

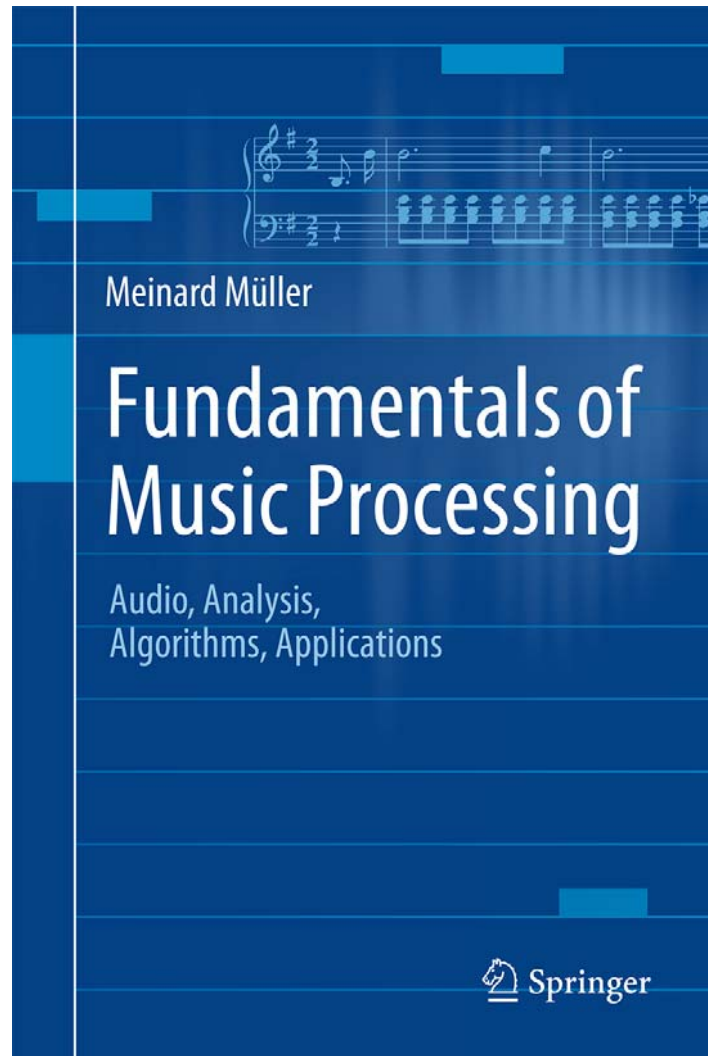
Lecture
Music Processing

Tempo and Beat Tracking

Meinard Müller

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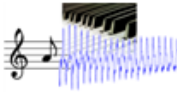
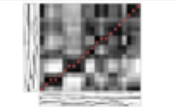
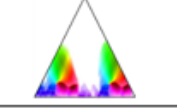
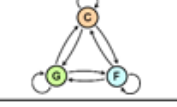
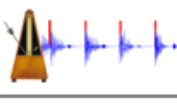
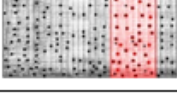
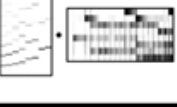
Book: Fundamentals of Music Processing



Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
483 p., 249 illus., hardcover
ISBN: 978-3-319-21944-8
Springer, 2015

Accompanying website:
www.music-processing.de

Book: Fundamentals of Music Processing

Chapter		Music Processing Scenario
1		Music Representations
2		Fourier Analysis of Signals
3		Music Synchronization
4		Music Structure Analysis
5		Chord Recognition
6		Tempo and Beat Tracking
7		Content-Based Audio Retrieval
8		Musically Informed Audio Decomposition

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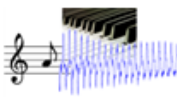
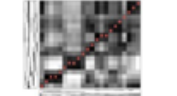
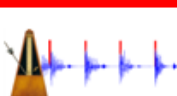
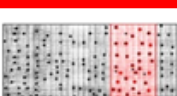
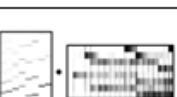
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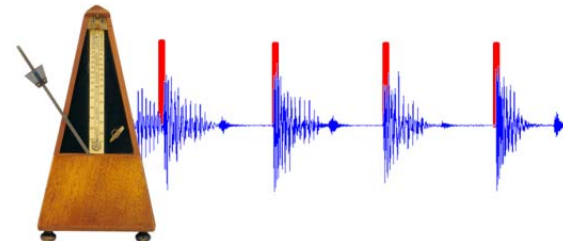
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Chapter 6: Tempo and Beat Tracking

- 6.1 Onset Detection
- 6.2 Tempo Analysis
- 6.3 Beat and Pulse Tracking
- 6.4 Further Notes



Tempo and beat are further fundamental properties of music. In Chapter 6, we introduce the basic ideas on how to extract tempo-related information from audio recordings. In this scenario, a first challenge is to locate note onset information—a task that requires methods for detecting changes in energy and spectral content. To derive tempo and beat information, note onset candidates are then analyzed with regard to quasiperiodic patterns. This leads us to the study of general methods for local periodicity analysis of time series.

Introduction

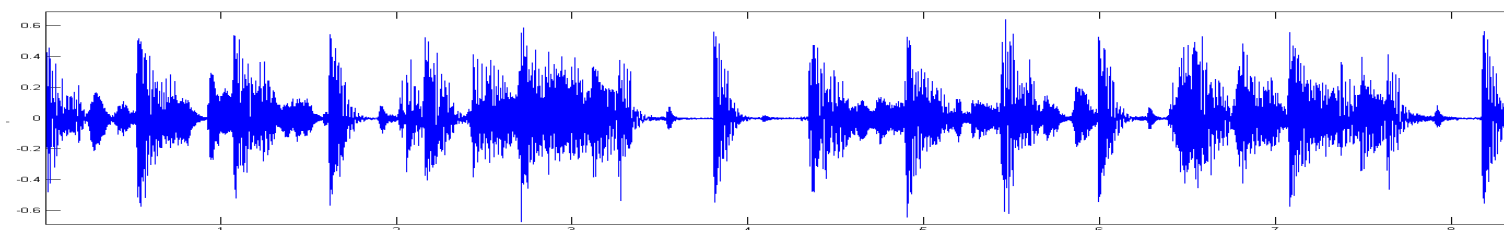
Basic beat tracking task:

Given an audio recording of a piece of music,
determine the periodic sequence of beat positions.

“Tapping the foot when listening to music”

Introduction

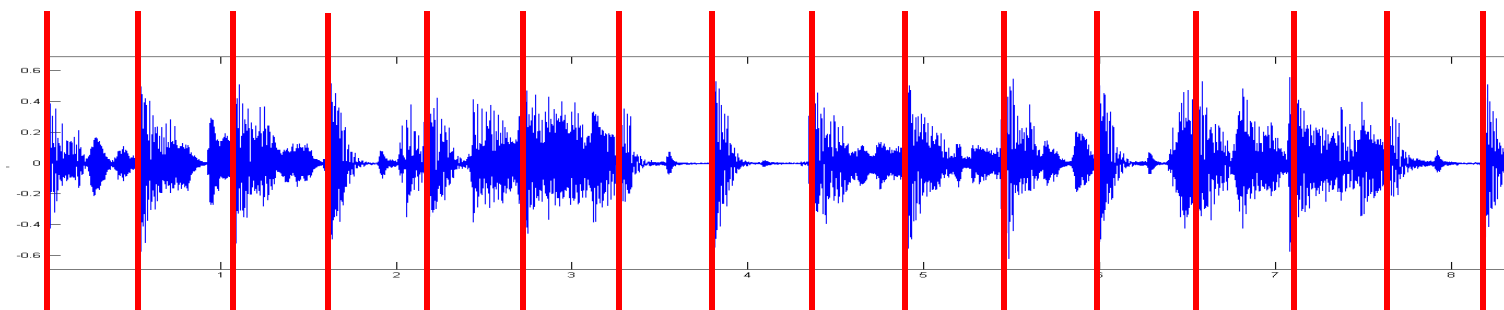
Example: Queen – Another One Bites The Dust



Time (seconds)

Introduction

Example: Queen – Another One Bites The Dust



Time (seconds)



Introduction

Example: Happy Birthday to you

Pulse level: **Measure**

The image displays a musical score for the song "Happy Birthday to you" in 3/4 time. The score is written on two staves. The first staff contains the lyrics "Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py". The second staff contains the lyrics "Birth - day dear _____, Hap - py Birth - day to you!". Four red arrows point down to the first four measures of the first staff, indicating the pulse level at the measure level.

Introduction

Example: Happy Birthday to you

Pulse level: **Tactus (beat)**

The image displays a musical score for the song "Happy Birthday to you" in 3/4 time. The score is written on two staves. The first staff contains the melody for the first line of the song, with lyrics "Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py". The second staff contains the melody for the second line, with lyrics "Birth - day dear _____, Hap - py Birth - day to you!". Red arrows point down to the first staff, indicating the pulse level (Tactus) at the beginning of each measure. The key signature is one sharp (F#) and the time signature is 3/4.

Introduction

Example: Happy Birthday to you

Pulse level: **Tatum (temporal atom)**

The image displays a musical score for the song "Happy Birthday to you" in 3/4 time, written in G major (one sharp). The score is presented on two staves. The first staff contains the lyrics "Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py" and is marked with 21 red arrows pointing down to the notes, indicating the pulse level. The second staff contains the lyrics "Birth - day dear _____, Hap - py Birth - day to you!" and also has red arrows pointing down to the notes. The music is written in a simple, clear style, suitable for a classroom or educational setting.

Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: ???



Introduction

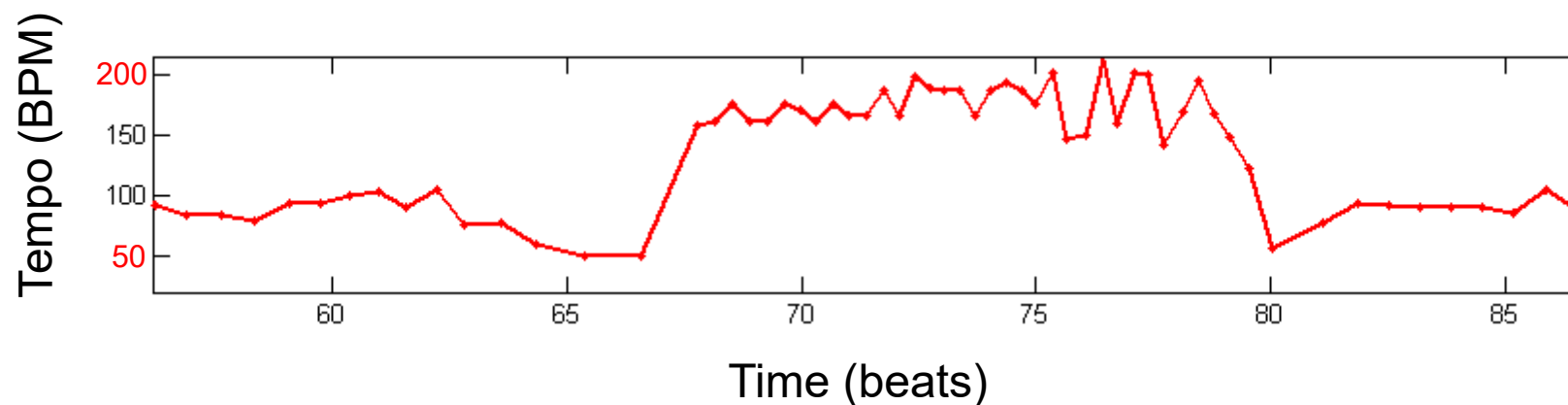
Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: 50-200 BPM



Tempo curve



Introduction

Example: Borodin – String Quartet No. 2

Pulse level: Quarter note

Tempo: 120-140 BPM (roughly)

Beat tracker without any prior knowledge



Beat tracker with prior knowledge on
rough tempo range



Introduction

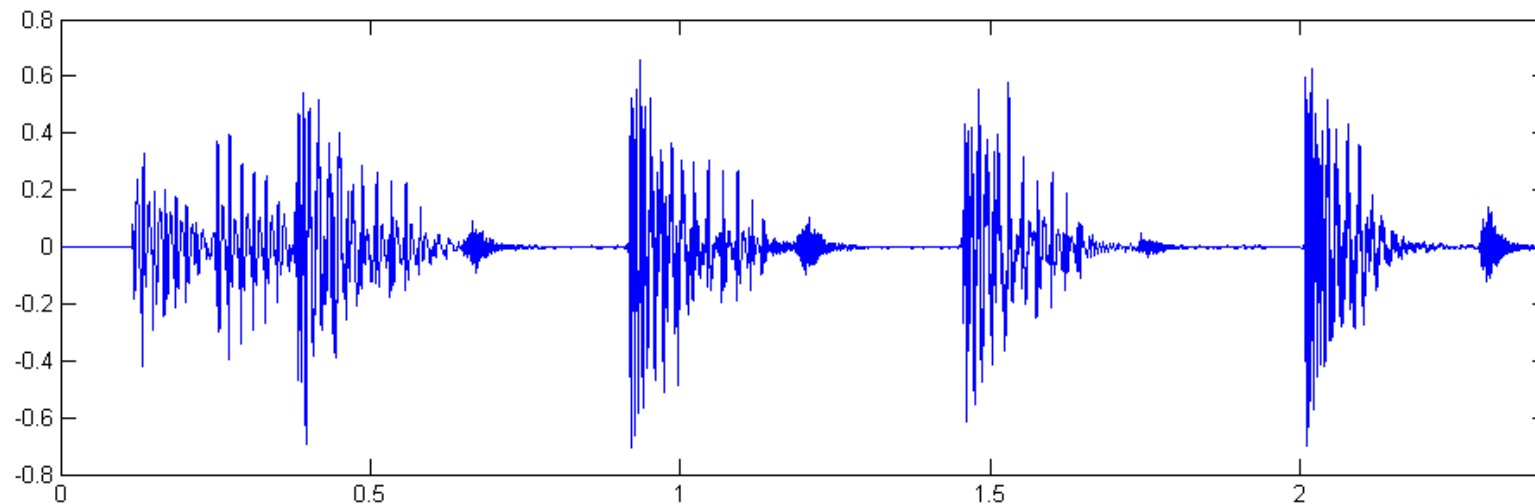
Challenges in beat tracking

- Pulse level often unclear
- Local/sudden tempo changes (e.g. rubato)
- Vague information
(e.g., soft onsets, extracted onsets corrupt)
- Sparse information
(often only note onsets are used)

Introduction

Tasks

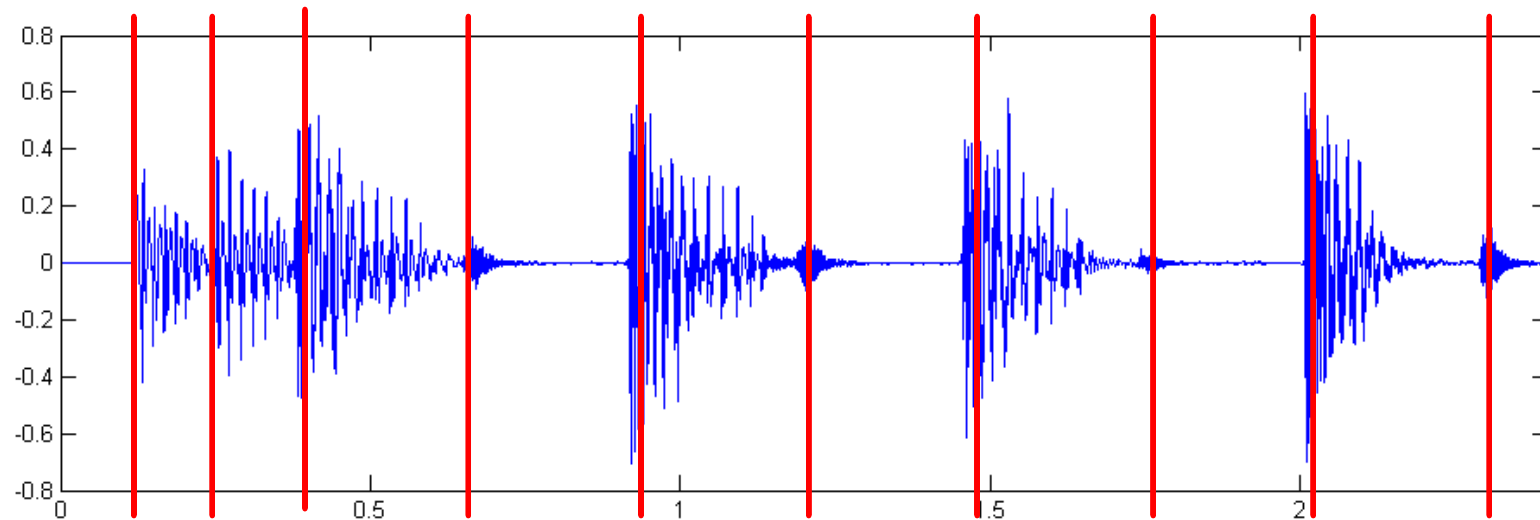
- Onset detection
- Beat tracking
- Tempo estimation



Introduction

Tasks

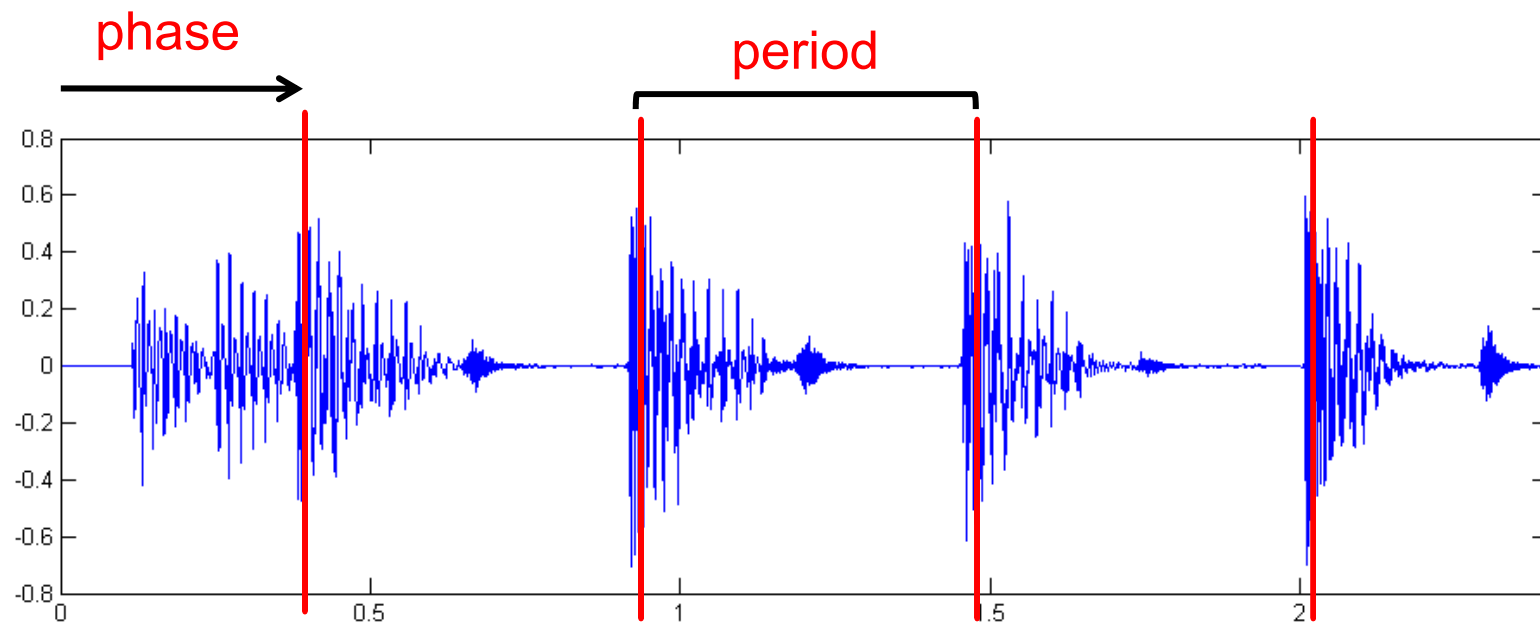
- Onset detection
- Beat tracking
- Tempo estimation



Introduction

Tasks

- Onset detection
- **Beat tracking**
- Tempo estimation



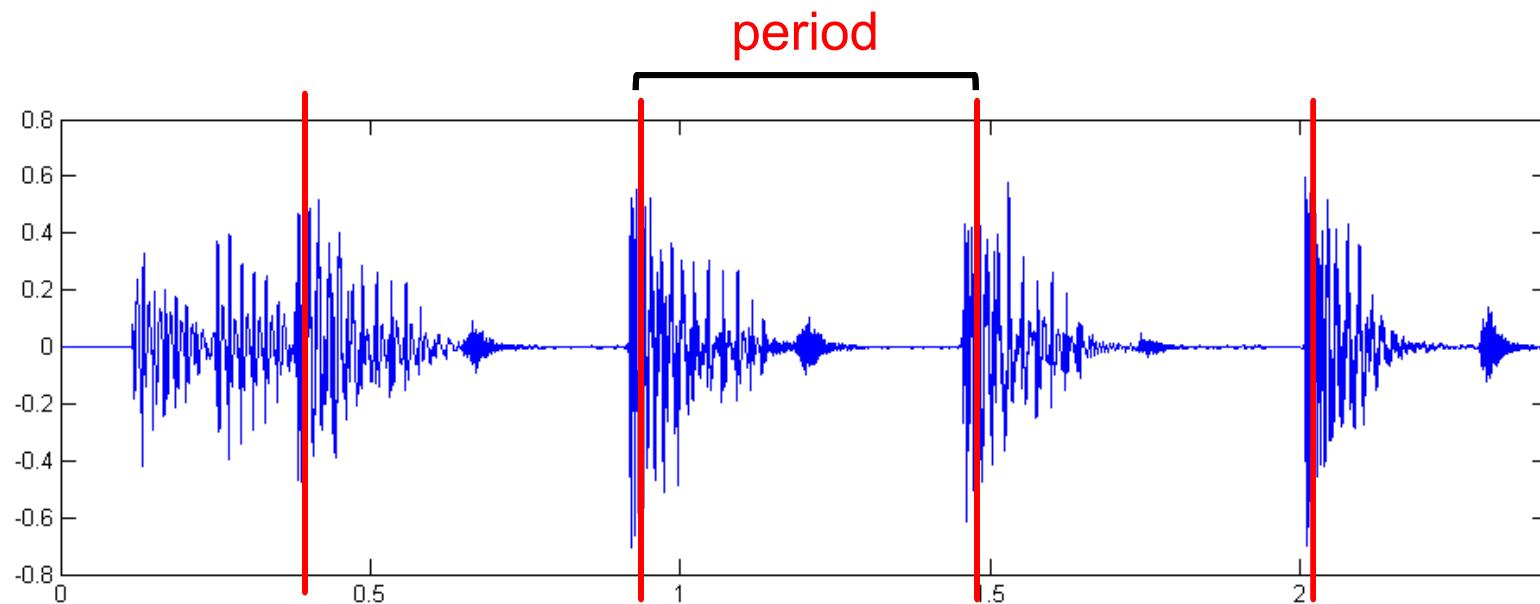
Introduction

Tasks

- Onset detection
- Beat tracking
- Tempo estimation

Tempo := 60 / period

Beats per minute (BPM)

