Earthquake Energy Balance, Stress Levels, and Heat Production in Rough Fault Simulations

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Or, how faults can host earthquakes at “high” background stresses \( \tau_b \approx 0.6(\sigma - p) \) with negligible heat production

frictional strength \( \tau \) on main fault reduced to nearly zero by dynamic weakening during coseismic slip

resistance to slip comes primarily from geometric complexity → load transferred to secondary faults and microfractures in off-fault material (causing plastic strain), which remains strong because resolved shear rates are too small for dynamic weakening (but off-fault heat production also remains small)
State of Stress in Earth’s Crust

a few faults, like San Andreas, operate at very low stresses...

...but rest of crust, and most of the faults within it, are strong

Southern California Earthquake Center Community Fault Model
Why are stress levels on SAF and other faults so different?

We first have to carefully define two related, but subtly different stresses:

• local tractions on fault surface, $\tau$

• overall or average stress driving slip, $\tau_b$ (remote tectonic stress)
Dynamic weakening* permits self-sustaining propagation at very low stress and minimal heating as illustrated in dynamic rupture simulation with thermal pressurization on planar fault in elastic medium.

*dynamic weakening=extreme reduction in fault strength during coseismic sliding (slip rates ~ 1 m/s) from thermal pressurization, decomposition reactions, flash heating, etc.
Ruptures with *Thermal Pressurization on Planar Faults in Elastic Medium*

- Open and light shaded circles = growing ruptures
- Dark shaded circles = arresting ruptures

**Boundary above which propagation becomes self-sustaining**

**Nucleation from region of decreased effective normal stress \( \sigma - p \)**

- Nucleation from region of increased shear stress \( \tau \)

**Background stress,**

\[ \frac{\tau_b}{(\sigma - p)} \]

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[Schmitt, Segall, and Dunham, 2015]
Dynamic weakening permits self-sustaining propagation at very low stress and minimal heating.

Average stress doing work on fault (i.e., generating heat) in our simulations is

\[
\tau_{\text{heat}} = \frac{\int \tau V \, dt}{\int V \, dt} = \frac{\text{work/area done on fault}}{\text{slip}} \approx 10 \, \text{MPa}
\]

with even smaller values expected for larger events due to additional pressurization. Thus, heat production from frictional sliding of fault satisfies constraints from heat flow:

If most of the fault motion is accomplished by periodic earthquakes, the lack of an anomaly both at Hollister and other locations along the San Andreas fault, establishes an upper limit of about 100 bars for the average frictional stress operating during faulting.

[Brune et al., 1969; see also Lachenbruch and Sass, 1980]

This is how the San Andreas (and other mature faults) appear to operate; thus, dynamic weakening potentially resolves “stress / heat flow paradox”
Why do other faults require \( \tau_b \approx 0.6(\sigma - p) \) for rupture?

Two possibilities, both with identical background stress levels, but quite different dynamics:

1. **dynamic weakening is unique to mature faults**
   Having well-developed cores prone to shear localization, heating, and thermal weakening

2. **dynamic weakening eliminates frictional resistance**, but **some other mechanism provides resistance to slip in addition to friction**

   **HEAT PRODUCTION**
   - **high** (from sliding at elevated \( \tau \)), though distributed shear can prevent melting
   - **low** (from sliding at negligible \( \tau \), and minimal off-fault heat production during plastic strain)

Second explanation appealing because dynamic weakening is ubiquitous in experiments and localization develops rapidly in sheared gouge.
Additional Resistance from Structural Complexities

like surface roughness, segmentation, branching, etc.

This geometric complexity introduces resistance to slip, even when faults are frictionless.

Corona Heights fault, San Francisco
(former quarry, front face stripped away)

[Fig. 2C]

[Candela et al., 2012]

[1992 Landers earthquake]

[Yann Klinger, IPGP]
Dynamic Ruptures on Geometrically Complex Faults with Dynamic Weakening

**structural complexity:** idealized here as fractal roughness of single fault surface, quantified with amplitude-to-wavelength ratio $\alpha$

$\alpha \sim 10^{-2}$ for immature faults, $\alpha \sim 10^{-3}$ for mature faults

[Power and Tullis, 1991; Brodsky et al., 2011; Candela et al., 2013]

**hypothesis:** roughness introduces resistance to slip, in addition to friction, and “roughness drag” $\tau^{\text{drag}}$ is dominant resistance for all but smoothest faults

Slip on rough fault causes massive near-fault strains, and (elastic) restoring stresses push back on fault in direction opposing average slip that “backstress” or “roughness drag” $\tau^{\text{drag}}$ increases with structural complexity, but is ultimately bounded by finite strength of off-fault material (in damage zones extending $\sim 100$ m from major faults)
Inelastic Deformation using Drucker-Prager Plasticity
(very similar to Mohr-Coulomb)

plastic strain essential to prevent otherwise unrealistically large stresses in vicinity of fault

...but plastic strain also produces heat (frictional sliding on secondary faults and microcracks), and we must check that this additional heating doesn’t violate heat flow constraints

[Dunham et al., 2011]
If earthquake nucleates at some $\tau_b/(\sigma-p)$, will it become large?

roughness drag substantially increases threshold stress for self-sustaining rupture

...but note that stress levels have not yet reached observed $\tau_b/(\sigma-p) \approx 0.6$ (because we only model roughness wavelengths $>100$ m)

Figure 6. Probabilities of rupture to reach an extent of 20 km (in either direction) from the hypocenter as a function of background stress level $\tau^b$. Rougher faults require larger $\tau^b$ to reach the same extent.

