

Symmetrically Adapted Data Summaries

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Contents

Standard Symmetric Summaries

Measurements and Data

Symmetrically Adapted Summaries

Example: Cyclic Symmetries

Two Concluding Remarks

References

Related References and External Links

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Standard Symmetric Summaries

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

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Standard Symmetric Summaries.

Statistics is about reductions and summaries. Most analyses of variance, for example,

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
rows	1	104.17	104.17	16.89	0.0544
columns	2	54.33	27.17	4.41	0.1850
Residuals	2	12.33	6.17		

follow from design/sampling decompositions

$$R^6 = V = V_1^x \oplus V_1^r \oplus V_2^c \oplus V_2^{\text{res}}$$

of the data (vector) space V into direct-sums of invariant subspaces V_i , in the sense that

$$(P x) = P (x) \in V_i ; \quad x \in V_i ;$$

where V_i is defined by projection P_i and P_i is any one of its defining design / sampling permutation symmetries.

Standard Symmetric Summaries.

This is a consequence of the canonical projections theorem stating that for an arbitrary representation of a finite group G with irreducible characters, the linear transformations

$$P = \frac{n}{g} \sum_{g \in G} \chi(g^{-1}) \rho(g);$$

are such that:

$$P^2 = P; \quad P^T = P; \quad I = \sum_{\chi} P_{\chi};$$

where g is the order of G and n is the dimension of ρ , thus allowing for the analysis of variance

$$\sum_{j=1}^n x_j^2 = \sum_{\chi} \chi(1) P_{\chi} x$$

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Standard Symmetric Summaries.

The algebra behind these standard summaries often follows from very broad (full) symmetric groups S_n (and their design/sampling products so that the ANOVAs are then (tensor) products of the standard (full-symmetric) decompositions

$$I = A + Q$$

in which the A matrices are $n \times n$ matrix with all entries equal to $1/n$ and $Q = I - A$.

Although in general

$$S_n \Rightarrow I = P_1 + P_2 + \dots + P_r$$

it follows that when S_n acts on $L = f_1; f_2; \dots; f_n$, ALL projections P but A and Q are null.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Standard Symmetric Summaries.

In the particular sampling case, when S_n acts on $L = \{1; 2; \dots; n\}$, we obtain

$$\begin{aligned} \sum_{i=1}^n x_i^2 = \|x\|^2 &= x^0 A x + x^0 Q x \\ &= n(\bar{x})^2 + \sum_{i=1}^n (x_i - \bar{x})^2. \end{aligned}$$

This is why the sample mean and the sample variance are the twin gods (**Dioskouroi**) of statistics !

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Standard Symmetric Summaries.

There are, consequently, design symmetries (G_s), (sampling symmetries (G_s)), and measurement symmetries (G). In this seminar we will discuss the reduction of measurement data independently of its sampling and design symmetries, which are often of the broad type mentioned in the previous slides. We will emphasize the role of certain symmetries intrinsic to measurement (and instrumentation) in generating a decomposition

$$G \Rightarrow I = P_1 \underbrace{P_2 \dots P_r}_{\{Z\}} P_r$$

in which measurement variance component(s) can be identified.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Standard Symmetric Summaries.

In summary,

$$G_{\text{design}} \Rightarrow I_d = P_1 P_2 \dots P_m;$$

$$G_{\text{sampling}} \Rightarrow I_s = A + Q;$$

$$G \Rightarrow I = P_1 P_2 \dots P_r;$$

can be combined to decompose the overall data space identity matrix I

$$I = I_d I_s I$$

in conformity with the canonical projections theorem.

(*) of our interest here.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements and Data

Contents

Standard Symmetric
Summaries

**Measurements and
Data**

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Measurements.

Measurements, such as the base quantities and dimensions used in the International System of Units (SI):

- | Length, L ;
- | Mass, M ;
- | Time, duration, T ;
- | Electric current, I ;
- | Thermodynamic temperature, T ;
- | Amount of substance, N ;
- | Luminosity intensity, J ,

or any of their derived quantities:

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements.