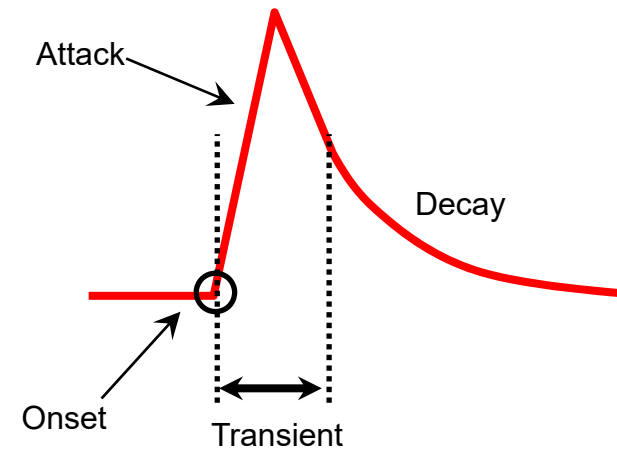
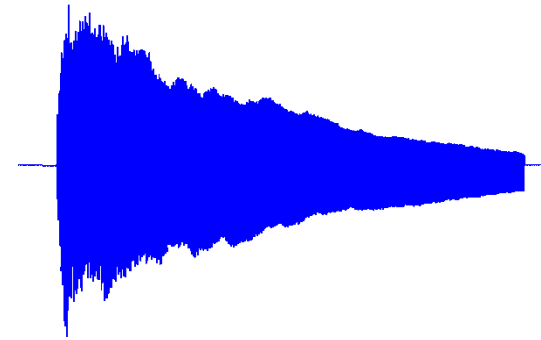


Onset Detection

- Finding start times of perceptually relevant acoustic events in music signal
- Onset is the time position where a note is played
- Onset typically goes along with a change of the signal's properties:
 - energy or loudness
 - pitch or harmony
 - timbre

Onset Detection

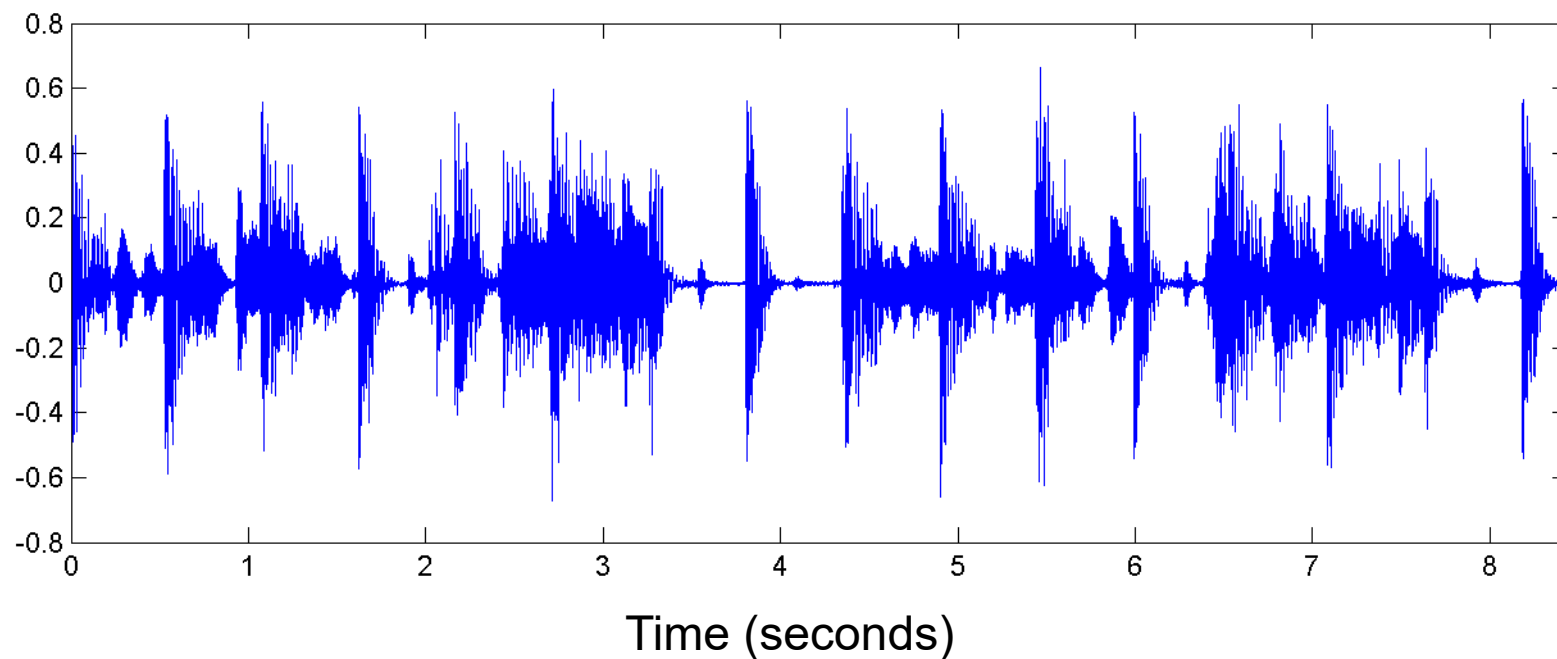
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 - timbre



Onset Detection (Energy-Based)

Steps

Waveform

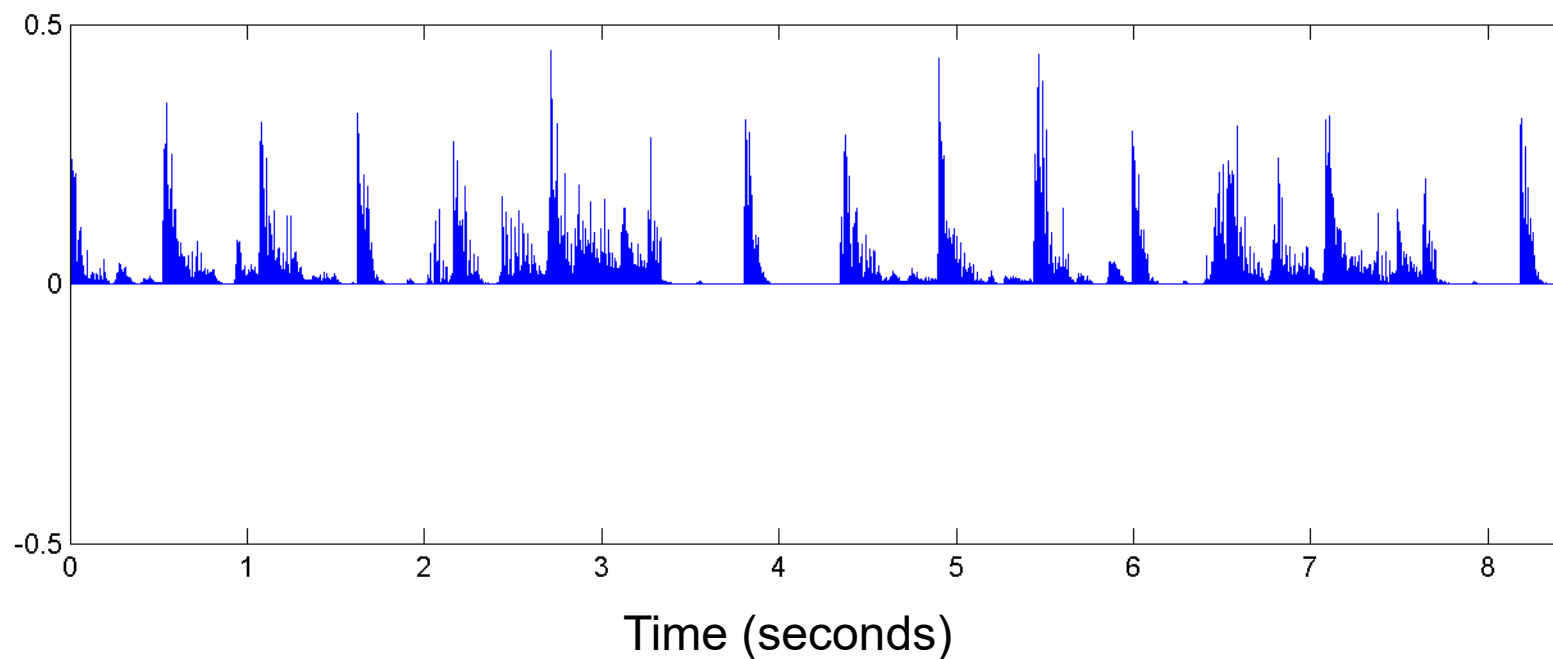


Onset Detection (Energy-Based)

Steps

1. Amplitude squaring

Squared waveform

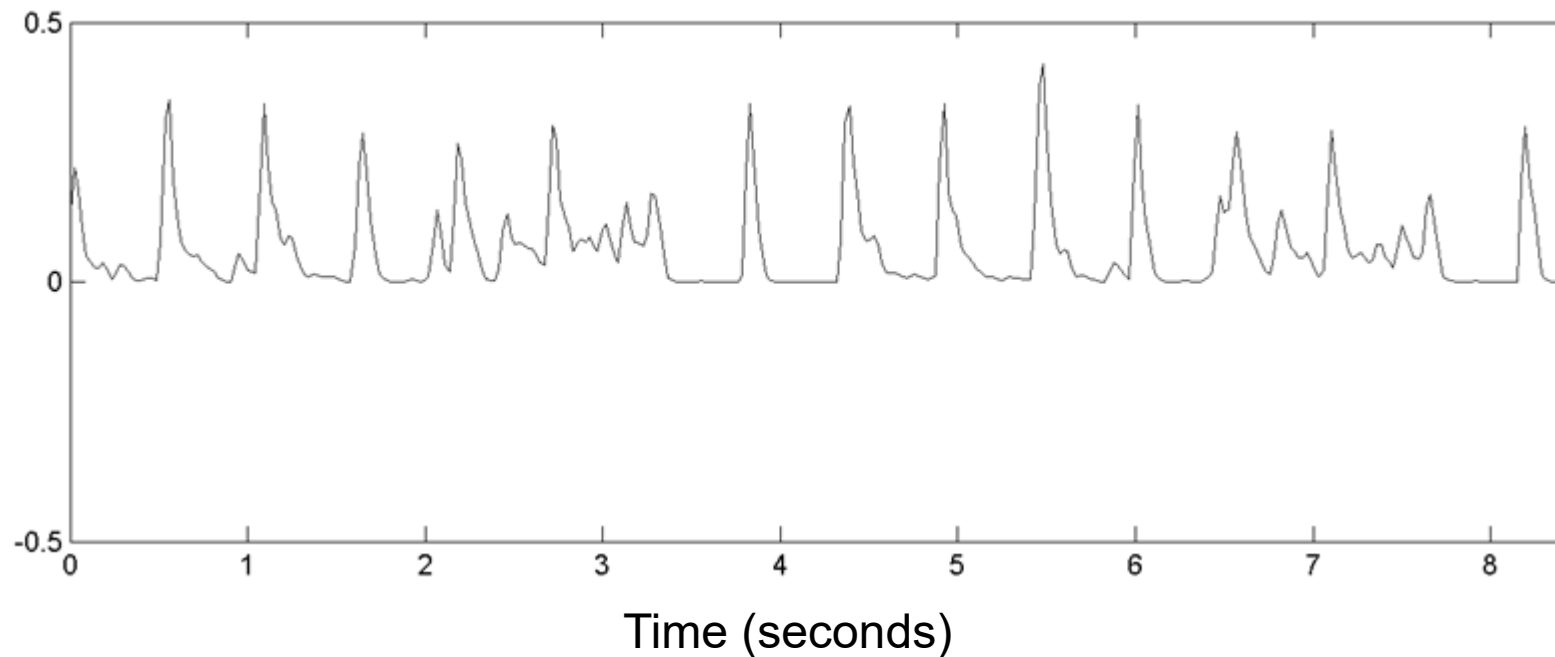


Onset Detection (Energy-Based)

Steps

1. Amplitude squaring
2. Windowing

Energy envelope



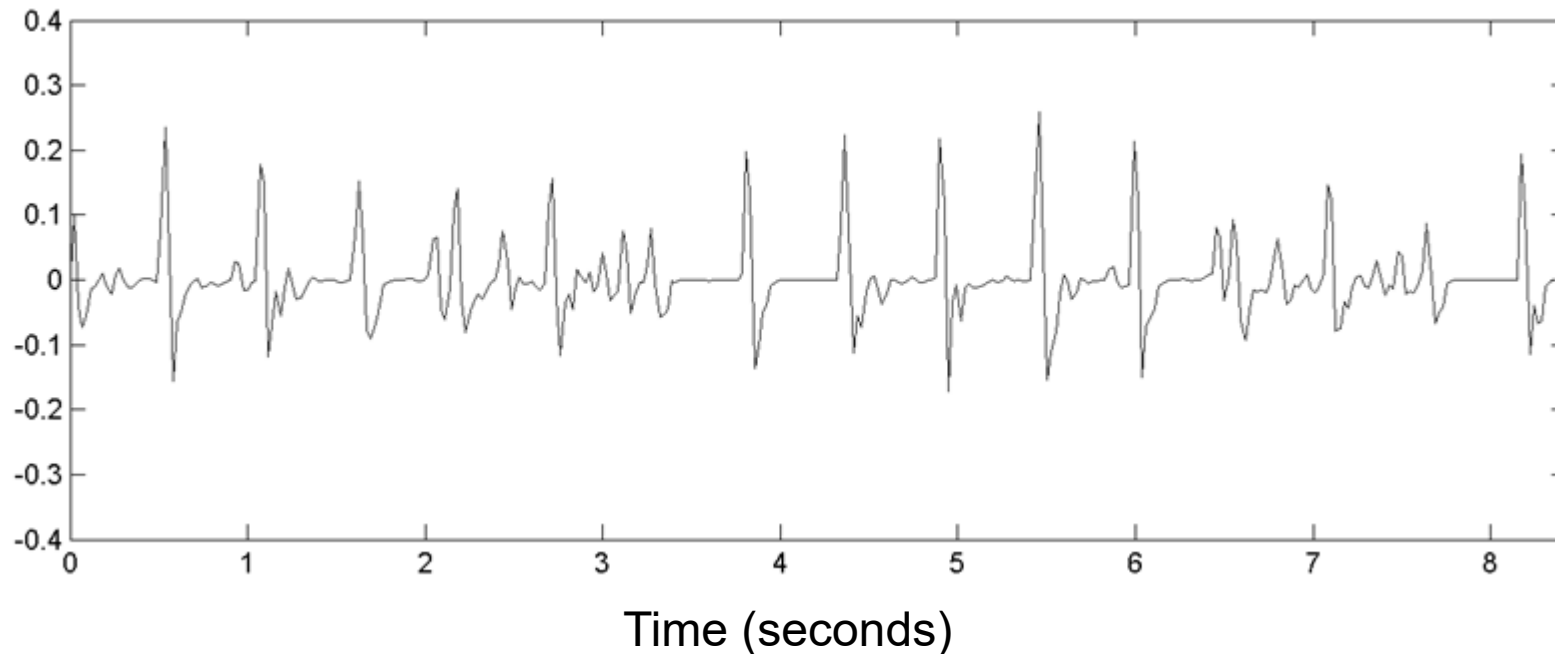
Onset Detection (Energy-Based)

Steps

1. Amplitude squaring
2. Windowing
3. Differentiation

Capturing energy changes

Differentiated energy envelope



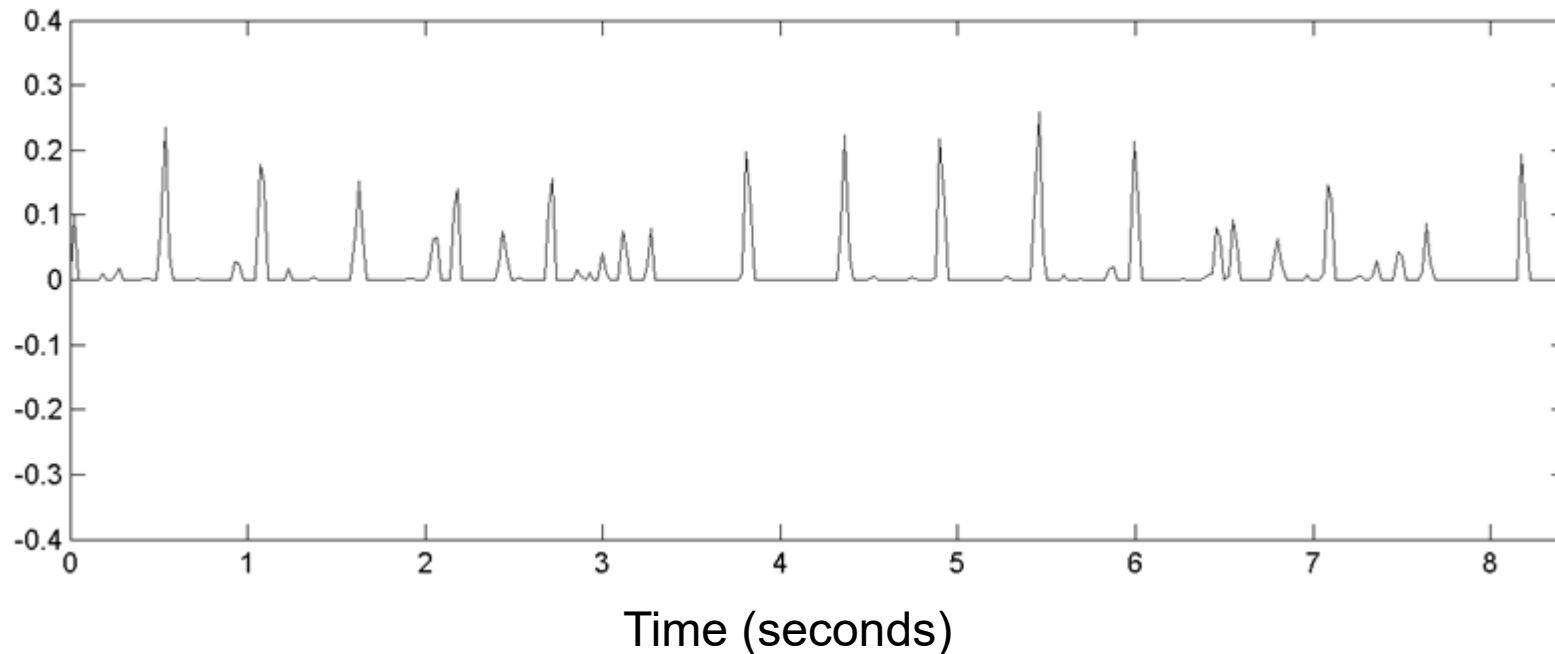
Onset Detection (Energy-Based)

Steps

1. Amplitude squaring
2. Windowing
3. Differentiation
4. Half wave rectification

Only energy increases are relevant for note onsets

Novelty curve

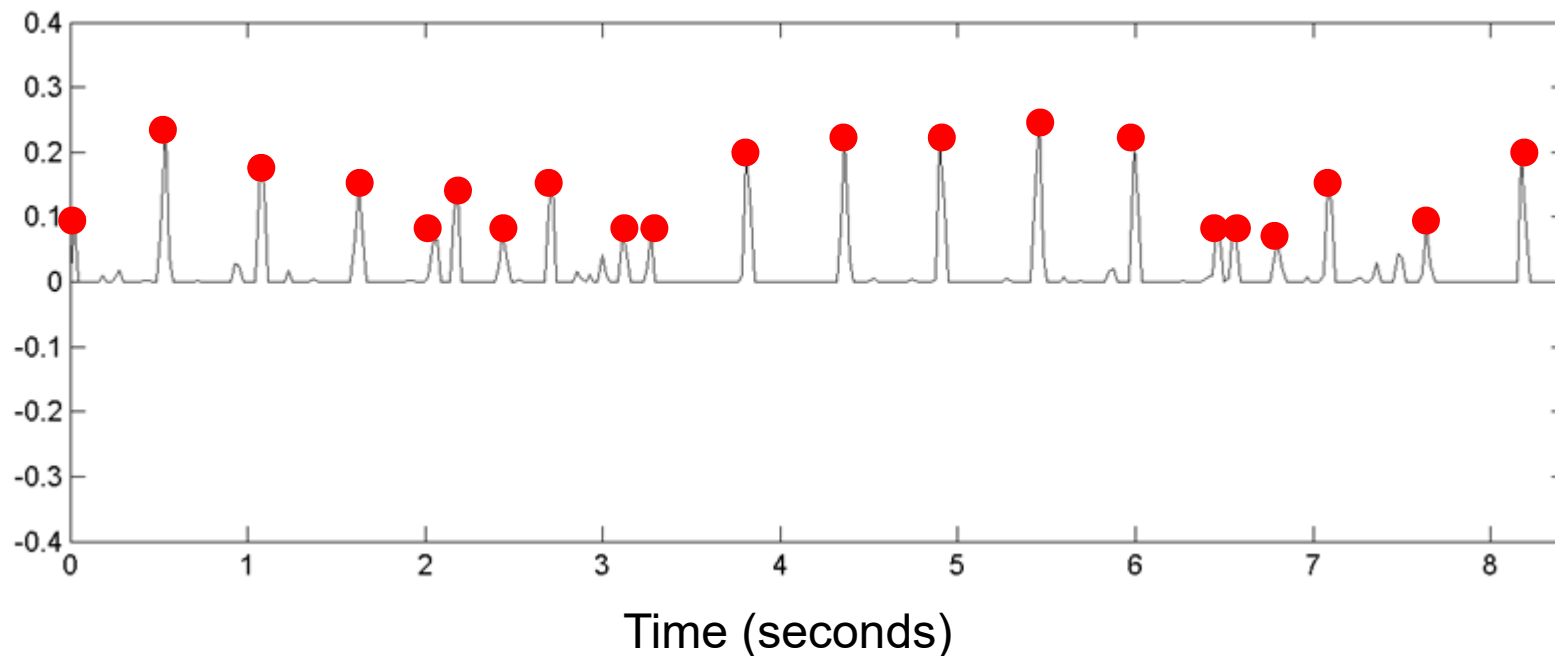


Onset Detection (Energy-Based)