[Fang and Dunham, 2013]
Bringing Simulations Closer to Reality (by accounting for shorter roughness wavelengths)

predicted threshold stresses \( \tau^b \) based on analytical estimate \([\text{Fang and Dunham, 2013}]\) of \( \tau^{\text{drag}} \) and hypothesis that \( \tau^b \approx \tau^{\text{pulse}} + \tau^{\text{drag}} \)

smooth faults: \( \tau^{\text{drag}} \) negligible, so \( \tau^b \approx \tau^{\text{pulse}} \approx 0.25(\sigma - p) \) (like SAF)
rough faults: \( \tau^{\text{drag}} \) dominates resistance, ultimately bounded by finite strength of off-fault material, so \( \tau^b \approx 0.6(\sigma - p) \)

[Fang and Dunham, 2013]
Heat Production by Off-Fault Plastic Deformation

work/volume done by plastic strain = \( \int \sigma_{ij} \dot{\varepsilon}_{ij}^p \, dt \)

\[ \sim (\text{deviatoric stress})(\text{plastic strain}) \sim (100 \ \text{MPa})(10^{-3}) = 0.1 \ \text{MJ/m}^3 \]

or, integrating over off-fault dimension, \( \sim 3 \ \text{MJ/m}^2 \), consistent with seismically inferred fracture energy for events of this size [e.g., Abercrombie and Rice, 2005]

\[ \text{temperature change} = \frac{\int \sigma_{ij} \dot{\varepsilon}_{ij}^p \, dt}{\rho c} \sim \frac{0.1 \ \text{MJ/m}^3}{3 \ \text{MJ/m}^3 K} \sim 0.03 \ \text{K} \]

(and contribution to total heat production is about 3 to 10 times less than Brune et al. [1969] upper bound)
Taken at face value, these results present a dilemma:

A fault operating at high stress (~40 MPa), and experiencing a small stress drop (~4 MPa), should have a huge amount of energy flowing into it.

I’ve also suggested that
• not much is dissipated as heat during frictional sliding on fault
• even less is dissipated as heat during plastic straining

So where is the energy going?

let’s examine one simulation in more detail
What do we mean by “stress drop”? 

$\sigma_{xy}$ (MPa) at $t = 0.63$ s
What do we mean by “stress drop”? 

\[ \sigma_{xy} \text{ (MPa) at } t = 1.33 \text{ s} \] 

[Diagram of rupture propagation]
What do we mean by “stress drop”? 

$\sigma_{xy}$ (MPa) at $t = 2.03$ s

rupture propagation
What do we mean by “stress drop”? 

\[ \sigma_{xy} \text{ (MPa) at } t = 2.73 \text{ s} \] 

rupture propagation
What do we mean by “stress drop”?

\[ \sigma_{xy} \text{ (MPa) at } t = 3.43 \text{ s} \]

The stress field immediately around the fault is incredibly heterogeneous, but farther away it is more uniform.
We can define stress drop in terms of background stress $\tau^b$ using values of $\sigma_{xy}$ outside of near-fault region of heterogeneous stress.

\[ \Delta \tau^b = 4 \text{MPa} \]

(consistent with what would be inferred from moment-length scaling, observable geodetically or with far-field seismic waves)
Do the usual earthquake energy balance, but expand the “fault” to include the near-fault region, too.

work (per area) done by surrounding solid on near-fault region \( \sim \tau^b \Delta u \sim (40 \text{ MPa}) \times (4 \text{ m}) = 160 \text{ MJ/m}^2 \)

I’ll discuss this in terms of stress doing work on near-fault region \( \sim \tau^b \sim 40 \text{ MPa} \)
stress doing work on near-fault region $\sim \tau^b \sim 40$ MPa (quantifies flux of strain + kinetic energy into region)

Where does this energy go?