Contents

Standard Symmetric
Summaries

**Measurements and
Data**

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Measurements.

are always the quantification of an object or a person under certain conditions and specific instruments.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

object () measurement () data

A scalar (basic) measurement is a mapping

$$x : L \rightarrow R$$

in which its image x is the observed data and
its domain L is a set of reference labels.

Every measurement is subject to two basic types of functional actions:

$$! \quad \perp \quad x \quad \mathbb{R} \quad s \quad \mathbb{R}$$

- | $(x)(\cdot) = x \cdot$, symmetry studies [1{3};
- | $(sx)(\cdot) = s(x \cdot)$, measurement theory [4{6].

(*) of our interest here.

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Measurements and Data.

$$! \quad \mathbb{L}^x \quad \mathbb{R}$$

| $= \mathbb{L}; \quad 2 \text{ Aut}(\mathbb{L})$ will be outlined here;

| is a G -set and $2 G$.

(*) of our interest here.

Starting with

$$\mathbb{L} \quad \mathbb{L} \overset{x}{V} \quad (\mathbb{R}); \quad 2 \text{ G} \quad \text{Aut}(\mathbb{L});$$

$$O_x = f(x \cdot) : 2 \text{ G}g = f x : 2 \text{ G}g$$

is the G -orbit of x .

- | G Abelian vs. G non-Abelian;
- | $x(\cdot)$ global symmetry vs $x(\cdot)$ localized symmetry (quantum chemistry, qNMR applications).

(*) of our interest here.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements and Data.

Example: A visual eld color-coded data(L)
produced by automated instrumentation:

[Contents](#)

[Standard Symmetric
Summaries](#)

**[Measurements and
Data](#)**

[Symmetrically Adapted
Summaries](#)

[Example: Cyclic
Symmetries](#)

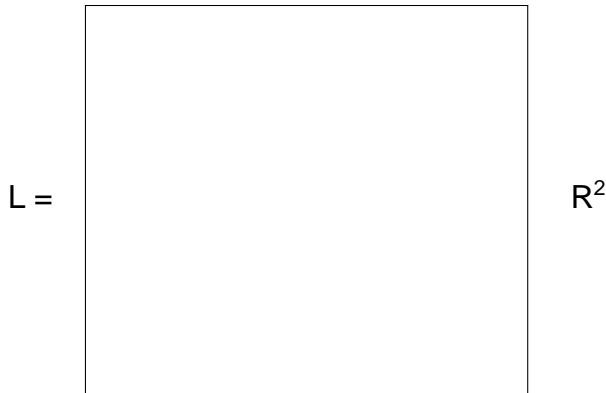
[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

Measurements and Data.

The visual eld data are (physically) labeled in the instrument by the set of points as points in a person's retina:



The trivial automorphism group:

$$G = (f \ 1g;)$$

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

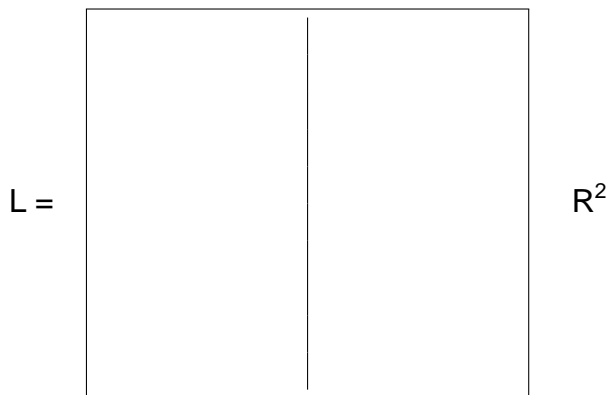
[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements and Data.

