

Airborne Gravity Smoothing by Kalman Filter with Adaptive 2nd Order Gauss-Markov Stochastic Gravity Model

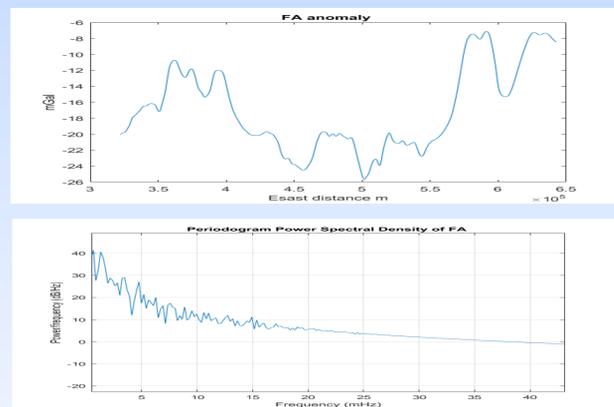
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Introduction

Airborne gravimetry requires the use of low pass filtering to obtain the gravity anomalies (GA) due to the small signal to noise ratios generated by the aircraft motions and the fact that in general the power of the various sources of noise concentrates in the higher frequencies of the measured data. In this work a Kalman smoother is designed based on a second order Gauss-Markov for free air stochastic gravity model. The optimum parameters for the Gauss-Markov gravity model are determined/tuned from the gravity meter data. The performance of the Kalman smoother is then compared with a finite impulse response (FIR) windowed exact Blackman and a Gaussian window design using simulated data.

Airborne Synthetic Data

A test gravity profile line is generated using public domain ground gravity data, from near Fort Collins CO, and upward continuing the profile line to 1000 m. The line is then resampled in order to simulate an airplane speed of 250 km/hr.



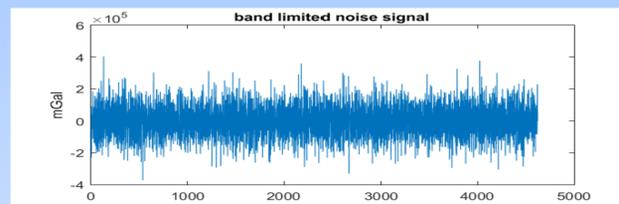
The PDS of this gravity profile line contains frequential energy in the band from DC to 100 seconds at the current airplane speed, which corresponds to a wave length of 6.2km.

Noise Sources

The biggest kinematic corrections to the output measurement of the gravity meter are the Eötvös correction and the vertical accelerations. The magnitude of the Eötvös correction can have typically hundreds of milliGals and up to few thousand for high speed and elevation flights. Vertical acceleration can be in the range of a few tens to hundreds of thousands of milliGals. The gravimeter measures the specific force of gravity plus kinematic accelerations. In order to isolate the free air anomaly these kinematic effects on the sensitive axis need to be subtracted from the output of the gravity meter. This process generates errors due to GPS intrinsic position errors which get amplified in high frequencies by the double differentiation.

An excellent analysis of the GPS sources of noise can be found in [2]. Another important source of noise is the time synchronization mismatch between the GPS and gravity meter output. This mismatch can generate large errors, which can be spread in a broad band of frequencies, and therefore they may be hard to filter by simple low pass filtering.

In order to simulate the different sources of noise a 20,000mGal band limited noise level was added to the gravity signal. The frequency content in the lower frequencies is varied to study the differences in the filter performance for noise, especially when these frequencies are close to the cut-off frequency of the low pass filter applied to smooth the data.



2nd Order Gauss-Markov Stochastic Model GMP2

To implement the optimal Kalman smoother, a realistic model of stochastic variable has to be employed. The commonly employed white Gaussian noise does not describe the system dynamics. The objective is to use a stochastic model that could match as much as possible the auto-covariance or power spectral density of the GA that we intend to measure.

A Gauss-Markov 2nd order model in SS form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix} \omega(t)$$

The Autocovariance function for a GPM2 giving by [1]

$$\Psi_{nn}(\tau) = \sigma^2 e^{-\zeta\omega_n|\tau|} \{ \cos\beta|\tau| \} + \frac{\zeta\omega_n}{\beta} \sin\beta|\tau|$$

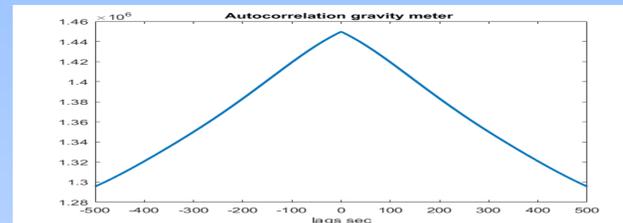
$$\text{Where } \beta = \omega_n \sqrt{1 - \zeta^2}$$

Selection of Proper Filter Parameters

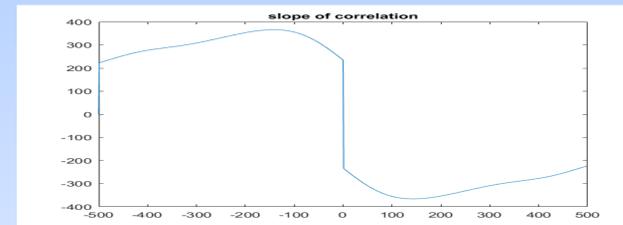
The equivalent ω_n or filter time for the FIR low pass filter has to be selected. In this study, since we are using synthetically generated noisy data based in a real gravity profile, we have the luxury of knowing the true gravity profile. Therefore we will be able to validate our parameter selection afterwards by calculating the error of the true gravity profile.