Steps

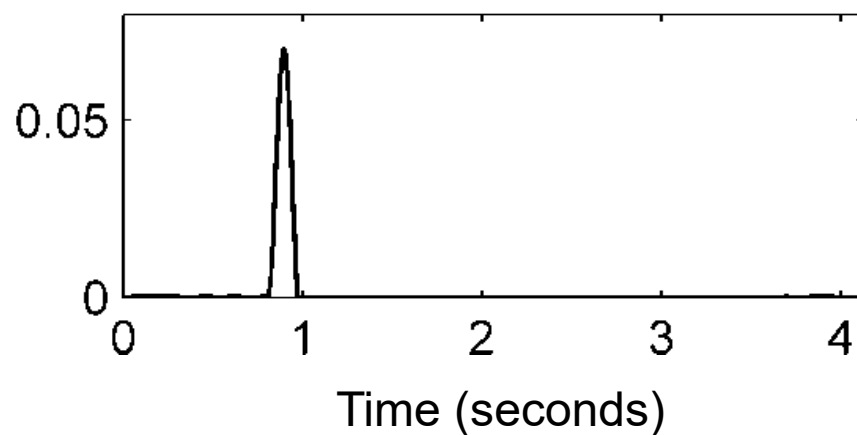
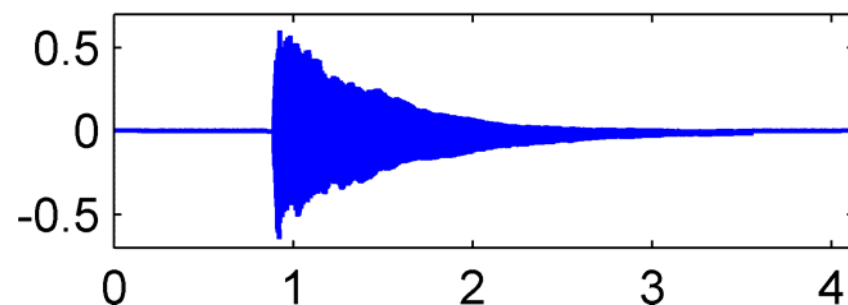
1. Amplitude squaring
2. Windowing
3. Differentiation
4. Half wave rectification
5. Peak picking

Peak positions indicate
note onset candidates



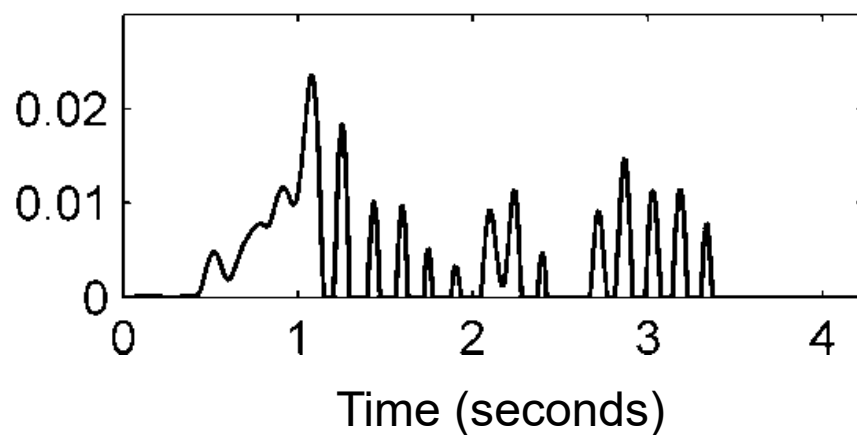
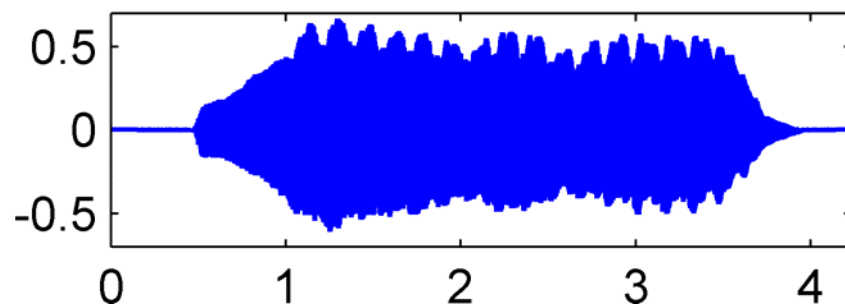
Onset Detection (Energy-Based)

Example: C4 played by piano



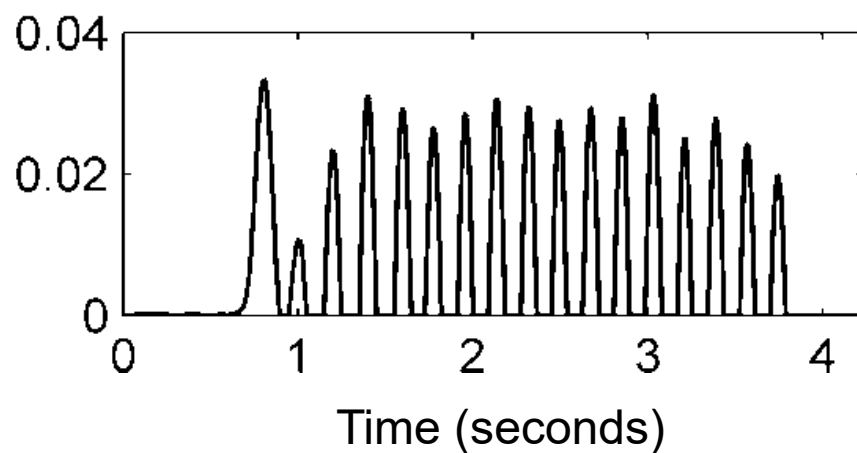
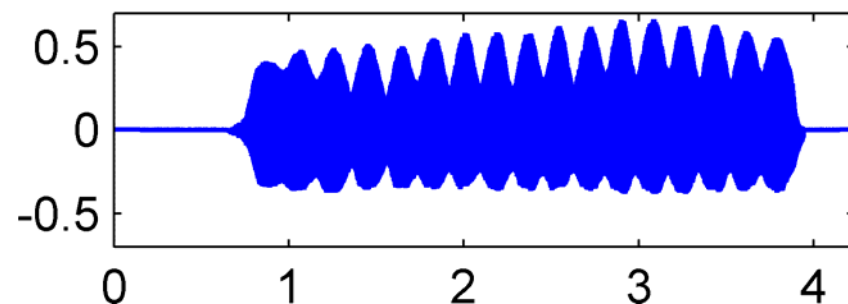
Onset Detection (Energy-Based)

Example: C4 played by violin



Onset Detection (Energy-Based)

Example: C4 played by flute



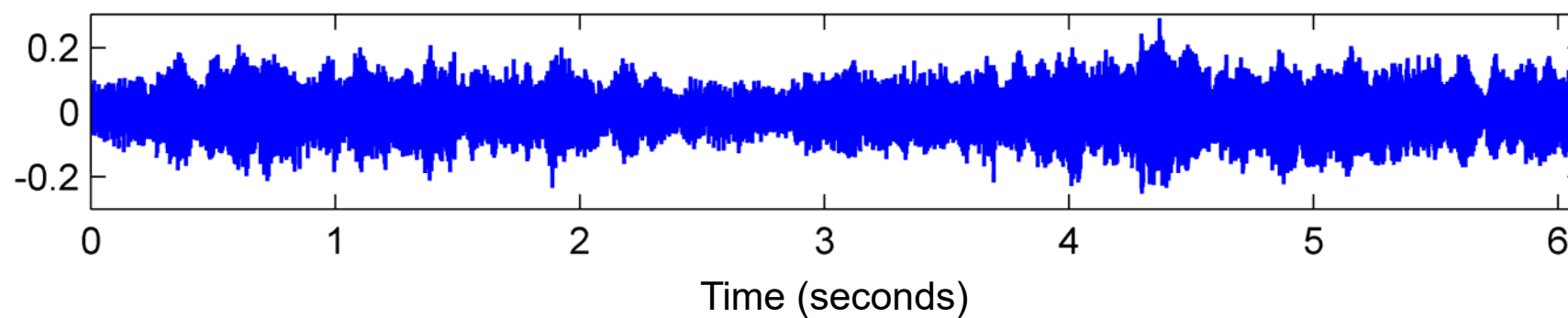
Onset Detection

- Energy curves often only work for percussive music
- Many instruments such as strings have weak note onsets
- No energy increase may be observable in complex sound mixtures
- More refined methods needed that capture
 - changes of spectral content
 - changes of pitch
 - changes of harmony

Onset Detection (Spectral-Based)



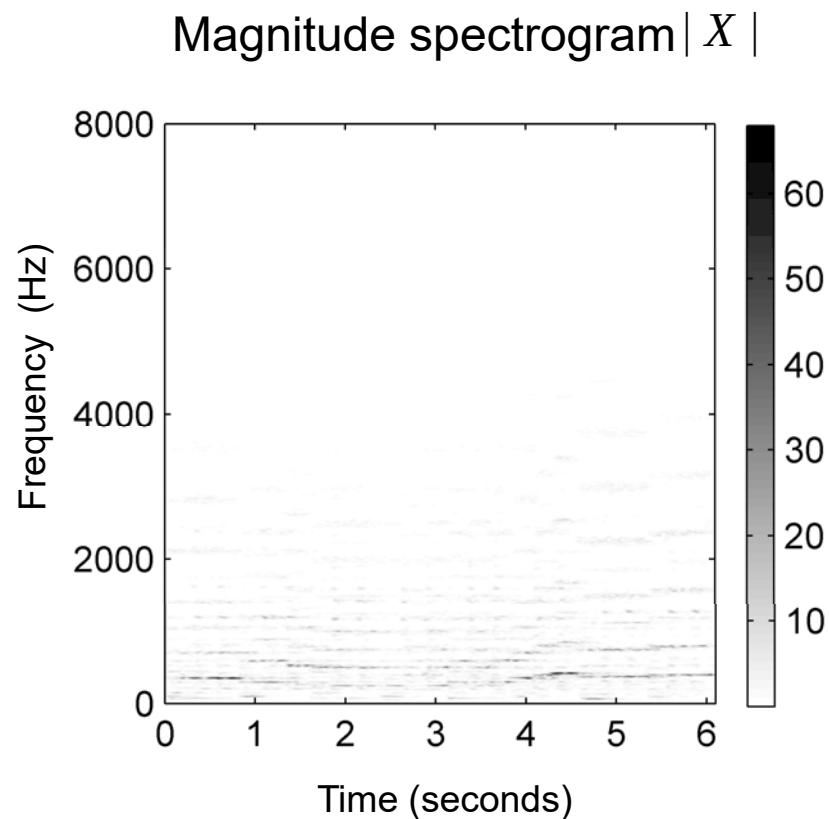
Audio recording



Onset Detection (Spectral-Based)

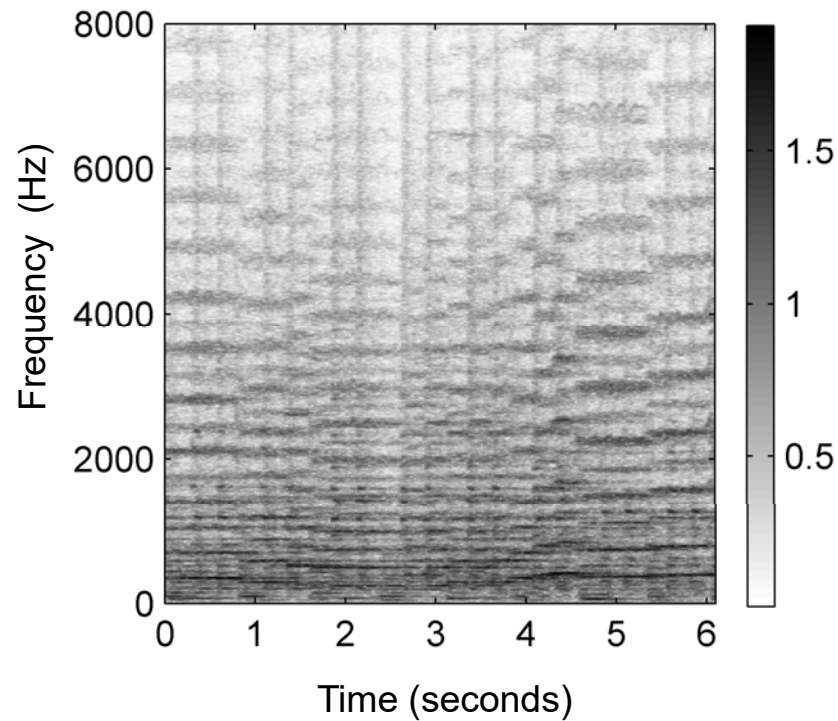
Steps:

1. Spectrogram



Onset Detection (Spectral-Based)

Compressed spectrogram Y



Steps:

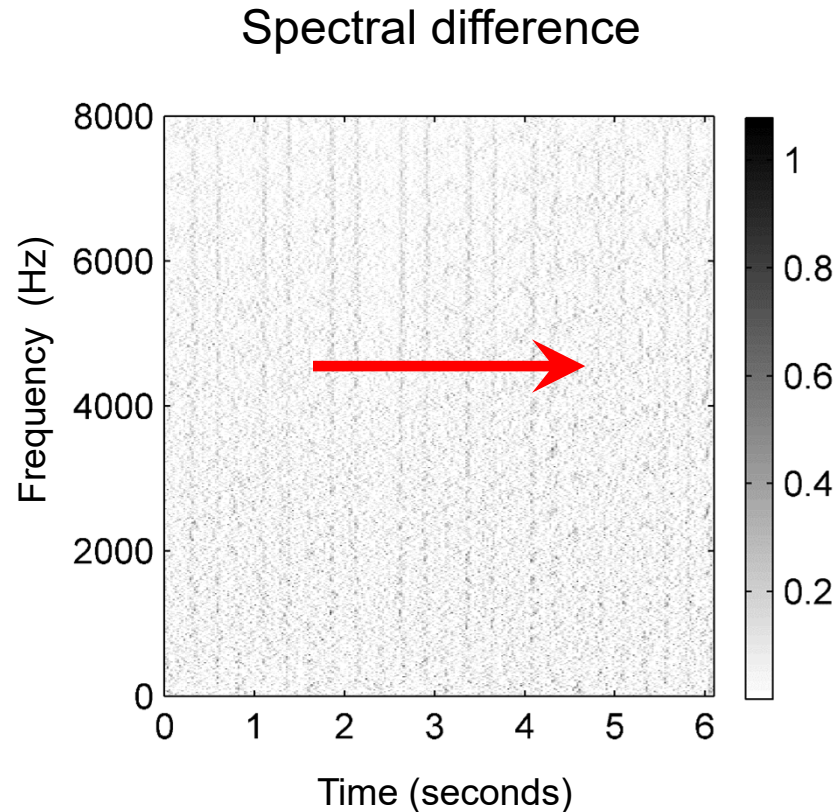
1. Spectrogram
2. Logarithmic compression

$$Y = \log(1 + C \cdot |X|)$$

Onset Detection (Spectral-Based)

Steps:

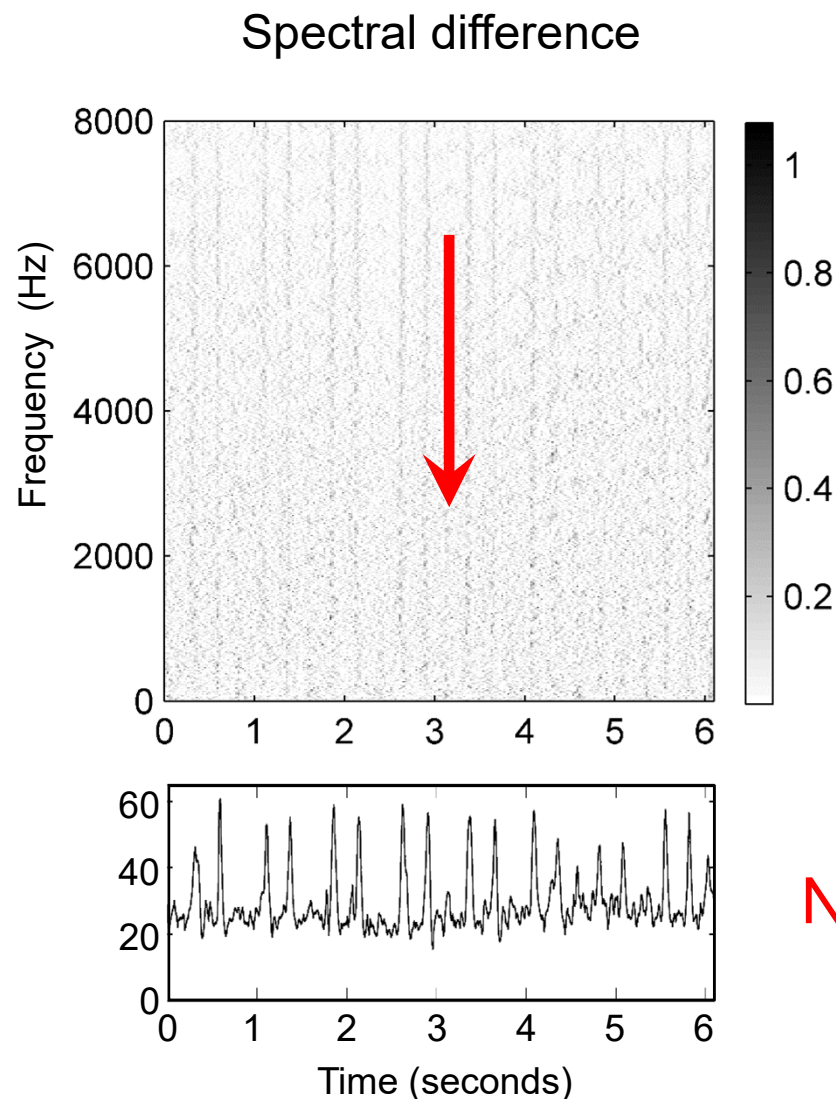
1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification



Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation



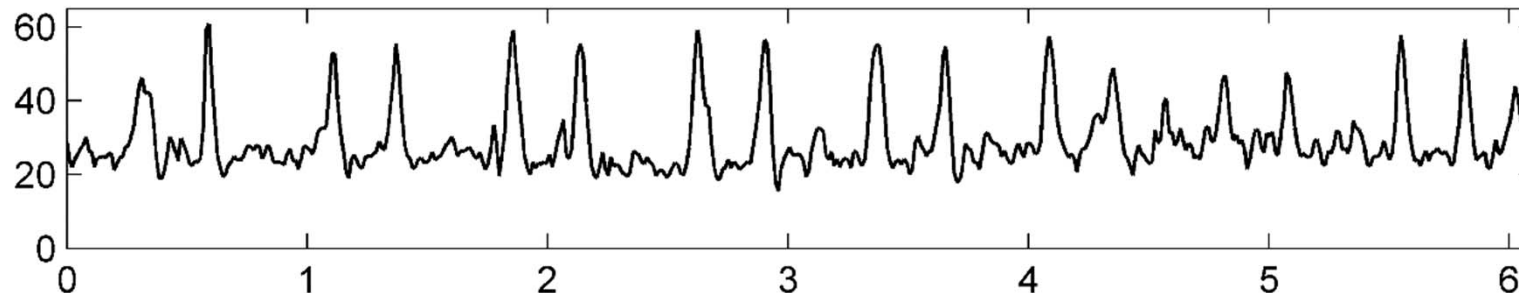
Novelty curve

Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation

Novelty curve



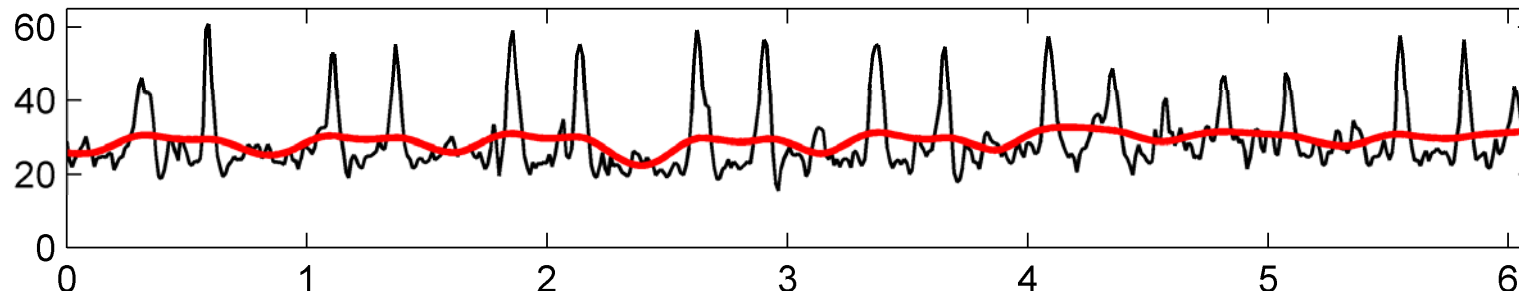
Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

Novelty curve

Subtraction of local average

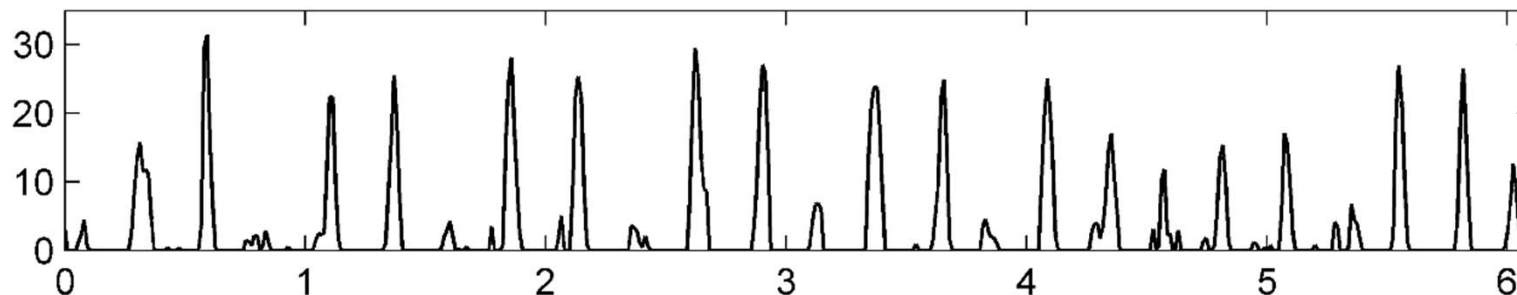


Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

Normalized novelty curve



Onset Detection (Spectral-Based)