A vertical line reflection ψ acting on L :



The automorphism group allowing to vertical eld isotropy:

$$G = (f 1; v g;)$$

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

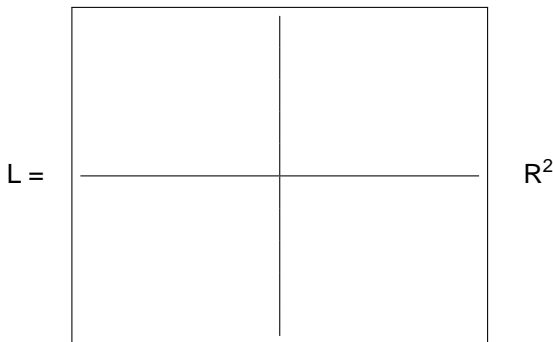
[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements and Data.

Vertical (v) and horizontal (h) line reflections, and the resulting central inversion (r) acting on L :



The automorphism group allowing to vertical, horizontal and rotational field isotropy:

$$G = (f 1; v; h; r; g;)$$

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements and Data.

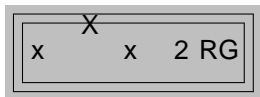
Here is the actual(L) with an G-orbit:

$$x(L) =$$

			26	27	24	21			
		23	23	26	28	25	22		
	27	29	27	27	28	29	29	28	
28	29	29	29	31	20	20	28	28	29
26	26	29	30	33	34	28	27	28	27
28	30	30	31	32	33	33	4	27	28
27	29	31	31	32	32	30	29	28	27
	30	31	31	28	29	32	30	30	
		28	32	29	28	29	28		
			26	27	28	29			

$$O_x = f(x) : 2 Gg, G = f1; v; h; rg;$$

$$x = (x_1; x_v; x_h; x_r)' (23; 25; 32; 29);$$



[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements and Data.

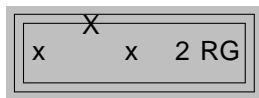
Each G -orbit such as

$$O_x = f(x \cdot) : 2Gg, G = f1; v; h; rg;$$

is an homogeneous space within which the data

$$x = (x_1; x_v; x_h; x_r)' \quad (23; 25; 32; 29);$$

can be directly label by the group of automorphisms. This construction says that



has the structure of a group algebra, thus facilitating the data-analytic operations, Points in a group algebra add as points in vector space and multiply as sequence convolutions. The $\mathbb{C}[G]$, which is a vector space allowing for the multiplication for its points

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Symmetrically Adapted Summaries

Contents

Standard Symmetric
Summaries

Measurements and
Data

**Symmetrically Adapted
Summaries**

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

The (scalar)symmetrically adapted summaries of data $\alpha \in \mathbb{R}^n$ are the linear combinations

$$T(x) = \sum_{i \in G} x_i$$

defined for all $\alpha \in \mathbb{R}^n$ by $T(x) = \sum_{i \in G} x_i$ satisfying

$$T(\alpha) = \sum_{i \in G} \alpha_i$$

for all symmetries; $\alpha \in \mathbb{R}^n$.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

T reduces as .

That is:

$$\begin{aligned}
 (T)(x) &= T(x) = X^{-1} (X) X = X^{-1} X \\
 &= X^{-1} X = X^{-1} X \\
 &= X^{-1} T(x):
 \end{aligned}$$

$$T = X^{-1} T$$

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Example (cont.): The Visual Field Global Orbit.

Symmetrically
Adapted Data
Summaries

Marios Viana

Here is the $G = f(1; v; h; r)$ -orbit of the visual field data

x :

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Symmetrically Adapted Summaries for the Visual Field Data.