The filter characteristics could be selected by knowing beforehand the wavelength of the geological features being surveyed. We propose to estimate the FA gravity signal waveband by an iterative method. With the iterative procedure applied, the resulting FA is filtered with the initial low pass filter time which is twice as long as the minimum wavelength we are trying to recover.

The raw FA signal contains the true gravity anomaly model by a GM(2) plus the band limited gaussian noise which does not have energy at the very low frequencies or DC. Otherwise it would be not possible to recover the true GA by low pass filtering. Therefore the initial low pass filter should attenuate the high frequency and leave relatively untouched the GA signal. Now from the autocorrelation plot we can find the natural frequency of the GM(2) model.



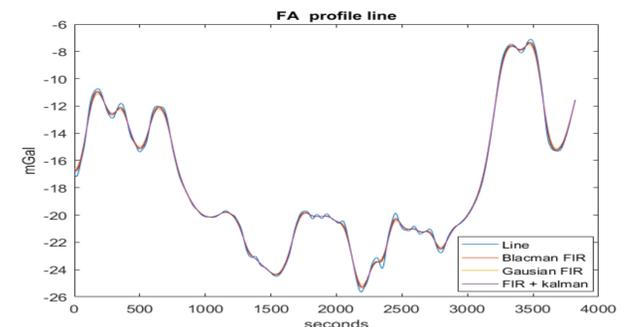
We can estimate the autocorrelation time by the first change in slope of the autocorrelation curve.



Filter Performance Comparison

The synthetic FA gravity profile is filtered with the three filters under test. Every filter response is evaluated for two cases; first for the case in which the frequencies of the added noise are relatively far from the stop frequency of the low pass filter, and secondly for the case in which the noise is relatively close to this stop frequency.

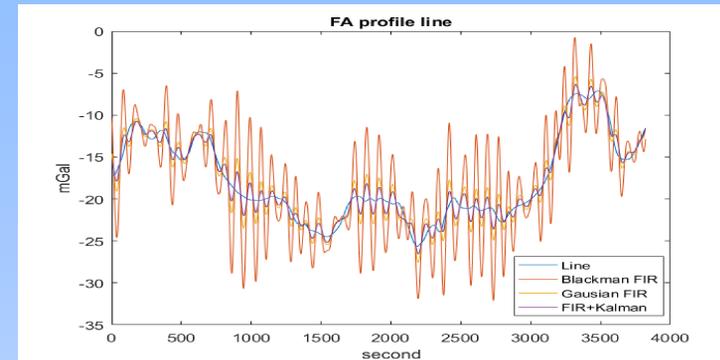
The next figure shows the comparison of the 3 filters with the real FA line for the case in which the noise only contains frequencies higher than 0.08 Hz, and the filter time is 100 seconds.



The three filters perform similarly for this case. The next table shows the performance for each filter

Noise f	FIR + Kalman Smoother	Blackman FIR	Gaussian FIR
> 0.08 Hz	0.19 mGal	1.15 mGal	0.26 mGal

For the case in which the noise has components of frequency in the range $f > 0.04$, the filters perform very differently.



Noise f	FIR + Kalman Smoother	Blackman FIR	Gaussian FIR
> 0.04 Hz	0.9 mGal	4.28 mGal	1.4 mGal

Conclusions

The results from filtering the test data with the three commonly used filters for airborne gravity data show that for the selected true line and for the simulated noise, the cascaded filter FIR followed by a Kalman smoother achieve the best results.

The error is calculated by subtracting the true gravity profile from the filtered data. In this way if a filter is over filtering the data it also increases the error since the test profile line has certain frequency components. In terms of this error, the Kalman smoother based on a 2nd order Gauss Markov has the best filtering performance in attenuating noise for the frequencies near to the stop band, but does not have enough attenuation in the stop band of the filter.

The two FIR filters have the most attenuation in the stop band, but they are not attenuating enough the frequencies near the stop band. As a consequence a cascade filter seems to yield the best results.

References

[1] Jason M. Leonard, Felipe G. Nievinski, George H. Born. "Gravity Error compensation using second-order Gauss-Markov processes". ASS 11-502

[2] Alexander Mark Bruton. "Improving the accuracy and resolution of SINS/ DGSP Airborne gravimetry". UCGE Reports number 20145