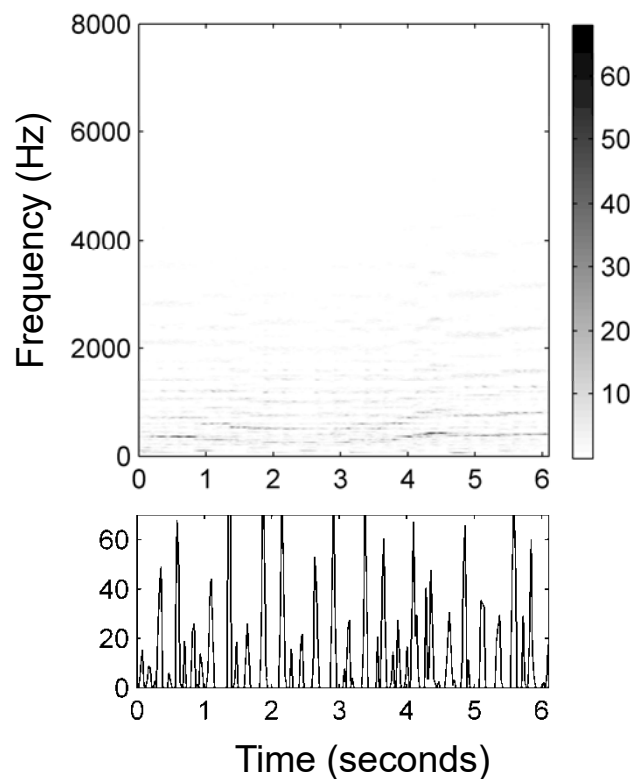
- Spectrogram $X = (X(t, k))_{t, k}$ $t \in [1 : T]$
 $k \in [1 : K]$
- Compressed Spectrogram $Y := \log(1 + C \cdot |X|)$ $C > 1$.
- Novelty curve $\Delta : [1 : T - 1] \rightarrow \mathbb{R}$:

$$\Delta(t) := \sum_{k=1}^K |Y(t+1, k) - Y(t, k)|_{\geq 0}$$

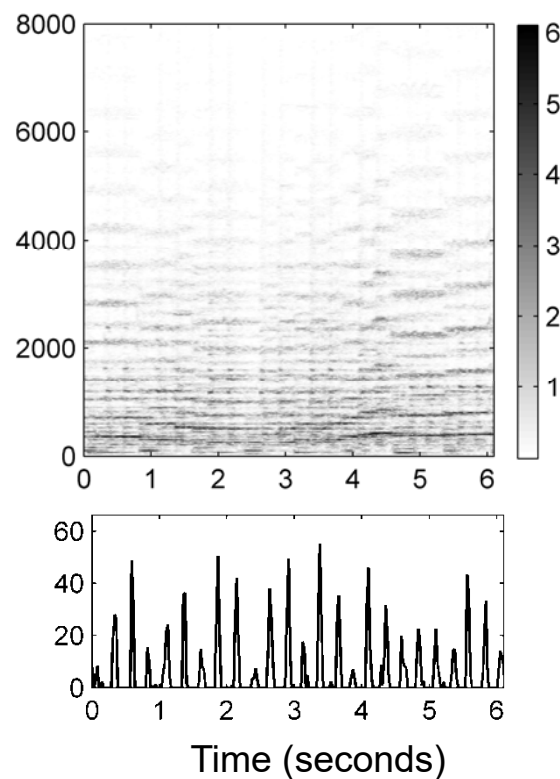
Logarithmic Compression

$$Y = \log(1 + C \cdot |X|)$$

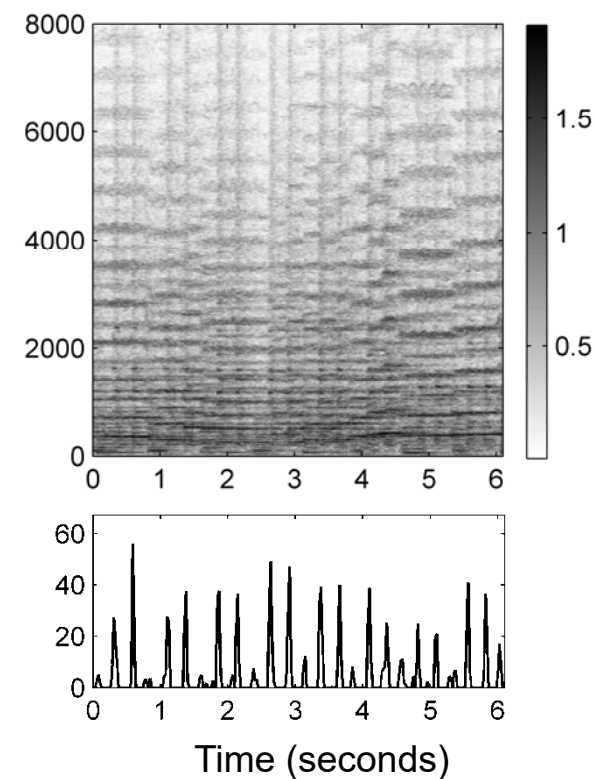
No compression



$C = 1$

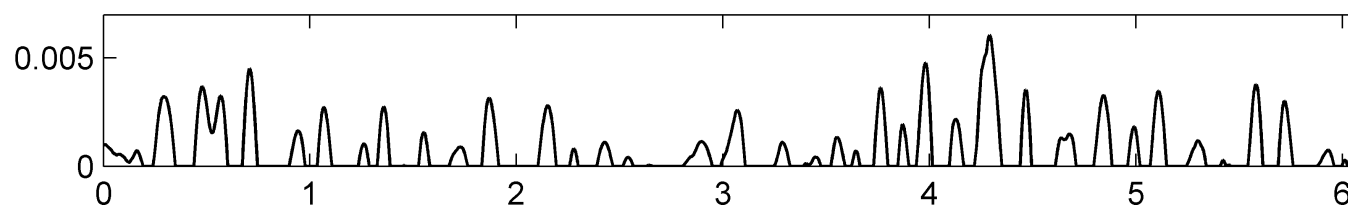


$C = 100$

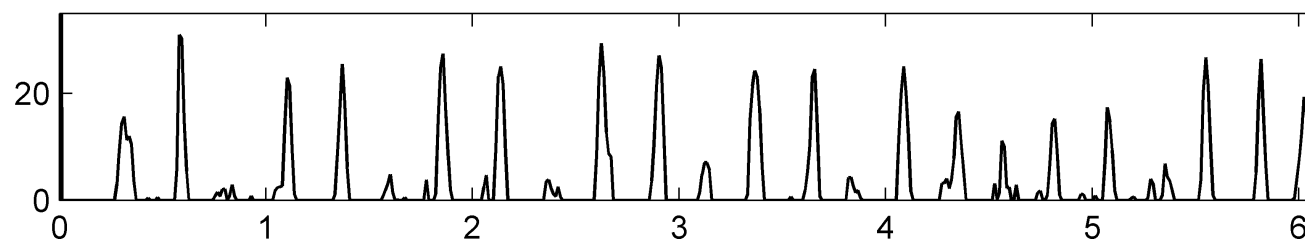


Onset Detection

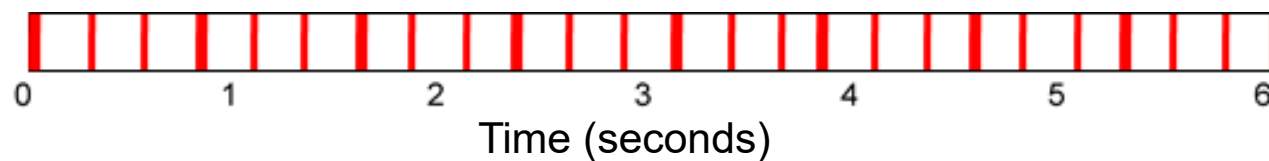
Energy-based novelty curve



Spectral-based novelty curve

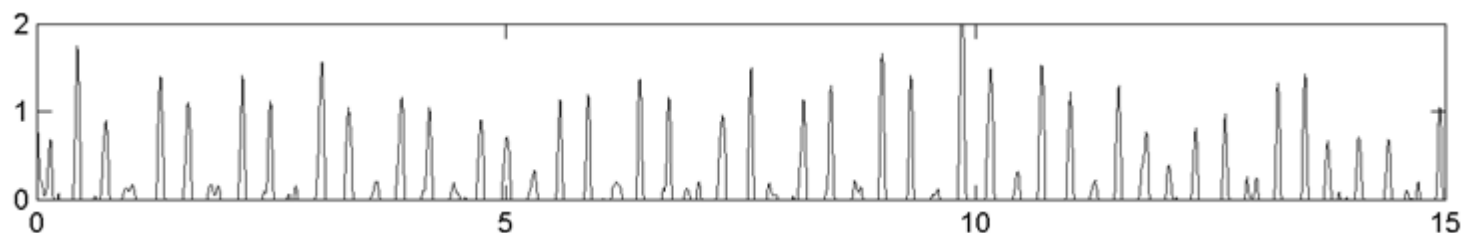


Manuel onset annotations

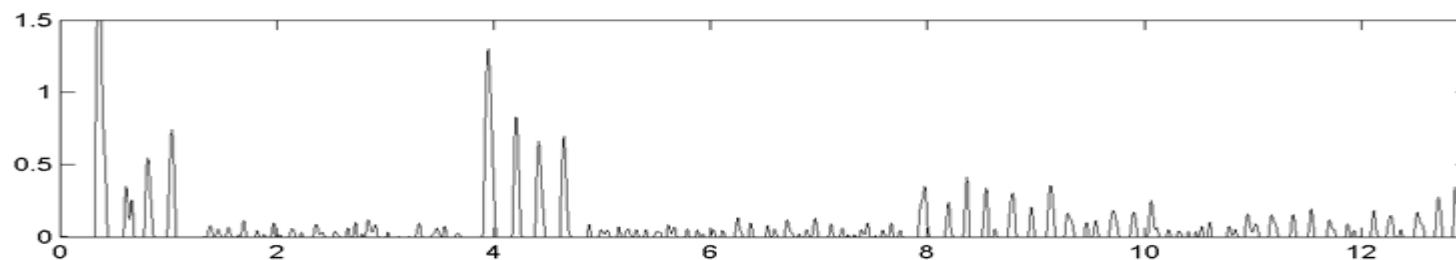


Onset Detection

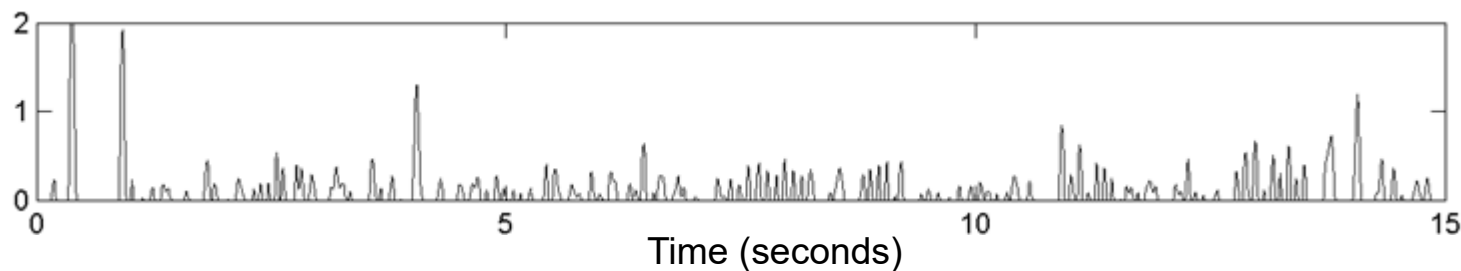
Shostakovich – 2nd Waltz



Beethoven – Fifth Symphony



Borodin – String Quartet No. 2



Onset Detection

Drumbeat



Going Home



Lyphard melodie



Por una cabeza



Donau



Beat and Tempo

What is a beat?

- Steady pulse that drives music forward and provides the temporal framework of a piece of music
- Sequence of perceived pulses that are equally spaced in time
- The pulse a human taps along when listening to the music

[Parncutt 1994]

[Sethares 2007]

[Large/Palmer 2002]

[Lerdahl/ Jackendoff 1983]

[Fitch/ Rosenfeld 2007]

The term **tempo** then refers to the speed of the pulse.

Beat and Tempo

Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

Beat and Tempo

Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

[Scheirer, JASA 1998]

Methods

[Ellis, JNMR 2007]

- Comb-filter methods
- Autocorrelation
- Fourier transform

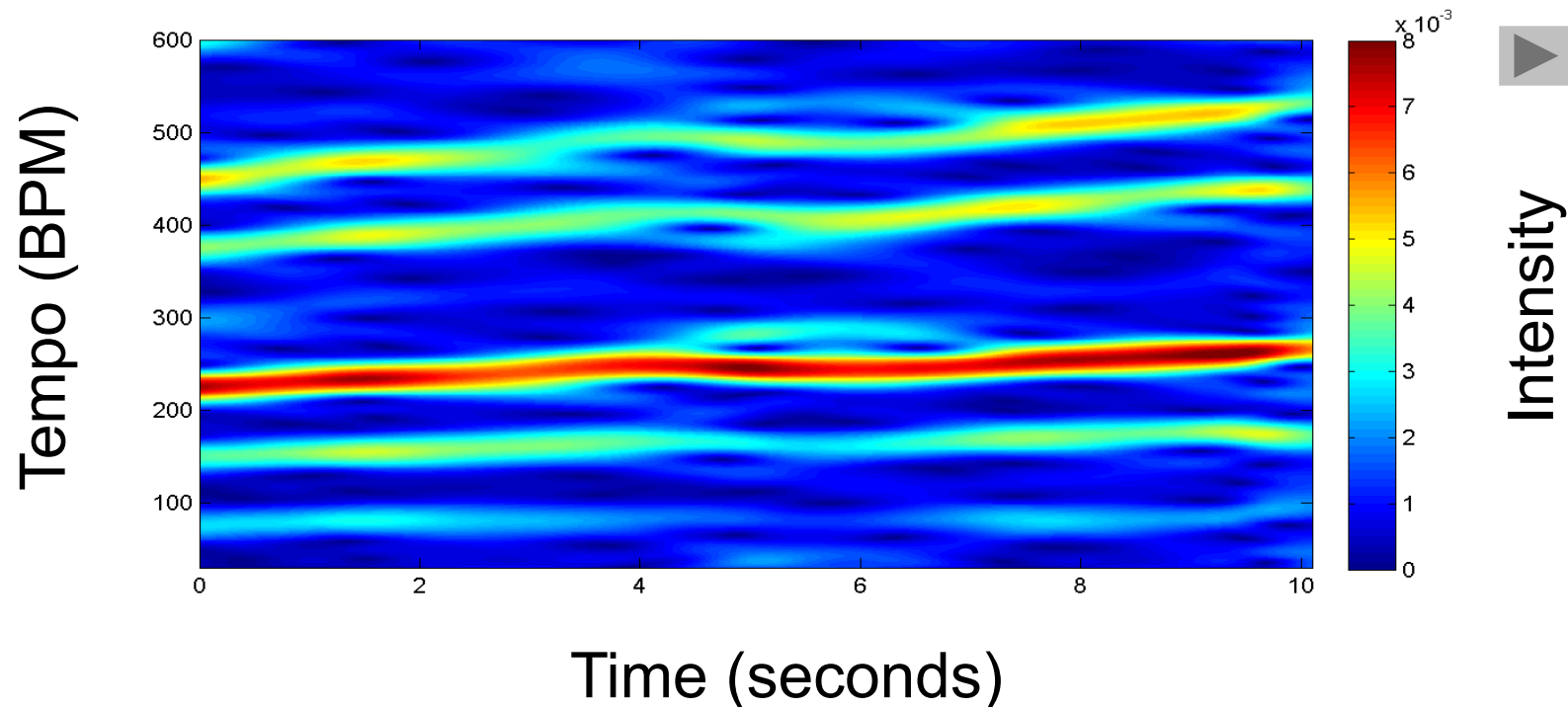
[Davies/Plumbley, IEEE-TASLP 2007]

[Peeters, JASP 2007]

[Grosche/Müller, ISMIR 2009]
[Grosche/Müller, IEEE-TASLP 2011]

Tempogram

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.



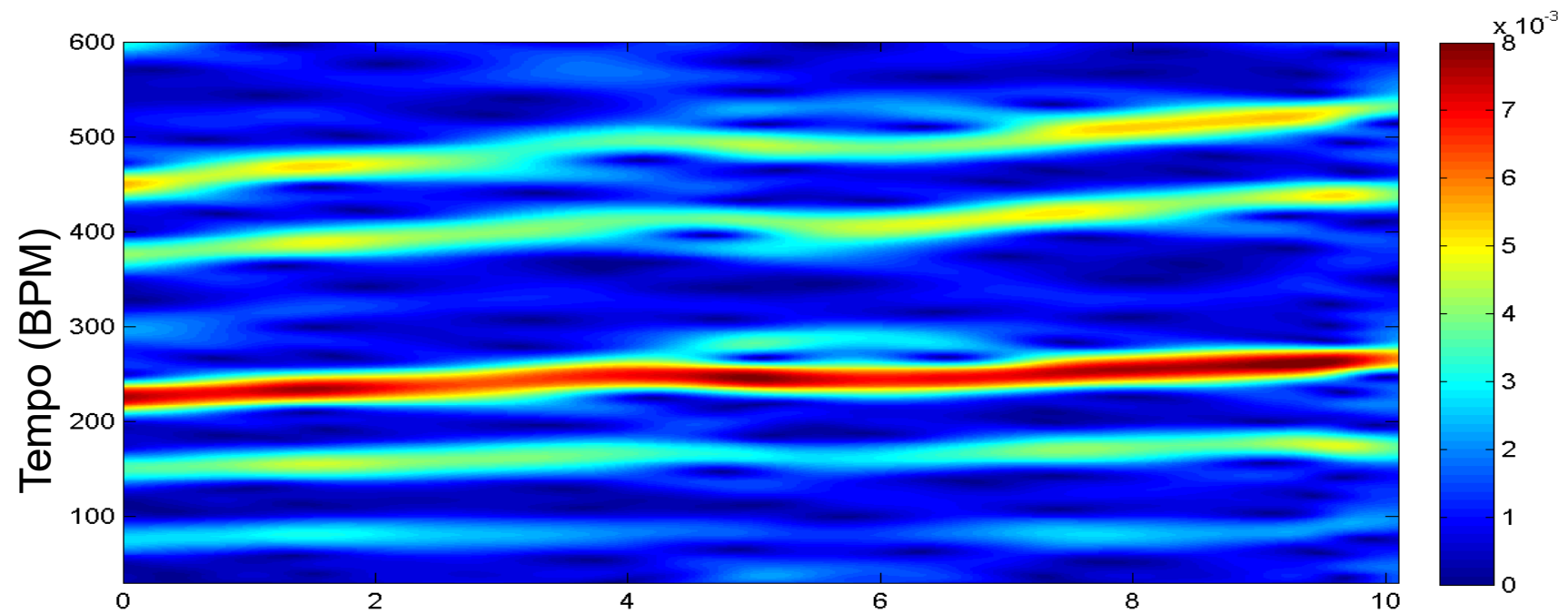
Tempogram (Fourier)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

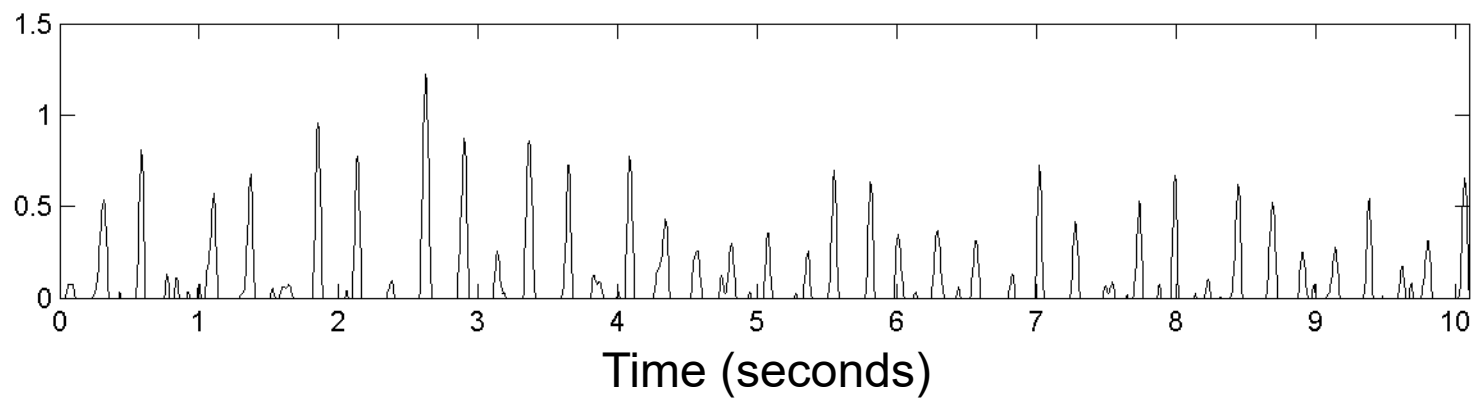
Fourier-based method

- Compute a spectrogram (STFT) of the novelty curve
- Convert frequency axis (given in Hertz) into tempo axis (given in BPM)
- Magnitude spectrogram indicates local tempo

