling correction

$$CC = G_1 ve + G_2 vcc + G_3 al + G_4 ax + G_5 Tilt \quad (13)$$

### ***drift correction***

Gravity drift is inherited by the spring-mass system. It may be caused by temperature change, elastic aging, and hysteresis of the system, etc. The drift rate is generally higher for new meters and usually gets better with time as the system matures. The drift rate can be obtained by occupation of the same special points (stations).

$$Drift = (g(t_1) - g(t_2)) / (t_2 - t_1) \quad [14]$$

Once the Drift rate is known the survey data can be corrected by adding a gravity correction proportional to t

$$gDriftcorrection = -Drift (t - t_0) \quad [15]$$

In general, the gDrift correction is very small for AT1M meters. Typical values of drift rates for a gravity a AT1M sensor are 1 to 3 mGals/month.

### ***Tidal correction***

For every position there is a temporal gravity variation which in general has small effect on Marine gravity surveys, but this variation can be corrected by using predictive models like Longmont and others.

It is the deformation caused by the moon-sun gravitational pull on the earth. The deformation can be up to about 10 cm. Tidal correction compensates for gravity effects caused by the attraction of the moon and the sun and the deformation of the earth so produced. The tidal correction causes a periodical (12 hour) gravity change which can reach a peak to peak of 300  $\mu$ Gals. For geophysical surveys the earth tide is usually not considered useful information and removed from the data. Fortunately, the earth tide can be predicted and corrected for to an accuracy of 1 to 3  $\mu$ Gals.

In the MATLAB script a Longman tide predictor is included.

## Free Air anomaly gravity

The most basic anomaly used in geologic studies Eliminates from the full field gravity the effects the latitudinal variations of gravity and elevation effects. The free air gravity anomaly is reduced to the Ellipsoid.

$$g_{FA} = g_{FF} - g_{norm} - \partial fh \quad [16]$$

The closed-form expression of the normal gravity Somigliana equation is given as:

	GRS67	GRS80	WGS84
$g_e(\text{m/sec}^2)$	9.7803184558	9.7803267715	9.7803253359
$K$	0.001931663383	0.001931851353	0.00193185265241
$e^2$	0.00669460532856	0.00669438002290	0.00669437999014

$$g_{norm} = g_e \frac{(1 + k \cdot \sin^2 \phi)}{\sqrt{1 - e^2 \cdot \sin^2 \phi}} \quad [17]$$

where  $k$  is the normal gravity constant;  $\phi$  is the latitude and  $g_e$  is the gravity at equator.

### **$\partial fh$ correction**

A standard value for calculating the gravity correction when the data is acquired over the ellipsoid is:

$$\partial fh = 0.3086 H \quad [18]$$

For the 1980 ellipsoid, the evaluation of the constants is given in Hinze et al. (2005) as

$$\partial f(h) = -(0.3087691 - 0.0004398 \sin^2 \phi) h + 7.2125 \times 10^{-8} h^2 \quad [19]$$

## Useful equations for the Marine gravity processing

### Geodetic Transformation

Using WGS84 ellipsoid parameters

$e^2 = 6.694379990141089e-003$  Eccentricity squared

$a = 6378137$  m Equatorial radius

$b = 6356752.31.41$  m Polar radius

$\phi$  = gravity meter latitude

$\lambda$  = gravity meter longitude

$f = \frac{(a-b)}{a} = 0.00335281068$  Earth flattening

$W = 7292114 e^{-11}$  rad/s Earth rotation rate

We can calculate East and North velocities in units of m/s by using from geodetic coordinates

$$V_N = \frac{a(1-e^2)\dot{\phi}}{(\sqrt{1-e^2}(\sin(\phi))^2)^3} \quad [20]$$

$$V_E = \frac{a\lambda\cos(\phi)}{\sqrt{1-e^2}(\sin(\phi))^2} \quad [21]$$

And the corresponding north and east acceleration can be calculated as

$$a_N = \dot{V}_N \quad \frac{m}{s^2}$$

$$a_E = \dot{V}_E \quad \frac{m}{s^2}$$

The North heading can be calculated as

$$\varphi = \arctg \left[ \frac{V_E}{V_N} \right] \quad [22]$$

### Projection to body frame Long and cross gravity meter axis

A rotation Matrix is used to project the North and East accelerations in the Long and cross body frame gravity meter axes

$$R_N^B = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}$$

Then

$$a_x = a_E \cos \varphi - a_N \sin \varphi \quad \text{cross acc in } \frac{m}{s^2} \quad [23]$$

$$a_y = a_E \sin \varphi + a_N \cos \varphi \quad \text{long acc in } \frac{m}{s^2} \quad [24]$$

## Filtering the Free air gravity FAA

Marine gravimetry data requires the use of low pass filtering to obtain the gravity anomalies (GA) due to the small signal to noise ratios generated by the ship motions and the fact in general the power of the noise concentrates in the higher frequencies of the measured data. A common xxx is not to use the vessel vertical position or acceleration as a correction and rely in the fact that the average vessel position must not change with time. That means that means that the output of the gravity meter signal will be mixed with a vertical acceleration signal very often of up to 100,000 mGal with periods up to 6-15 seconds.

Because the ship motions induced accelerations are of relatively higher frequency, the gravity signal can be recuperated by low pass filtering the output of the gravity meter.

Obviously, the longer filter times provide a smoother data, but also can attenuate the geological signal that we intend to recover.

For smoothing the gravity data, a L taps FIR Gaussian filter can be used, with the coefficients calculated:

$$w(n) = e^{-\frac{1}{2}\left(\alpha\frac{n}{\frac{L-1}{2}}\right)^2} = e^{-\frac{n^2}{2\sigma^2}} \quad [25]$$

Where:

$$-\frac{L-1}{2} \leq n \leq \frac{L-1}{2} \quad [26]$$

$$\sigma = \frac{L-1}{2\alpha} \quad [27]$$

We use a  $\sigma$ , and L= number of seconds to be filtered

The Gaussian window is applied using convolution with the gravity signal.

## References

[1] Mary F. Peters and Jonh M. Brozena. Methods to improve existing gravity meters for airborne gravimetry

[2] W.M. Telford, L.P. Geldard and R.E. Sheriff, 1990, Applied Geophysics 2<sup>nd</sup> edition. Cambridge University Press

[3] R.E. Sheriff, 1997, Encyclopedic Dictionary of Exploration Geophysics, 3<sup>rd</sup> edition, Society of Exploration Geophysicists

[4] R.J. Blakely, 1995, Potential theory in gravity and magnetic applications, Cambridge University Press

[5] W.E. Featherstone, 1998, A geodetic approach to gravity data reduction for geophysics, Computers & Geosciences, Vol. 23, NO. 10 1063-1070

[6] Lucien J. B. Lacoste. Cross correlation method for evaluating and correcting shipboard data, Geophysics vol 38 August 1973.

[7] L M Longman, Formulas for computing Tidal Accelerations Due to Moon and Sun, Journal of Geophysical Research vol64, n12, December 1959

[8] Raymond B Harlan 1968 "Eotvos corrections for Airborne Gravimetry", JGR 73,n14

### Note:

Dynamic Gravity Systems has available a set of MATLAB script programs to process and plot and export Free Air gravity for AT1M meters.