Contents

Standard Symmetric Summaries

Measurements and Data

Symmetrically Adapted Summaries

Example: Cyclic Symmetries

Two Concluding Remarks

References

Related References and External Links

$$T(x) = x_1 + x_v + x_h + x_r; \quad = (1; 1; 1; 1);$$

0	0	0	0	102	106	106	102	0	0	0	1
0	0	101	109	111	111	111	109	101	0	0	0
0	115	119	119	112	112	112	119	119	115	115	0
111	114	117	110	115	115	110	117	114	111	111	0
109	111	90	122	132	132	122	90	111	109	109	0
109	111	90	122	132	132	122	90	111	109	109	0
111	114	117	110	115	115	110	117	114	111	111	0
0	115	119	119	112	112	119	119	115	115	115	0
0	0	101	109	111	111	109	101	0	0	0	0
0	0	0	102	106	106	102	0	0	0	0	0

Symmetrically Adapted Summaries for the Visual Field Data.

$$T(x) = x_1 + x_v + x_h + x_r; \quad = (1; 1; 1; 1);$$

[Contents](#)

[Standard Symmetric
Summaries](#)

[Measurements and
Data](#)

[Symmetrically Adapted
Summaries](#)

[Example: Cyclic
Symmetries](#)

[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

Symmetrically Adapted Summaries for the Visual Field Data.

Contents

Standard Symmetric Summaries

Measurements and Data

Symmetrically Adapted Summaries

Example: Cyclic Symmetries

Two Concluding Remarks

References

Related References and External Links

$$T(x) = x_1 \quad x_v + x_h \quad x_r; \quad = (1; \quad 1; 1; \quad 1);$$

0	0	0	0	2	2	2	2	0	0	0	1
0	0	1	1	1	1	1	1	1	0	0	0
0	1	1	3	2	2	3	3	1	1	0	0
1	2	3	10	11	11	10	3	2	1	1	0
1	1	28	0	2	2	0	28	1	1	1	0
1	1	28	0	2	2	0	28	1	1	1	0
1	2	3	10	11	11	10	3	2	1	1	0
0	1	1	3	2	2	3	1	1	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	2	2	2	2	0	0	0	0	0

Symmetrically Adapted Summaries for the Visual Field Data.

Note that the isotropy of the symmetrically adapted summary is such that it either remains the same or has a change of sign when the field is re-orientated. This is the consequence of the fundamental property shown in Slide 26. The same can be observed in the remaining summaries that follow:

Symmetrically Adapted Summaries for the Visual Field Data.

$$T(x) = x_1 \quad x_v + x_h \quad x_r; \quad = (1; \quad 1; 1; \quad 1);$$

[Contents](#)

[Standard Symmetric
Summaries](#)

[Measurements and
Data](#)

[Symmetrically Adapted
Summaries](#)

[Example: Cyclic
Symmetries](#)

[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

Symmetrically Adapted Summaries for the Visual Field Data.

Contents

Standard Symmetric Summaries

Measurements and Data

Symmetrically Adapted Summaries

Example: Cyclic Symmetries

Two Concluding Remarks

References

Related References and External Links

$$T(x) = x_1 + x_v \quad x_h \quad x_r; \quad = (1; 1; 1; 1);$$

0	0	0	0	8	4	4	8	0	0	0	1
0	0	11	13	3	3	3	13	11	0	0	0
0	5	3	7	2	2	2	7	3	5	0	0
3	0	3	12	13	13	12	3	0	3	3	0
3	3	22	6	2	2	6	22	3	3	3	0
3	3	22	6	2	2	6	22	3	3	3	0
3	0	3	12	13	13	12	3	0	3	3	0
0	5	3	7	2	2	7	3	5	0	0	0
0	0	11	13	3	3	3	13	11	0	0	0
0	0	0	8	4	4	8	0	0	0	0	0

Symmetrically Adapted Summaries for the Visual Field Data.

$$T(x) = x_1 + x_v \quad x_h \quad x_r; \quad = (1; 1; 1; 1);$$

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

Symmetrically Adapted Summaries for the Visual Field Data.