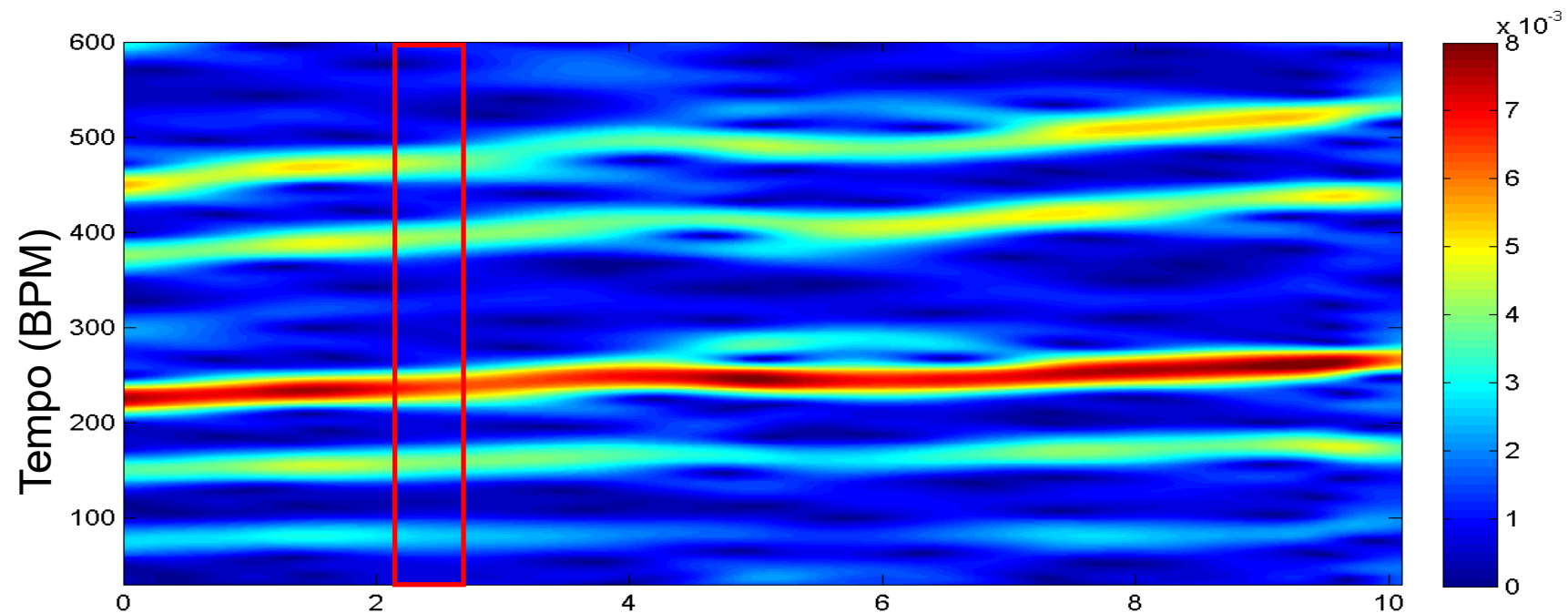
Tempogram (Fourier)



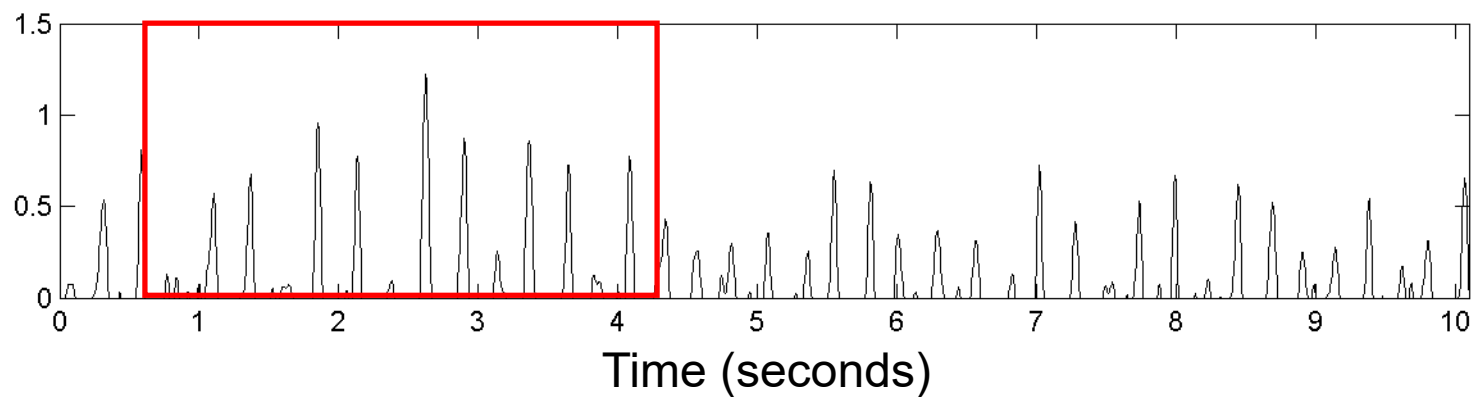
Novelty curve



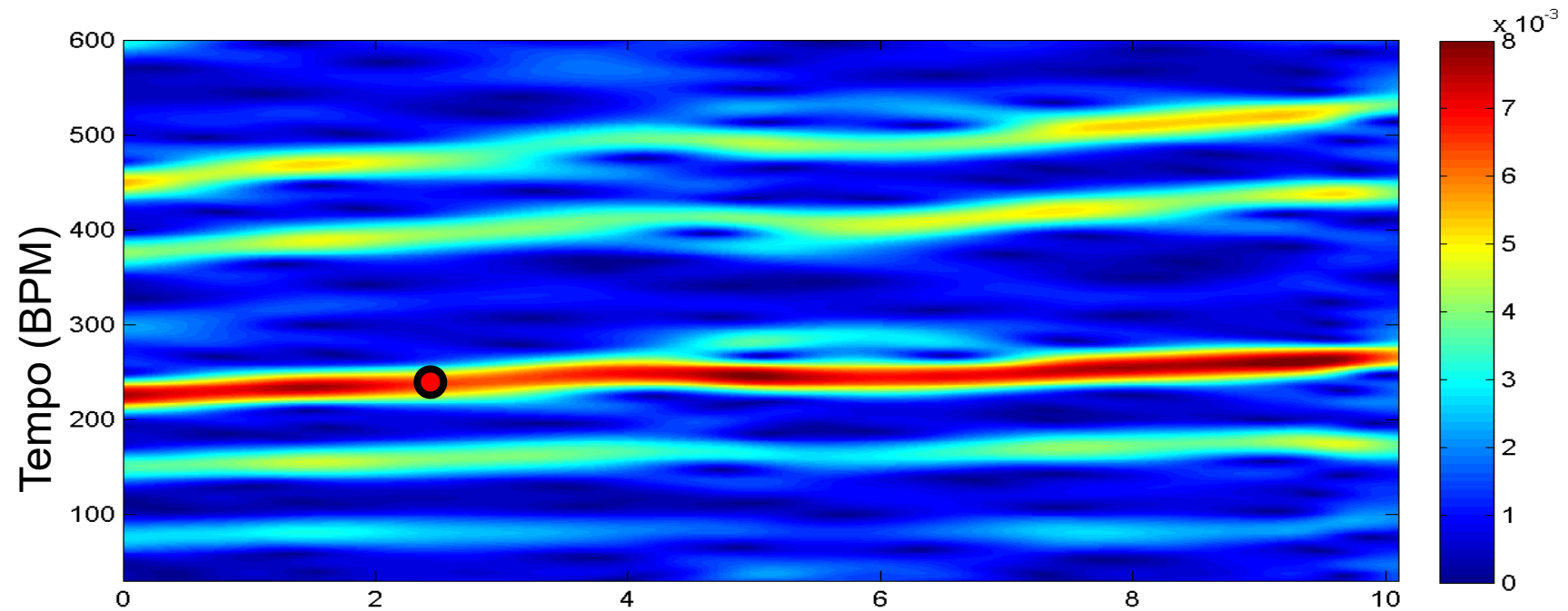
Tempogram (Fourier)



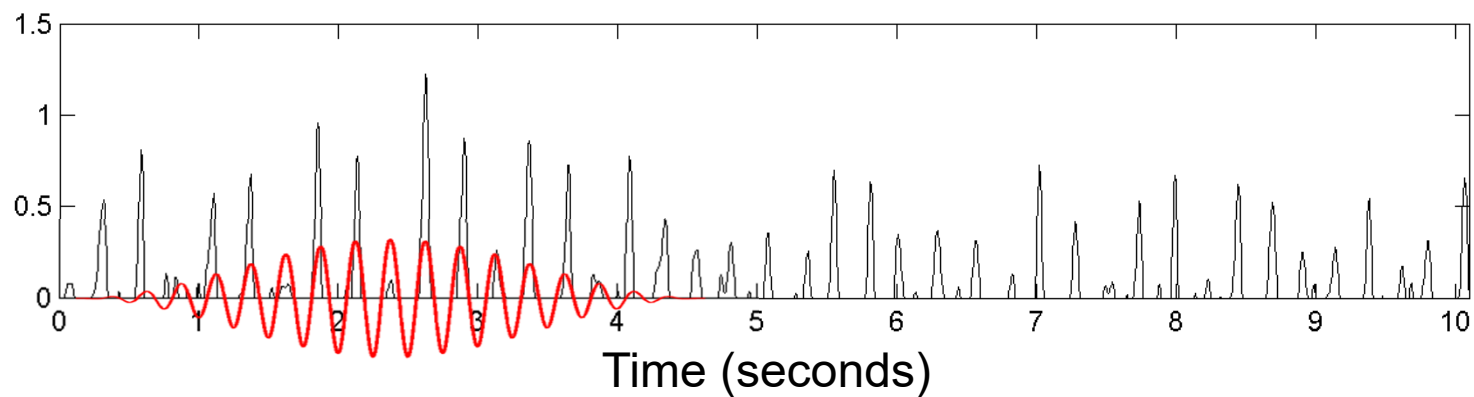
Novelty curve (local section)



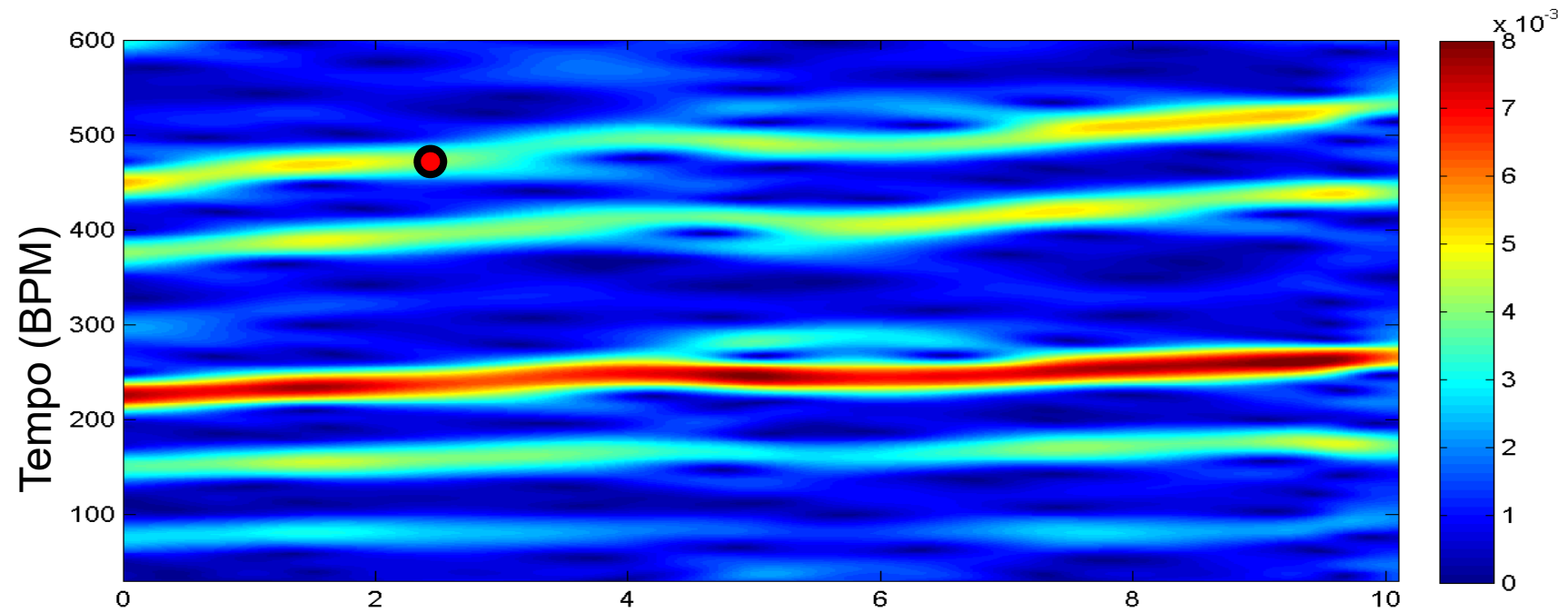
Tempogram (Fourier)



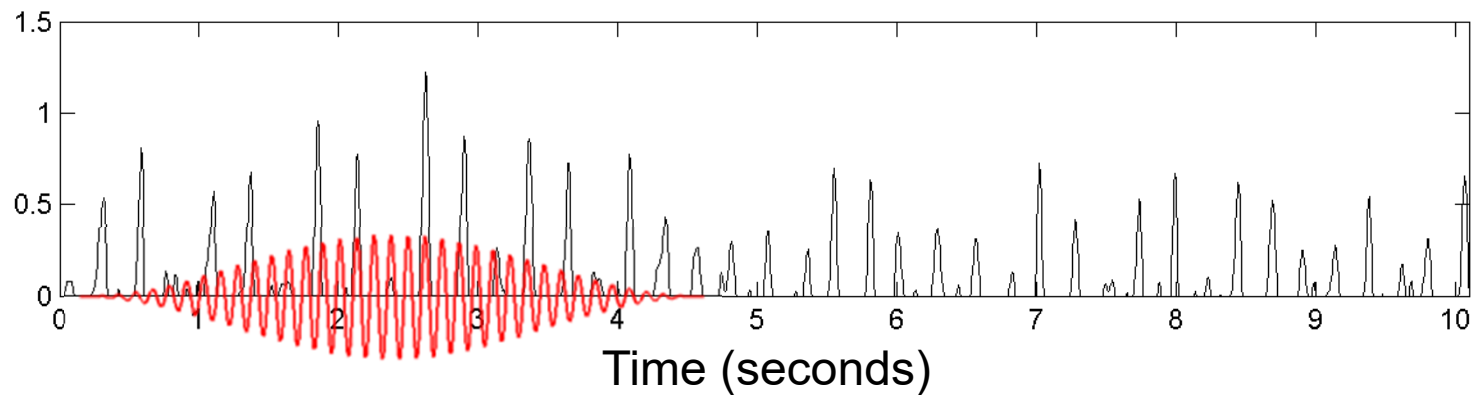
Windowed sinusoidal



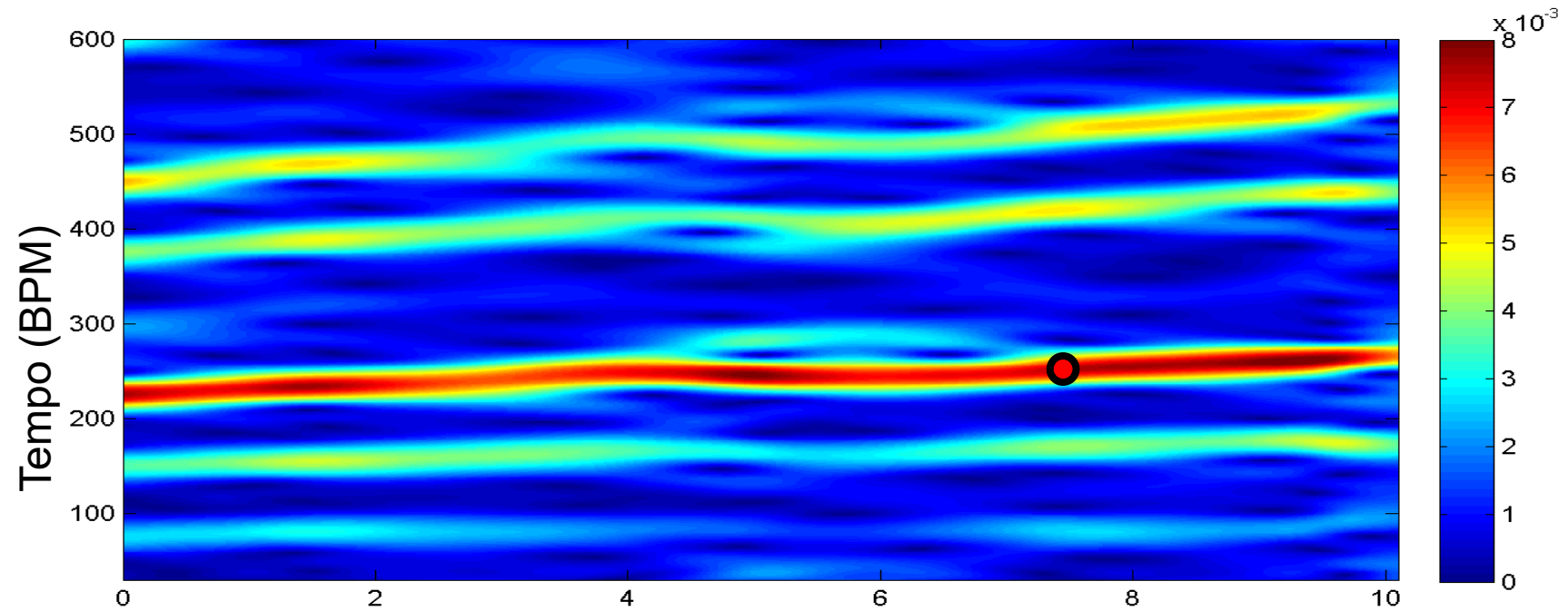
Tempogram (Fourier)



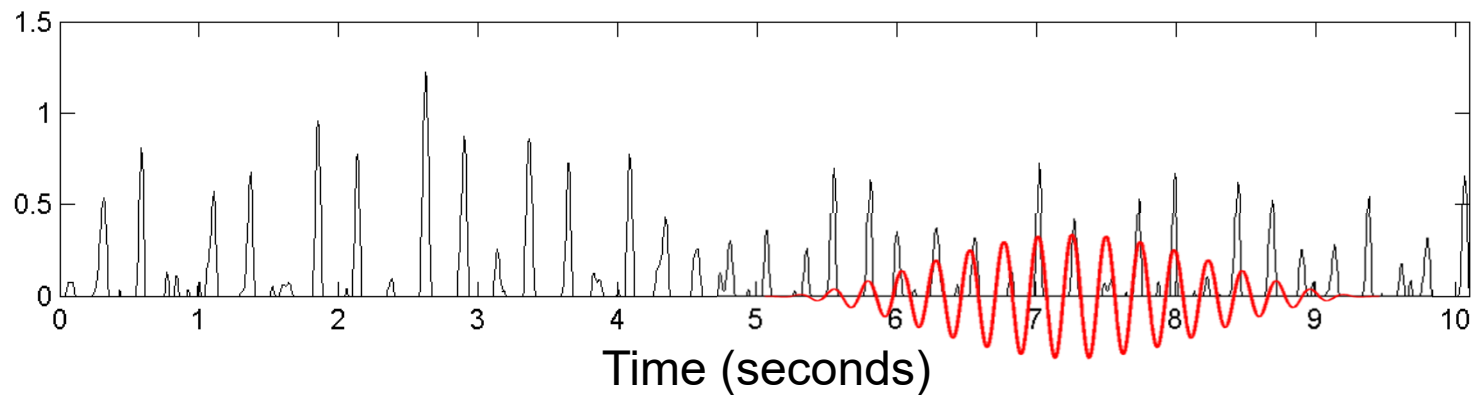
Windowed sinusoidal



Tempogram (Fourier)



Windowed sinusoidal



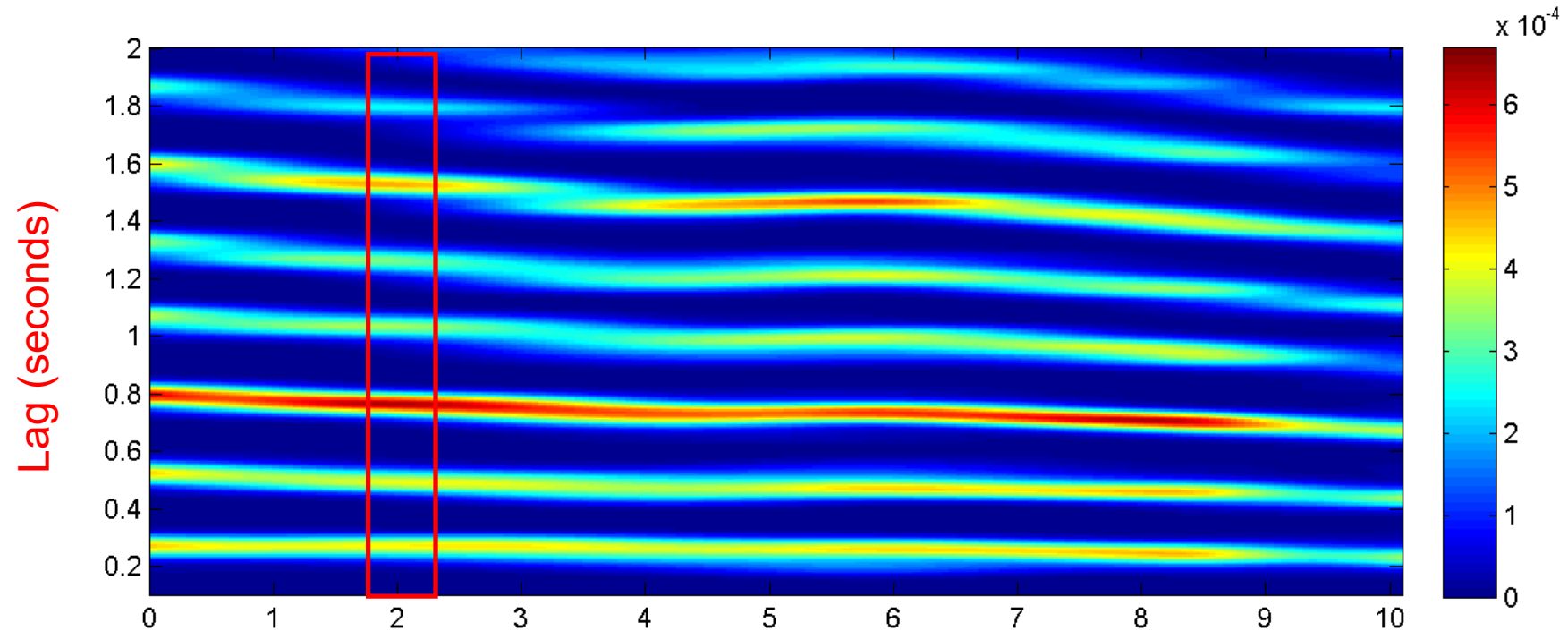
Tempogram (Autocorrelation)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