**frictional heat on fault:**
$\tau_{\text{heat}} \sim 20$ MPa in this example (but can be made *arbitrarily small* by appealing to more extreme dynamic weakening)

$$
\tau_{\text{heat}} = \frac{\int \tau V \, dt}{\int V \, dt}
$$
stress doing work on near-fault region $\sim \tau^b \sim 40$ MPa (quantifies flux of strain + kinetic energy into region)

**Where does this energy go?**

- **Frictional heat on fault:**
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- **Heat from plastic straining:**
  stress-equivalent dissipation $^*$
  $\sim 10$ MPa in this example (relatively insensitive to dynamic weakening)

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*stress equivalent dissipation =
  dissipated energy/volume,
  integrated over off-fault distance,
  divided by slip*
stress doing work on near-fault region $\sim \tau^b \sim 40$ MPa (quantifies flux of strain + kinetic energy into region)

Where does this energy go?

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heat from plastic straining:
stress-equivalent dissipation* $\sim 10$ MPa in this example (relatively insensitive to dynamic weakening)

strain energy in near-fault stress concentrations:
$\sim 10$ MPa in this example (relatively insensitive to dynamic weakening)

...but these stresses trigger earthquakes (aftershocks and background seismicity) on secondary faults, which are themselves fractal surfaces, repeating this process at ever smaller scales – with some additional energy loss via radiation
Conclusions

 Faults can host earthquakes at “high” background stresses $\tau_b \approx 0.6(\sigma - p)$ with negligible heat production if

- dynamic weakening reduces frictional resistance $\tau$ to nearly zero (eliminating on-fault heat production)
- structural complexity introduces additional resistance, elevating threshold background stress for rupture to $\tau_b \approx 0.6(\sigma - p)$, a value set by strength of off-fault material (i.e., frictional strength of secondary faults and microcracks at low slip/shear rates)
- off-fault heat production during off-fault plastic strain also within bound placed by lack of heat flow anomaly

Alternative explanation, distributed shear without dynamic weakening, predicts higher levels of heat production (not observed in drilling measurements from Chi-Chi, Wenchuan, Tohoku, etc.)
Extra Slides
And that’s how the San Andreas (and other mature faults) appear to operate – “stress / heat flow paradox”

maximum compressive stress at high angle to San Andreas, requiring $\tau_b/(\sigma - p) < 0.3$

absence of heat flow anomaly requires low dynamic friction

[Townend, 2006; see also ongoing work in Community Stress Model project]
And that’s how the San Andreas (and other mature faults) appear to operate – “stress / heat flow paradox”

dynamic rupture simulations on planar faults with thermal pressurization and flash heating [Noda, Dunham, Rice, 2009] show self-healing slip pulses at low $\tau_b$

threshold stress level $\tau_{\text{pulse}}$

Arresting pulse → Growing pulse → Crack

Background stress, $\tau^b/\bar{\sigma}_0$

![Graph showing slip pulses and stress evolution](image)
...but almost all other faults operate at higher stress

Collettini and Sibson [2001] compilation of active ($M_w$ 5.5-6) dipping faults, showing fault operation at $\tau_b/(\sigma_p - p) \approx 0.6$, based on Sibson [1985] lock-up angle concept

+ many other lines of evidence, including stress measurements in deep boreholes [Townend and Zoback, 2001]
Possible Discriminants

geometric resistance model predicts:

local near-fault stress changes $\sim$ absolute stress ($\sim$100 MPa)

$\Rightarrow \textit{diversity of aftershock focal mechanisms}$ [Smith and Dieterich, 2010]

as observed in Loma Prieta, Landers, and other events

(but seismically inferred stress drop, linking moment and fault dimension and controlling ground motion amplitude, is only $\sim$3 MPa $\ll$ absolute stress)

rough faults produce realistically flat acceleration spectra at high frequencies [Dunham et al., 2011];

see also comparison to GMPEs from 3D simulations [Shi and Day, 2013]
Earthquake Energy Balance

Local energy balance for elastic-plastic material:

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + U \right) + \nabla \cdot (\sigma \cdot v) = -\dot{W}^p
\]

\[
U = \frac{1}{2} \left( \sigma^{0}_{ij} + \sigma_{ij} \right) \varepsilon_{ij} \quad \text{(elastic strain energy density)}
\]

\[
\dot{W}^p = \sigma_{ij} \dot{\varepsilon}_{ij}^p \geq 0 \quad \text{(plastic work rate per unit volume)}
\]
Vary roughness, measure threshold stress for self-sustaining rupture...

...but no unique threshold stress for rupture on rough faults

Figure 4. Sequences of cumulative slip $\Delta$ every 0.28 s for ruptures occurring on the same fault but at three different $\tau^b$ levels. The extent of rupture and the amount of fault slip increase with background stress.

[Fang and Dunham, 2013]
Remarkable variability, despite statistically identical fault surfaces (same roughness $\alpha$) and same background stress $\tau_b$

**red=fault profile, blue=slip snapshots**

Fang and Dunham, 2013

\[ \alpha = 0.006, \frac{\tau_{\text{ave}}}{\sigma^0} = 0.2817, \frac{\tau}{\sigma^0} = 0.2976 \]