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

$$T(x) = x_1 \quad x_v \quad x_h + x_r; \quad = (1; \quad 1; \quad 1; 1);$$

0	0	0	0	8	4	4	8	0	0	0	1
0	0	1	5	3	3	5	1	0	0	0	0
0	1	1	1	0	0	1	1	1	1	0	0
1	0	1	8	11	11	8	1	0	1	1	0
1	5	24	4	0	0	4	24	5	1	1	0
1	5	24	4	0	0	4	24	5	1	1	0
1	0	1	8	11	11	8	1	0	1	1	0
0	1	1	1	0	0	1	1	1	1	0	0
0	0	1	5	3	3	5	1	0	0	0	0
0	0	0	8	4	4	8	0	0	0	0	0

Symmetrically Adapted Summaries for the Visual Field Data.

$$T(x) = x_1 \quad x_v \quad x_h + x_r; \quad = (1; \quad 1; \quad 1; 1);$$

[Contents](#)

[Standard Symmetric
Summaries](#)

[Measurements and
Data](#)

[Symmetrically Adapted
Summaries](#)

[Example: Cyclic
Symmetries](#)

[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

The Induced Symmetrically Adapted Covariance Structure.

Writing the data indexed by the components \mathbf{G} by $(x; y; z; w)$, it then follows that

$$= \frac{1}{4} \begin{matrix} X \\ 2G \end{matrix} \begin{matrix} 2 & & & & 3 \\ x^2 & xy & xz & xw \\ 6 & xy & y^2 & yz & yw \\ 4 & xz & yz & z^2 & zw \\ & xw & yw & zw & w^2 \end{matrix} \begin{matrix} 3 \\ 7 \\ 5 \\ 7 \\ 5 \end{matrix} = \begin{matrix} 2 & & & & 3 \\ F & B & C & D \\ 6 & B & F & D & C \\ 4 & C & D & F & B \\ & D & C & B & F \end{matrix} \begin{matrix} 3 \\ 7 \\ 5 \\ 7 \\ 5 \end{matrix}$$

is the G -centralized covariance structure, with

$$F = (x^2 + y^2 + w^2 + z^2)=4; \quad B = (xy + zw)=2;$$

$$C = (xz + yw)=2; \quad D = (xw + yz)=2;$$

Since the projections \mathbf{P} are linear combination of the \mathbf{G} , it follows that \mathbf{P} commutes with all the canonical projections- allowing for the invariances mentioned earlier in Slide 4.

[Contents](#)
[Standard Symmetric Summaries](#)
[Measurements and Data](#)
[Symmetrically Adapted Summaries](#)
[Example: Cyclic Symmetries](#)
[Two Concluding Remarks](#)
[References](#)
[Related References and External Links](#)

Symmetrically Adapted Summaries for the Visual Field Data.

The $G = f; v; h; rg$ -adapted summaries correspond to the direct-sum decomposition

$$V = \mathbb{R}^4 = V_1 \oplus V_2 \oplus \dots \oplus V_4$$

of the measurement space into invariant subspaces, all of which \mathbb{R} -dimension of 1.

Measurement and sampling decomposition:

$$(P_1 + \underbrace{P_2 + P_3 + P_4}_{\text{anisotropy components}}) (A + Q)$$

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

The Cyclic Symmetries Case

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

**Example: Cyclic
Symmetries**

Two Concluding
Remarks

References

Related References and
External Links

Models for the anterior surface of the human cornea:

[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Measurements (L) of the curvature of a human cornea, in diopter (m^{-1}) units, along a given central aperture.

44.5839, 44.2913, 44.3495, 44.2333, 44.1754, 44.2333,
44.4079, 44.1176, 43.8882, 44.0026, 43.8312, 43.7176,
43.7176, 43.4923, 43.5484, 43.3248, 43.3805, 43.3805,
43.6047, 43.6047, 44.1176, 44.5839, 44.8207, 45.1807,
45.424, 45.424, 45.6698, 45.6081, 45.4852, 45.6081,
45.5466, 45.302, 45.0601, 44.8207, 44.7613, 44.5839

Paradigma: Kùklikèc SummetrDec

The curvature $\kappa(L)$ and the underlying cyclic labels produced by the automated instrumentation (computerized corneal topographers).