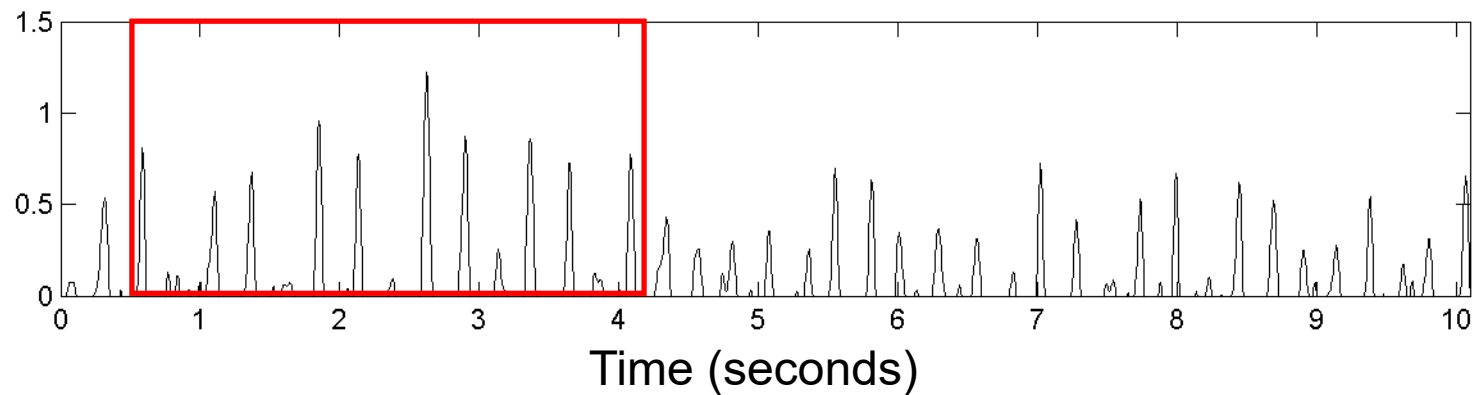
Autocorrelation-based method

- Compare novelty curve with time-lagged local sections of itself
- Convert lag-axis (given in seconds) into tempo axis (given in BPM)
- Autocorrelogram indicates local tempo

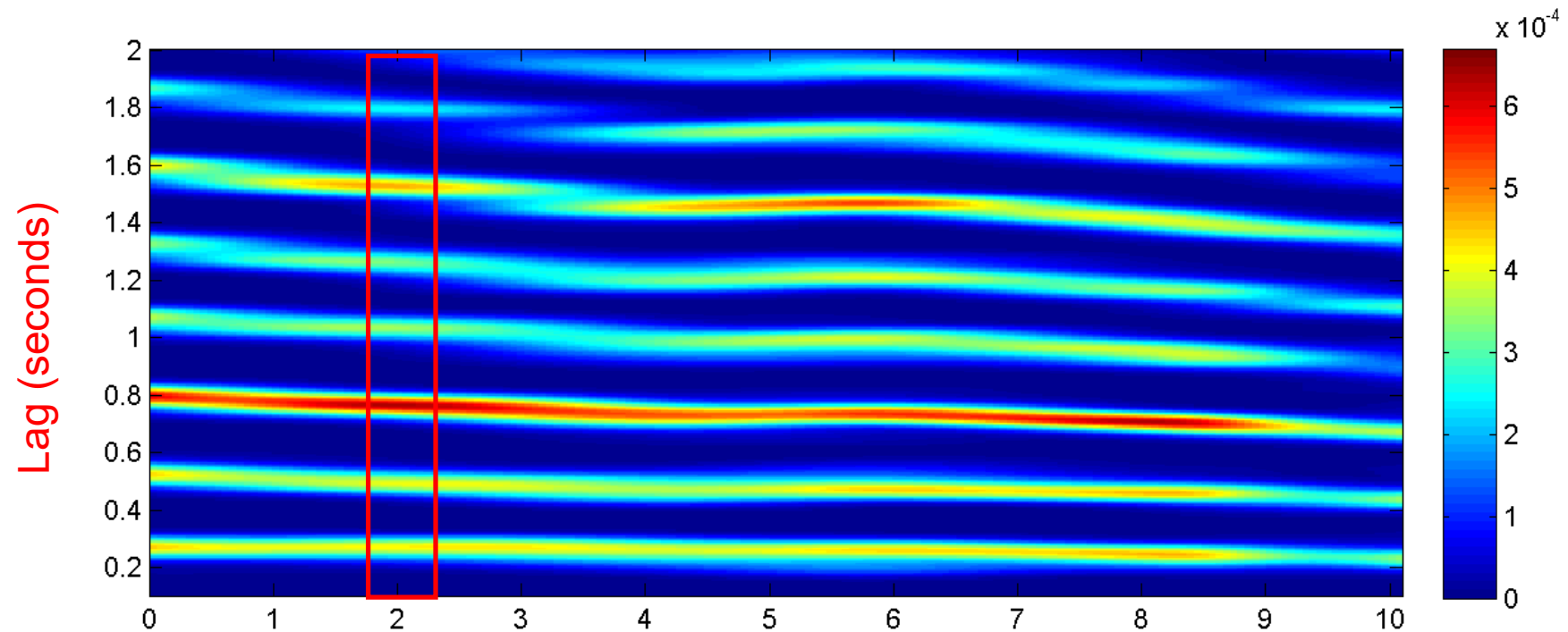
Tempogram (Autocorrelation)



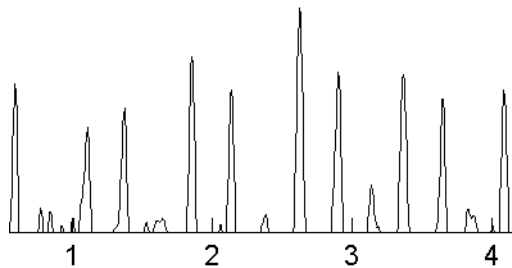
Novelty curve (local section)



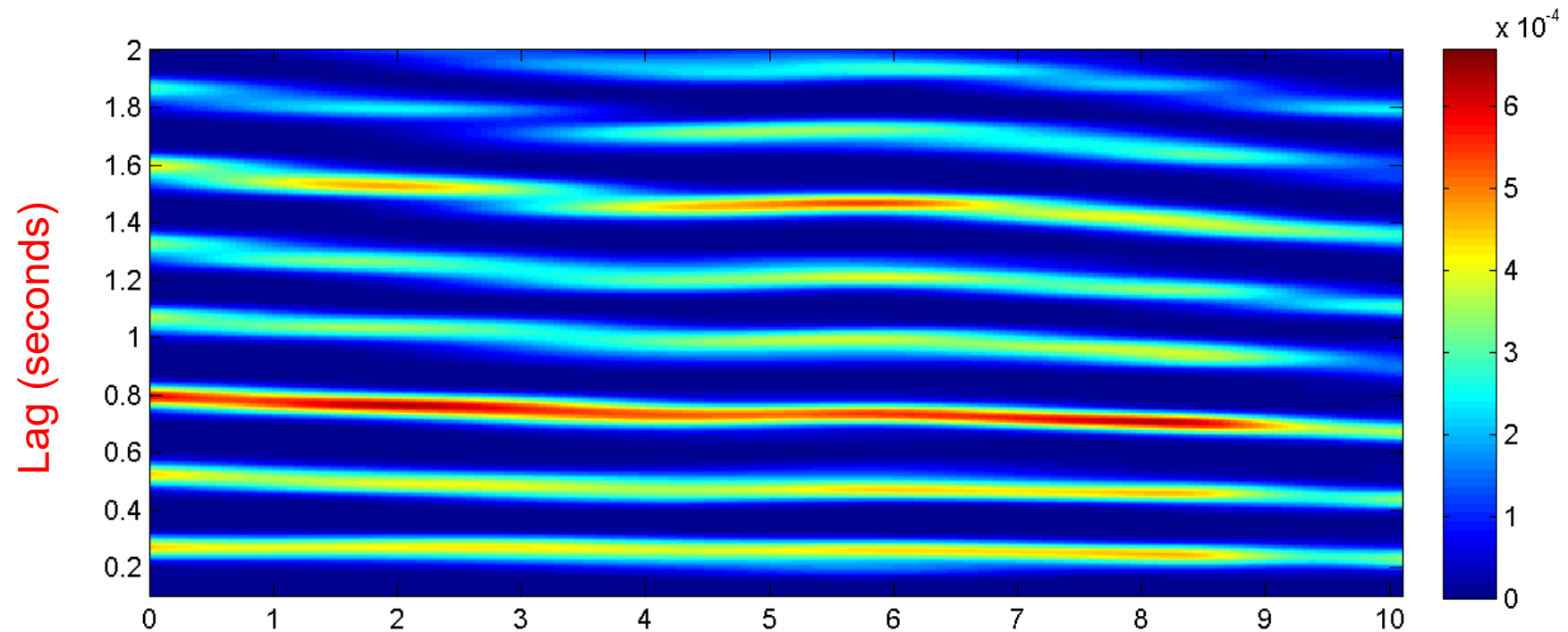
Tempogram (Autocorrelation)



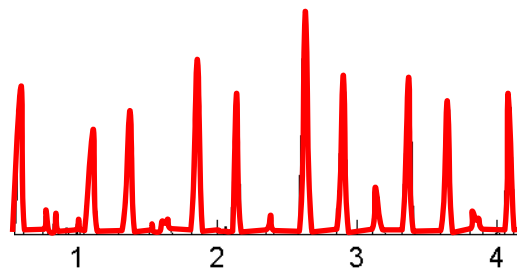
Windowed autocorrelation



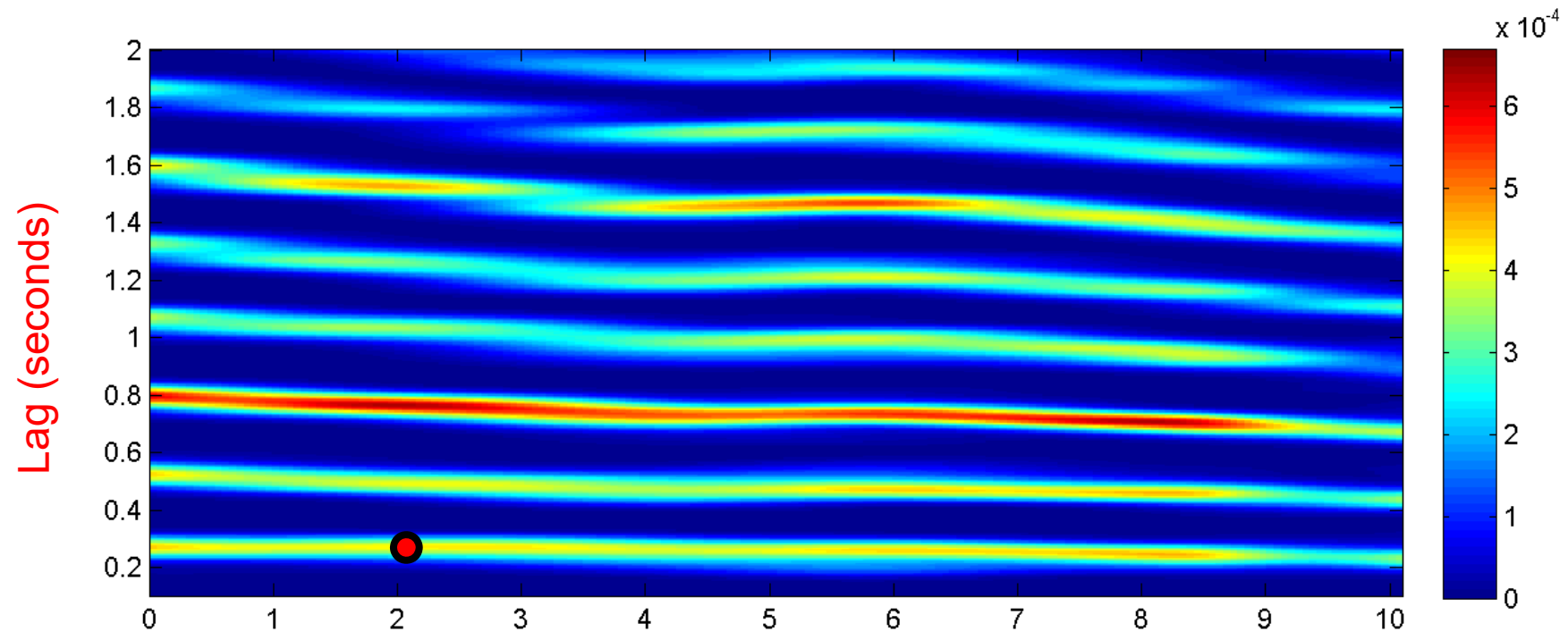
Tempogram (Autocorrelation)



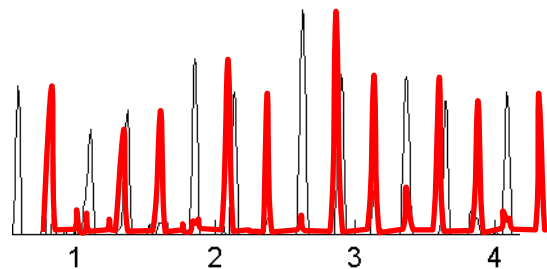
Lag = 0 (seconds)



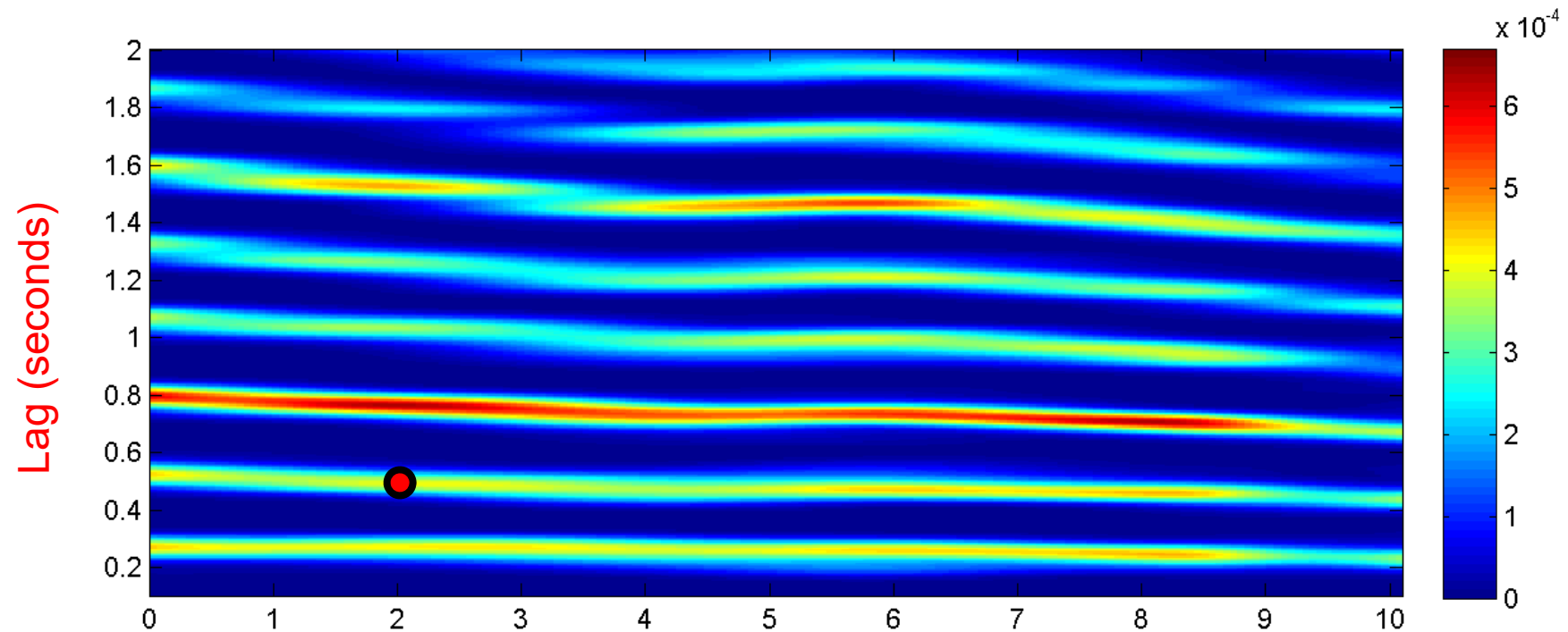
Tempogram (Autocorrelation)



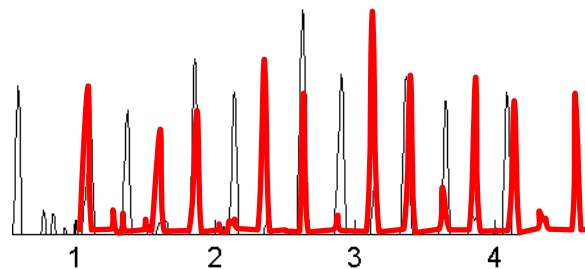
Lag = 0.26 (seconds)



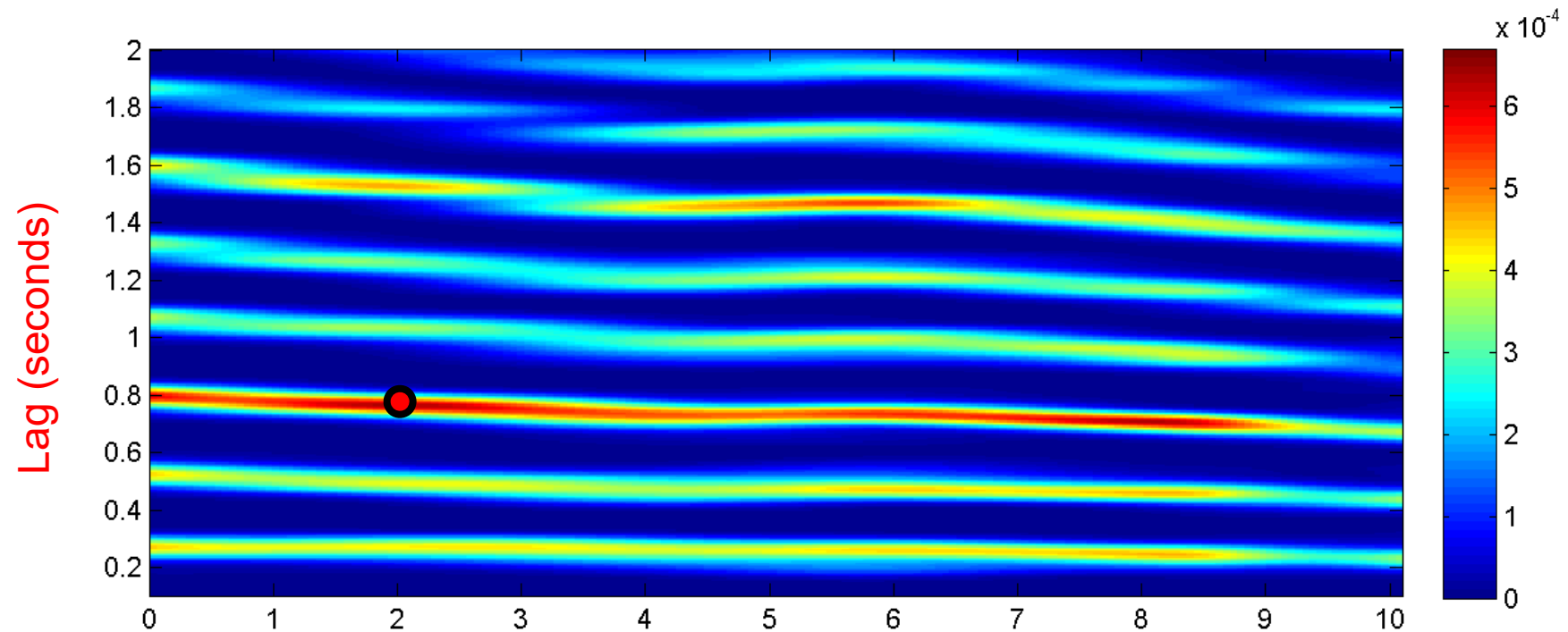
Tempogram (Autocorrelation)



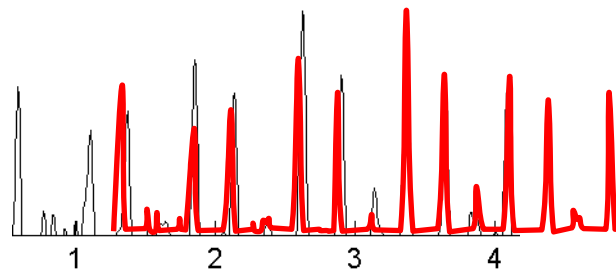
Lag = 0.52 (seconds)



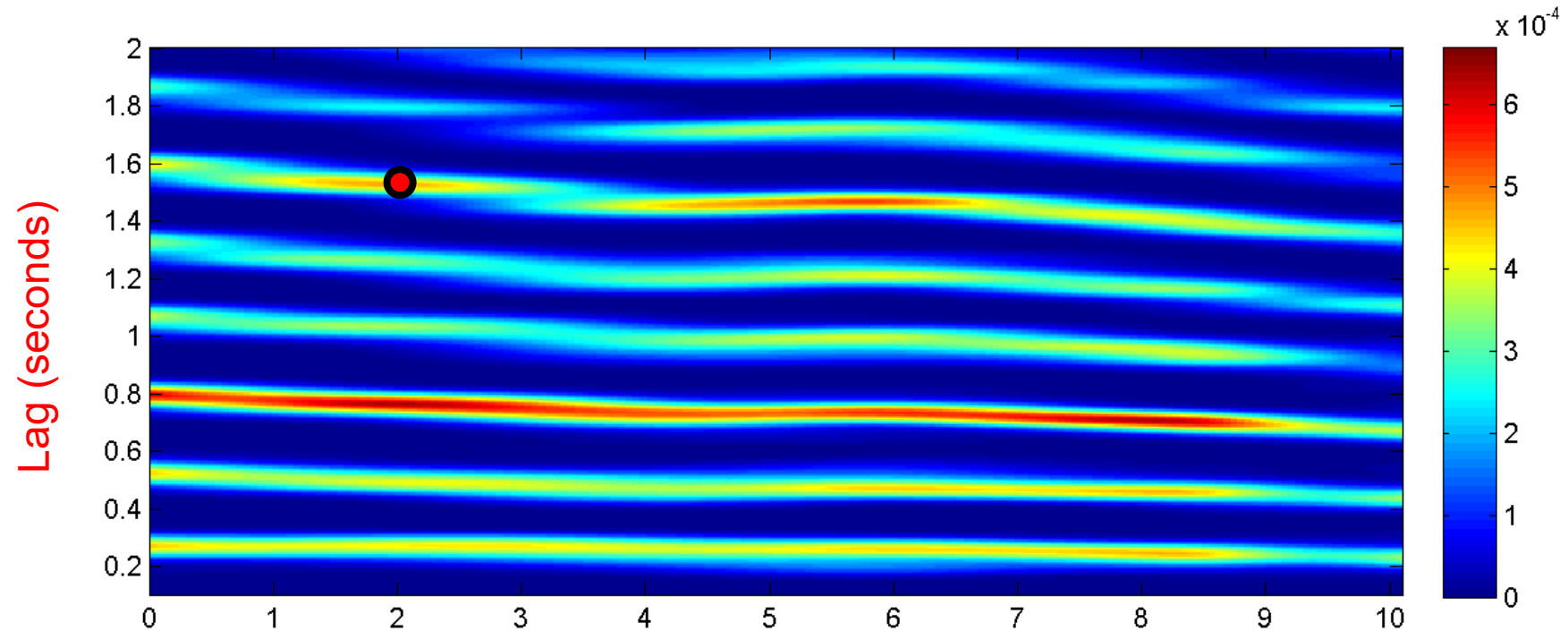
Tempogram (Autocorrelation)



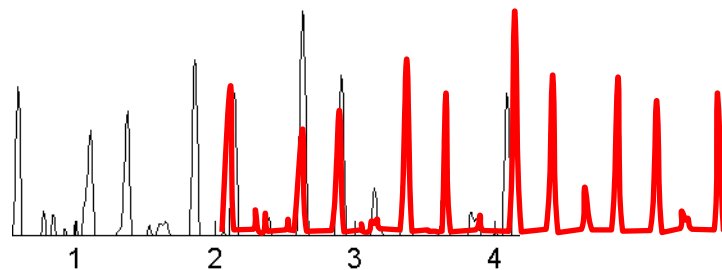
Lag = 0.78 (seconds)



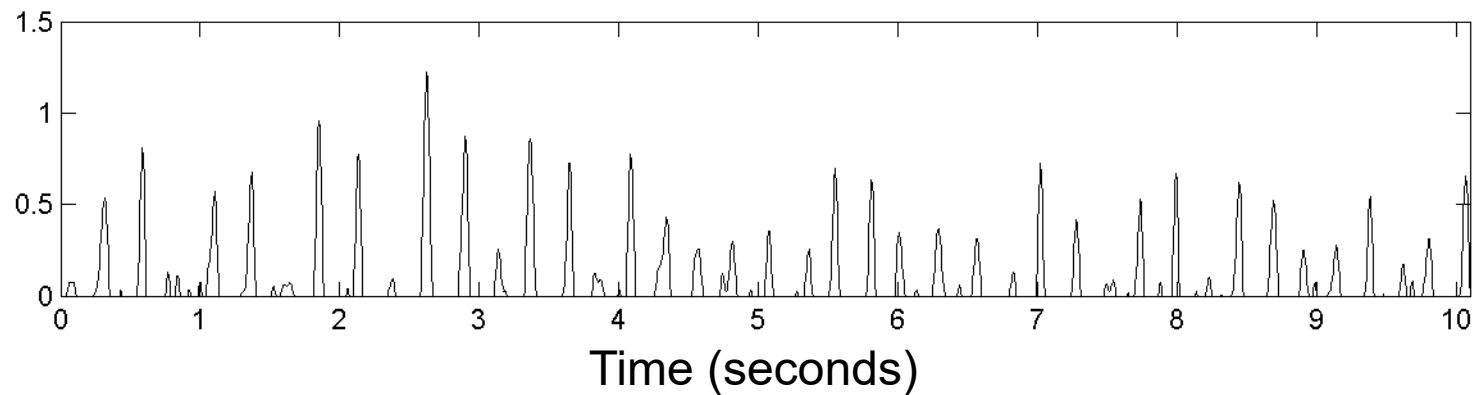
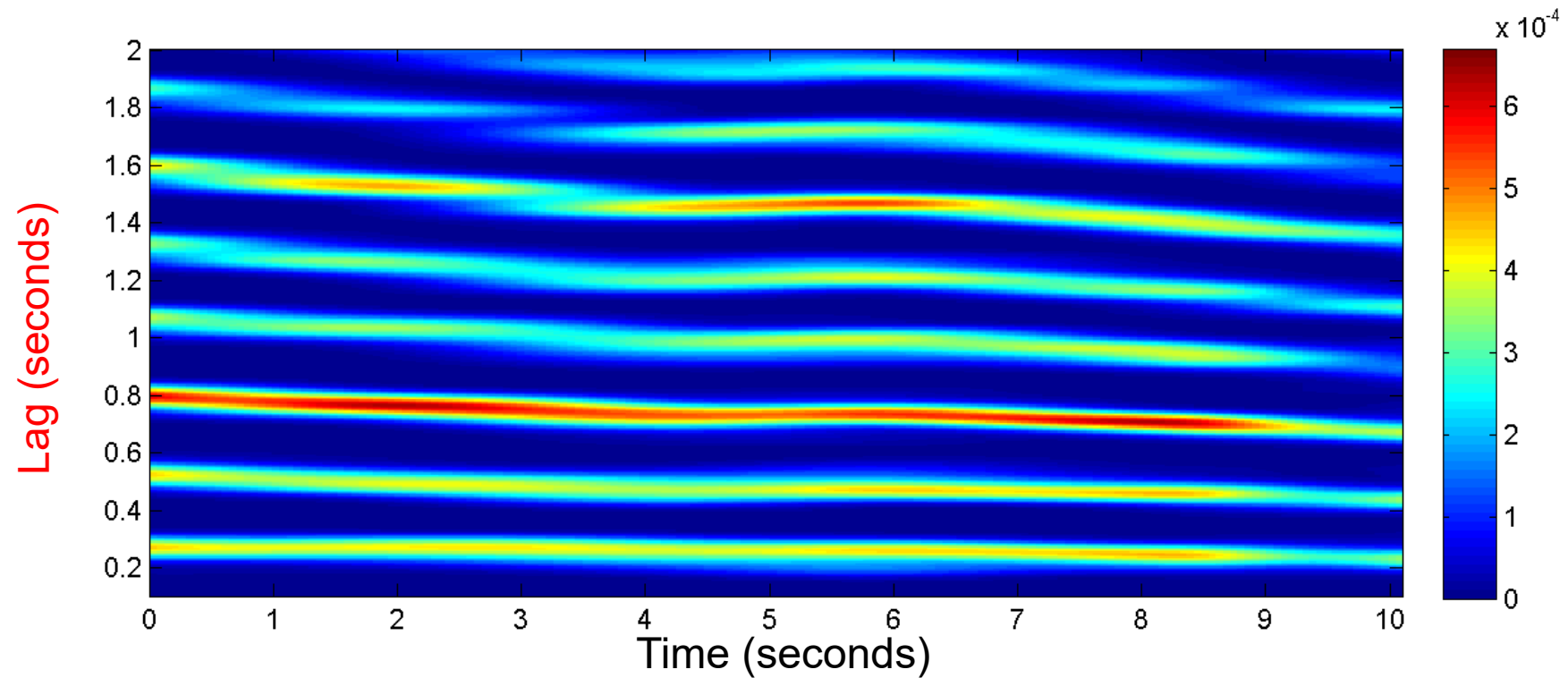
Tempogram (Autocorrelation)



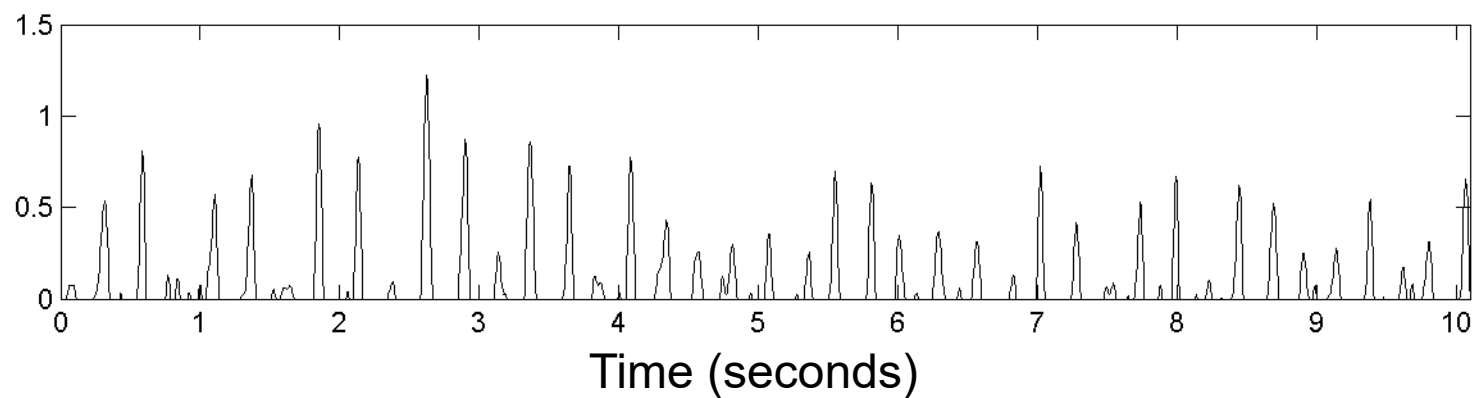
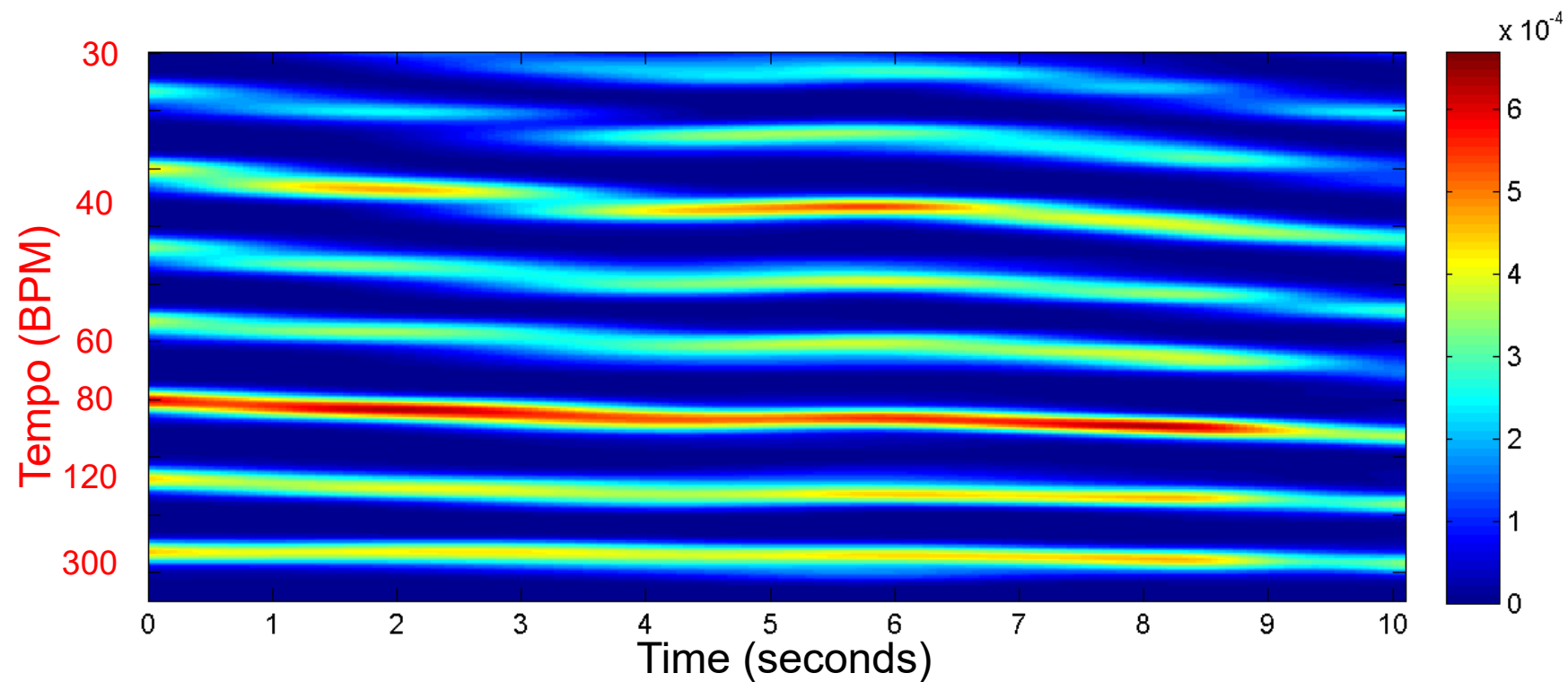
Lag = 1.56 (seconds)



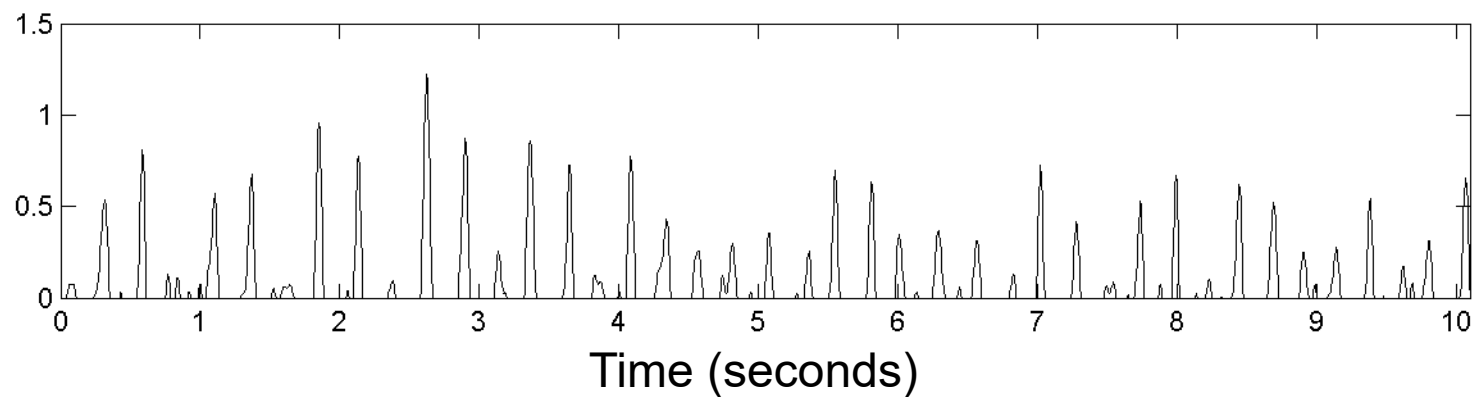
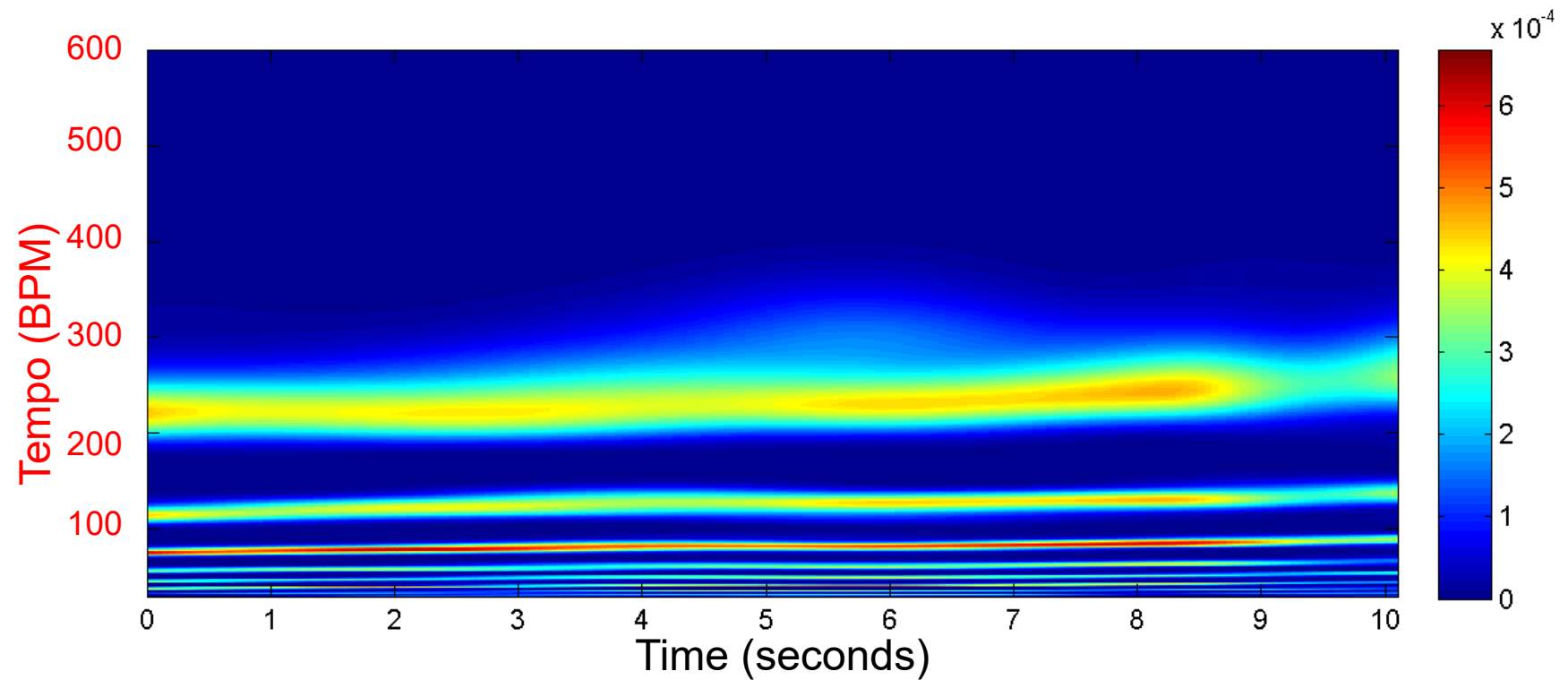
Tempogram (Autocorrelation)



Tempogram (Autocorrelation)

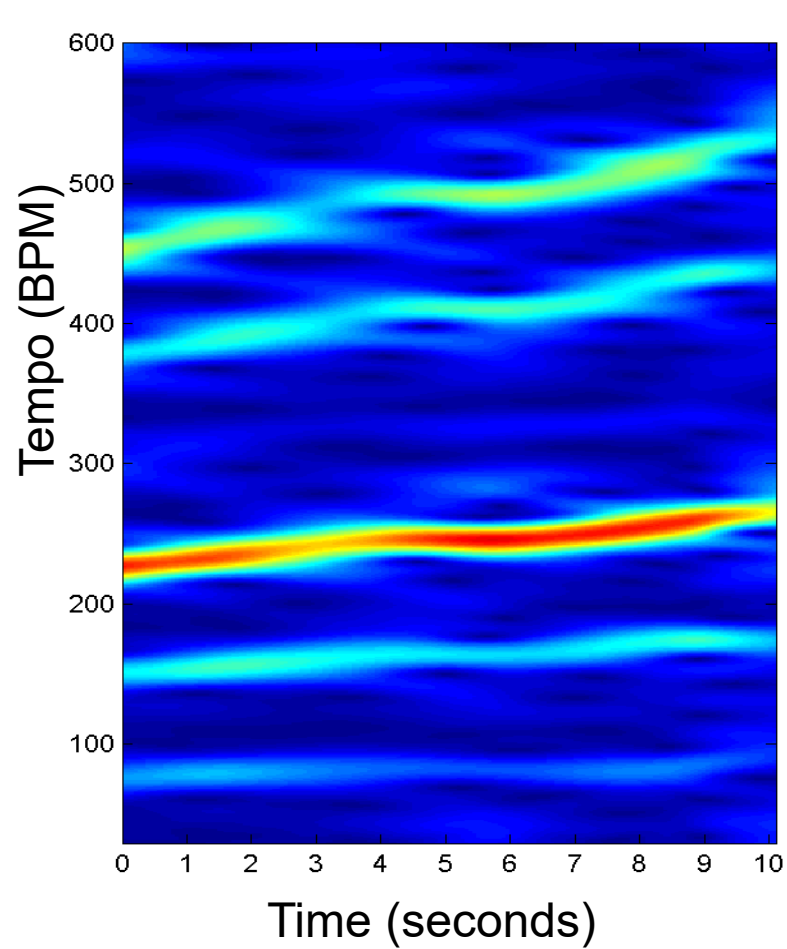


Tempogram (Autocorrelation)