[Contents](#)

[Standard Symmetric
Summaries](#)

[Measurements and
Data](#)

[Symmetrically Adapted
Summaries](#)

**[Example: Cyclic
Symmetries](#)**

[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

$$L = f \mid k : k = 0; \dots; 35g; \quad ! = 2 = 36;$$

$$G = Z_{36} \mid f \quad e^{i!k} : k = 0; \dots; 35g;$$

$$\overset{j}{k} = e^{i!kj}; \quad j = 0; \dots; 35;$$

$$\overset{j}{k} \overset{j}{m} = \overset{j}{k+m \pmod{36}};$$

so that cyclically adapted summaries of interest

$$T_j(x) = \sum_k x_k \overset{j}{k}; \quad j = 0; \dots; 35;$$

are just the discrete Fourier transforms of

Parâmetro: Kuklikèc SummetrDec

A theoretical model for the corneal curvature:

$$c(\theta; \alpha; \beta) = \alpha + \cos[2(\theta - \beta)]; \theta \in (0; 2\pi)$$

j	$T_j(c)$	$T_j(x)$
0(<)	36	159988
2(<)	18 cos(2)	7:15597
34(<)	18 cos(2)	7:15597
2(=)	18 sin(2)	2:47261
34(=)	18 sin(2)	2:47261

$$T(c) \setminus T(x) = \begin{matrix} \wedge = 44:44; \wedge = 0:42; \wedge = 0:16 \end{matrix}$$

$$\mathbb{X}_k = c(2 \cdot k=36 : \wedge; \wedge; \wedge); k = 0; \dots; 35$$

[Contents](#)

[Standard Symmetric
Summaries](#)

[Measurements and
Data](#)

[Symmetrically Adapted
Summaries](#)

[Example: Cyclic
Symmetries](#)

[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

Paradigma: Kullik's Summative

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

**Example: Cyclic
Symmetries**

Two Concluding
Remarks

References

Related References and
External Links

⌘ (blue), x (black).

Paradigma: Kullik's Summative

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

**Example: Cyclic
Symmetries**

Two Concluding
Remarks

References

Related References and
External Links

⌘ (blue), x (black).

Absolute Fitting Error Distribution.

[Contents](#)

[Standard Symmetric
Summaries](#)

[Measurements and
Data](#)

[Symmetrically Adapted
Summaries](#)

[Example: Cyclic
Symmetries](#)

[Two Concluding
Remarks](#)

[References](#)

[Related References and
External Links](#)

$\|x_j - 2(D) < 5(D) = \text{instrument error.}$

The cyclically adapted summaries correspond to the direct-sum decomposition

$$V = \mathbb{C}^{36} = V_1 \oplus V_2 \oplus \dots \oplus V_{36}$$

of the data space V into invariant subspaces, all of which in \mathbb{C} -dimension of 1.

Two Concluding Remarks

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

**Two Concluding
Remarks**

References

Related References and
External Links

If statistics is an integral part of the scientific method, then we must be aware that it cannot be adequately discussed without contact with the science to which it applies.

data analysis () scientific context

Without contact with experimentation,
we statisticians face the risk of
becoming like geometers who study
space and never measure it.

data analysis () measurement

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[Contents](#)

[Standard Symmetric Summaries](#)

[Measurements and Data](#)

[Symmetrically Adapted Summaries](#)

[Example: Cyclic Symmetries](#)

[Two Concluding Remarks](#)

[References](#)

[Related References and External Links](#)

Related References and External Links.

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Byzantine motifs: [BXM](#) The Byzantine and Christian Museum-Athens.

Contents

Standard Symmetric
Summaries

Measurements and
Data

Symmetrically Adapted
Summaries

Example: Cyclic
Symmetries

Two Concluding
Remarks

References

Related References and
External Links