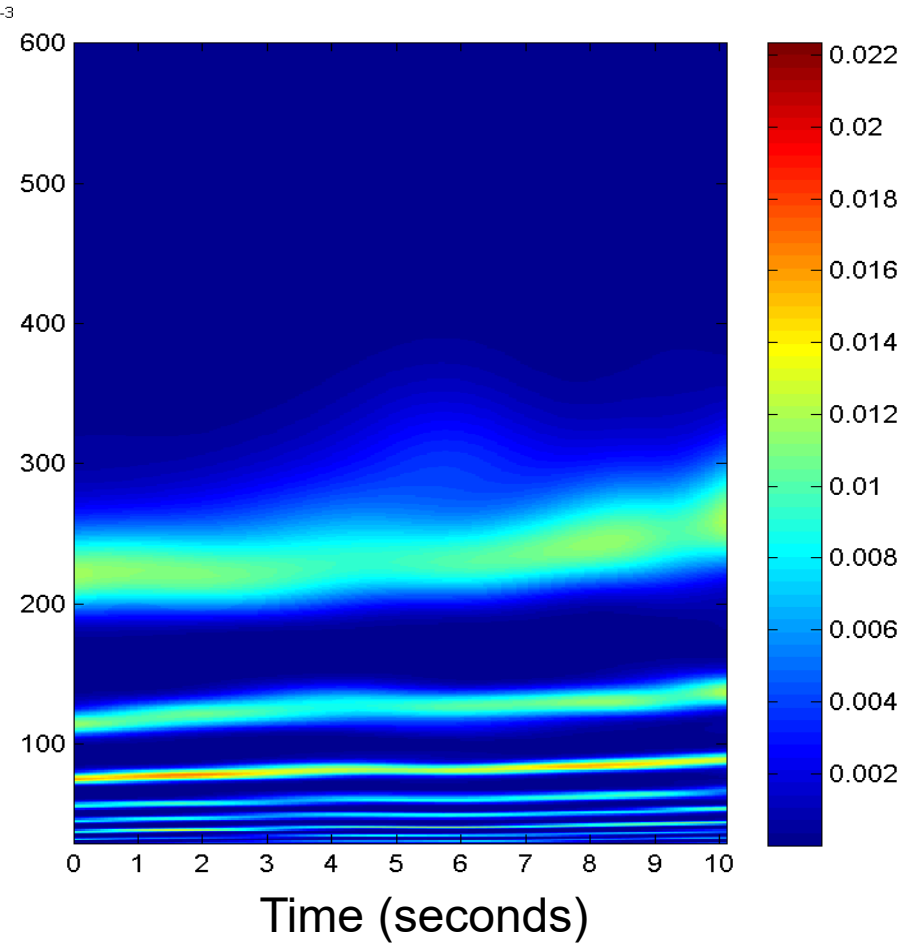


Tempogram

Fourier

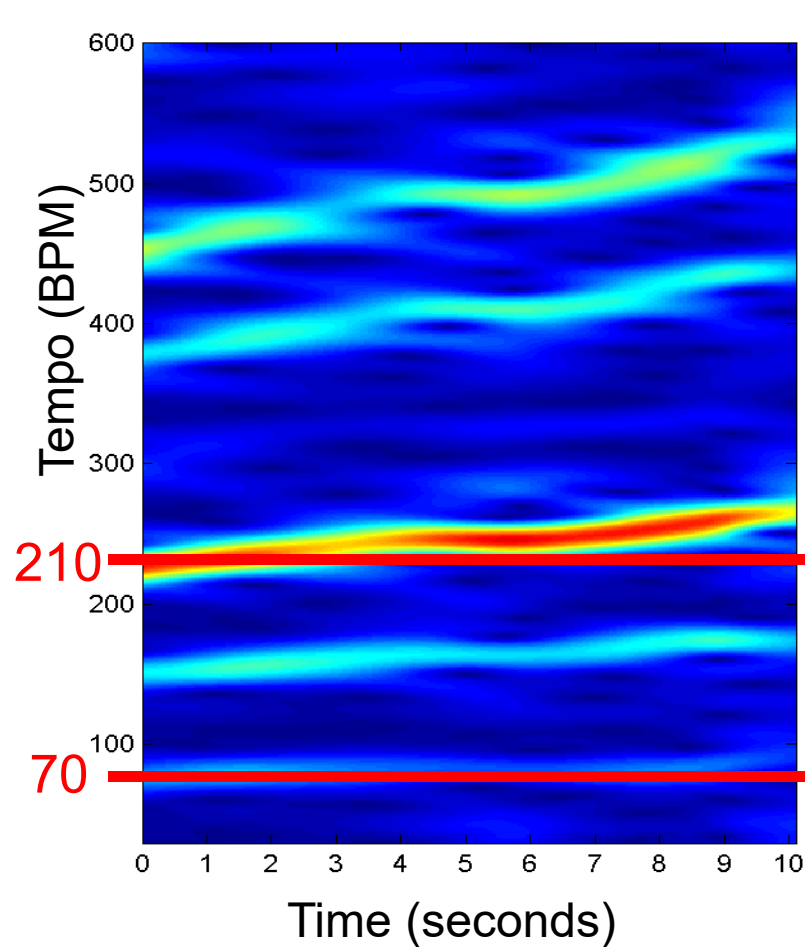


Autocorrelation

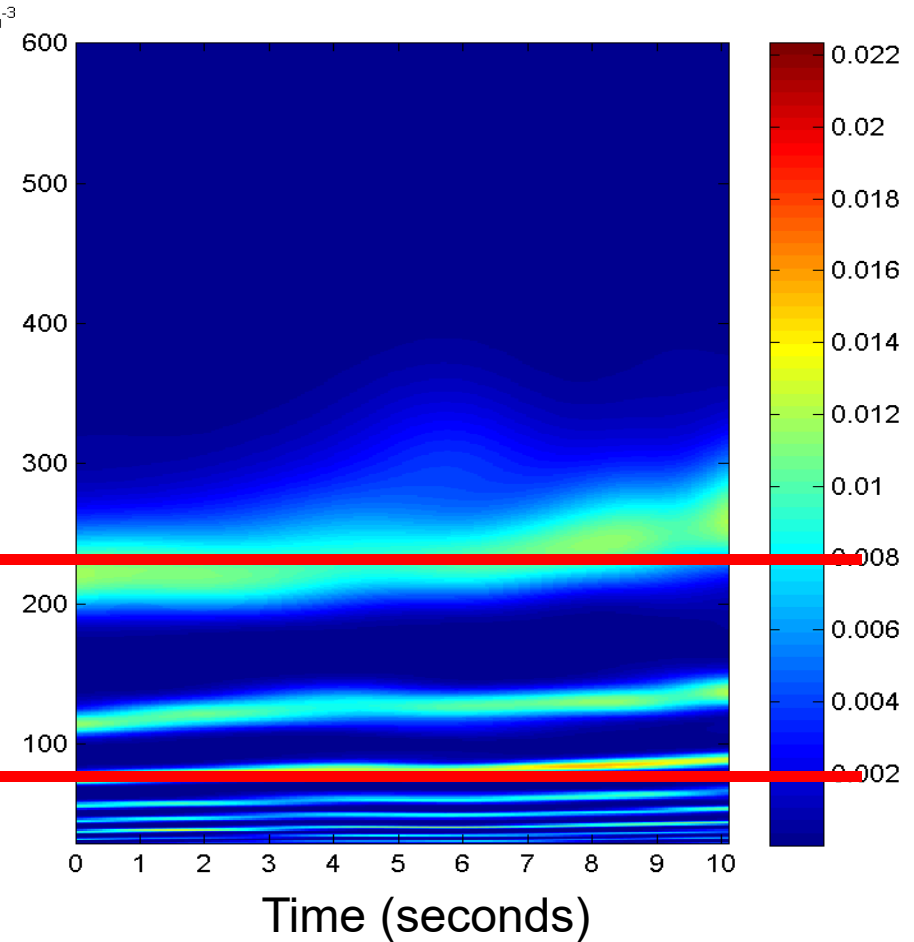


Tempogram

Fourier



Autocorrelation



Tempo@Tatum = 210 BPM

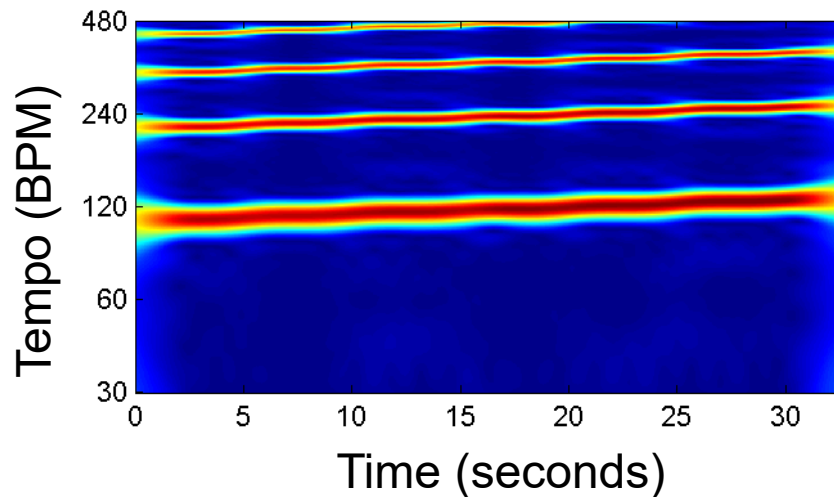
Tempo@Measure = 70 BPM

Tempogram



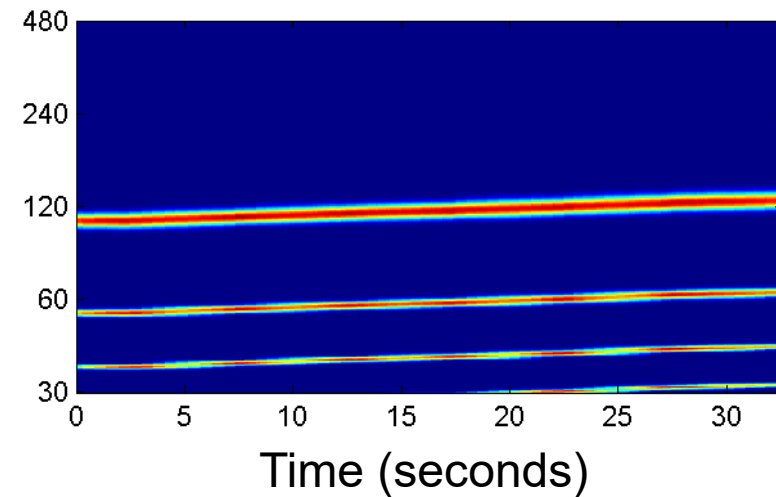
Time
(seconds)

Fourier



Emphasis of tempo harmonics
(integer multiples)

Autocorrelation



Emphasis of tempo subharmonics
(integer fractions)

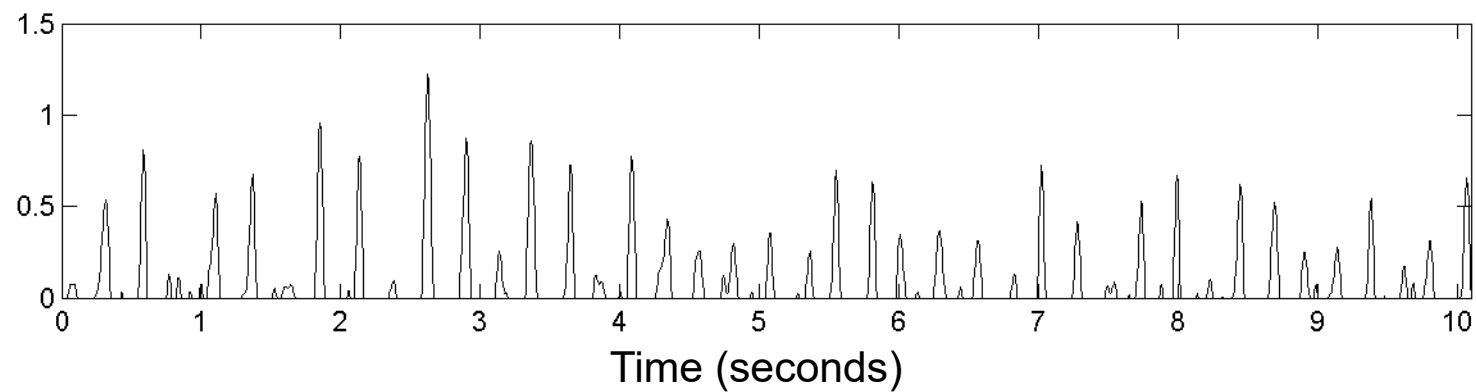
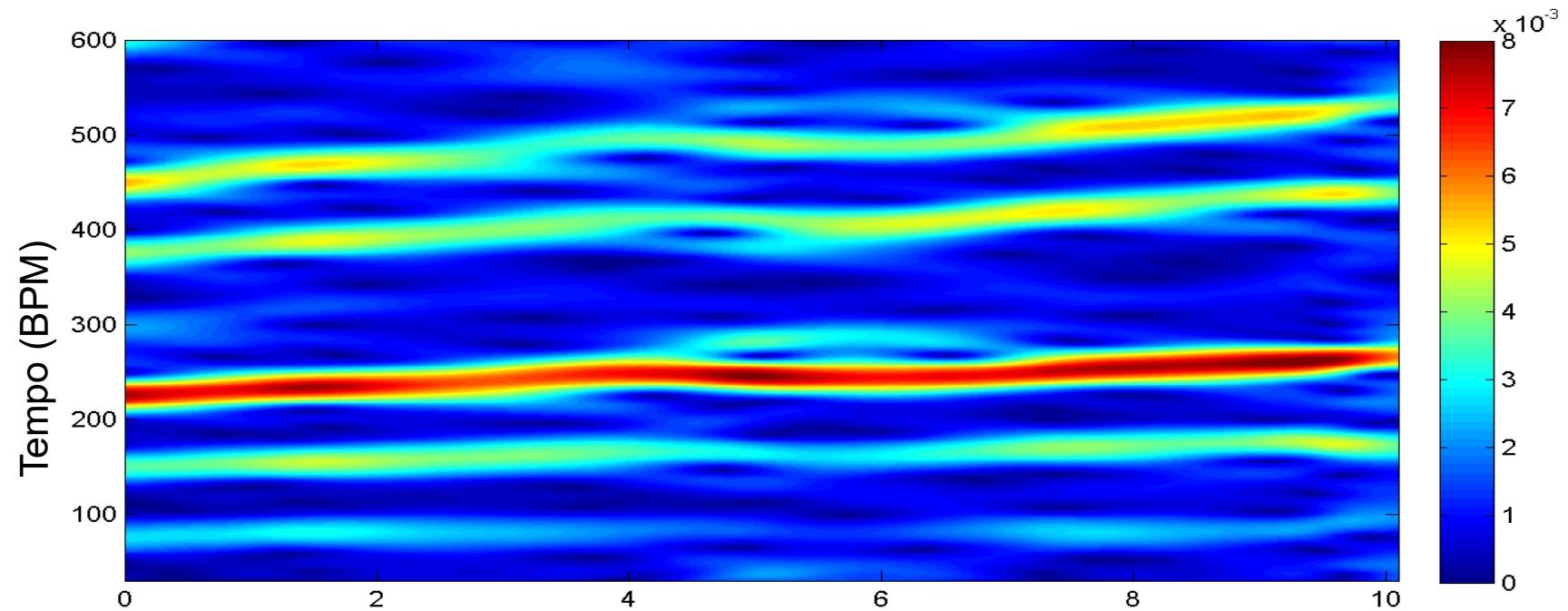
Tempogram (Summary)

Fourier	Autocorrelation
Novelty curve is compared with sinusoidal kernels each representing a specific tempo	Novelty curve is compared with time-lagged local (windowed) sections of itself
Convert frequency (Hertz) into tempo (BPM)	Convert time-lag (seconds) into tempo (BPM)
Reveals novelty periodicities	Reveals novelty self-similarities
Emphasizes harmonics	Emphasizes subharmonics
Suitable to analyze tempo on tatum and tactus level	Suitable to analyze tempo on tactus and measure level

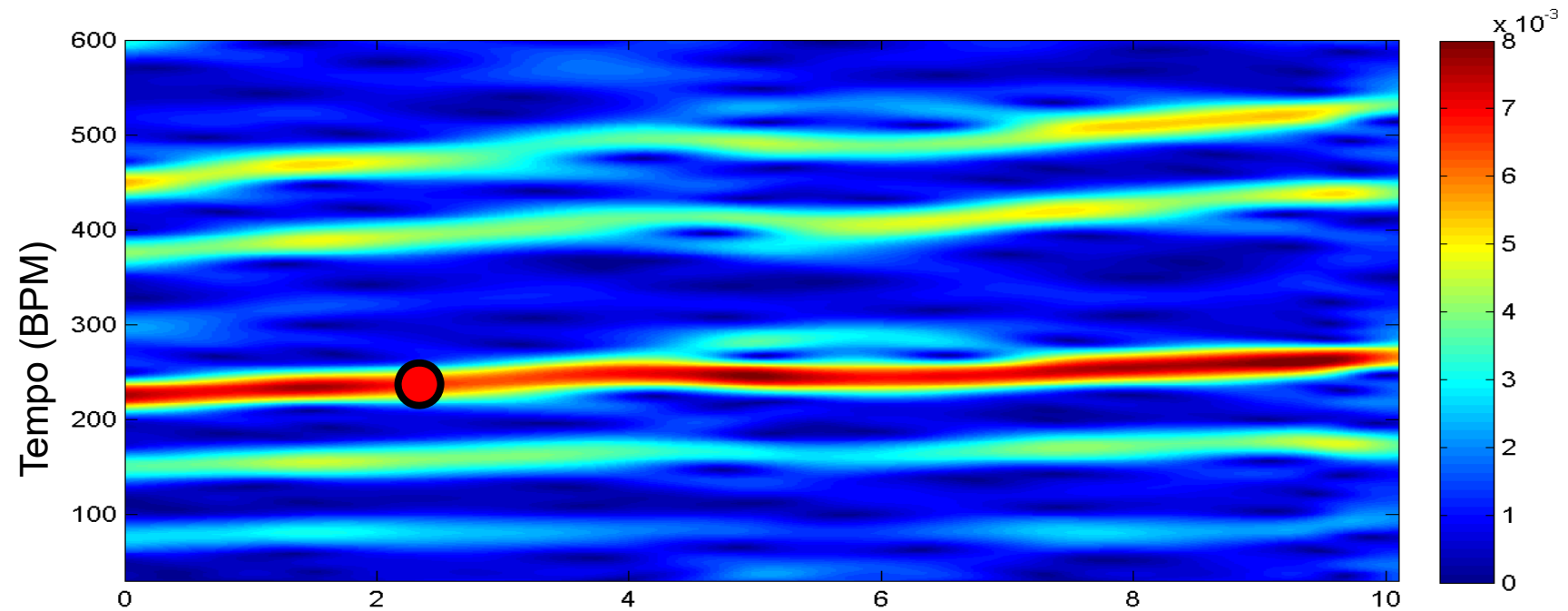
Beat Tracking

- Given the tempo, find the best sequence of beats
- Complex Fourier tempogram contains **magnitude** and **phase** information
- The **magnitude** encodes how well the novelty curve resonates with a sinusoidal kernel of a specific tempo
- The **phase** optimally aligns the sinusoidal kernel with the peaks of the novelty curve

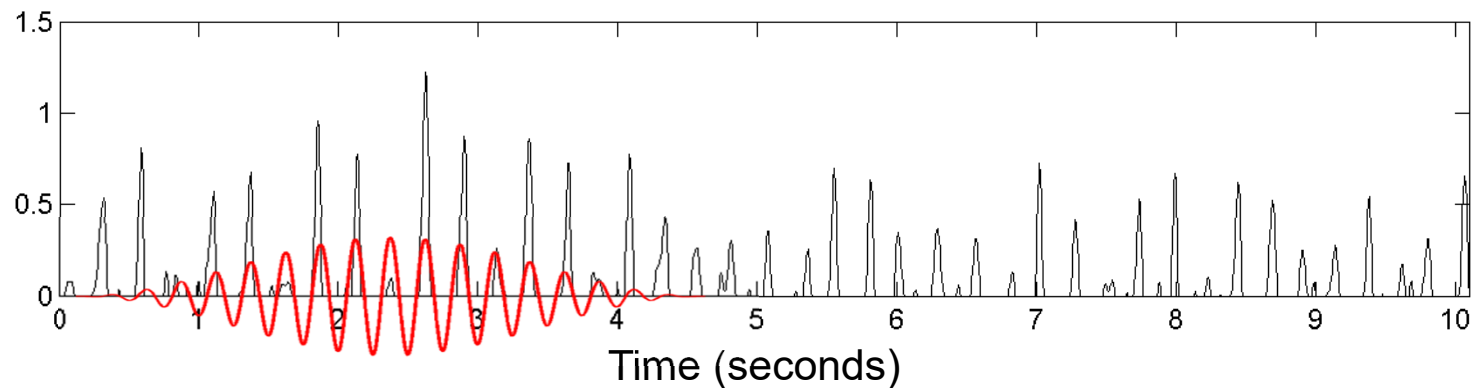
Local Pulse Tracking



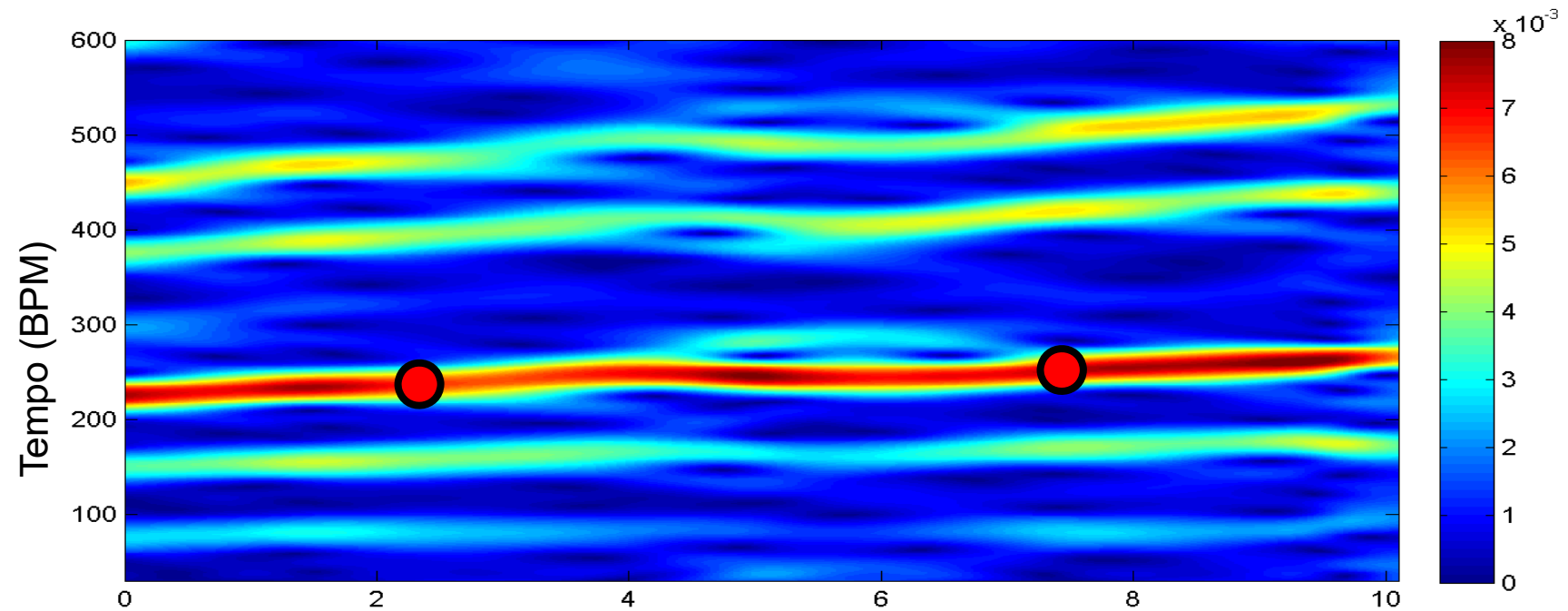
Local Pulse Tracking



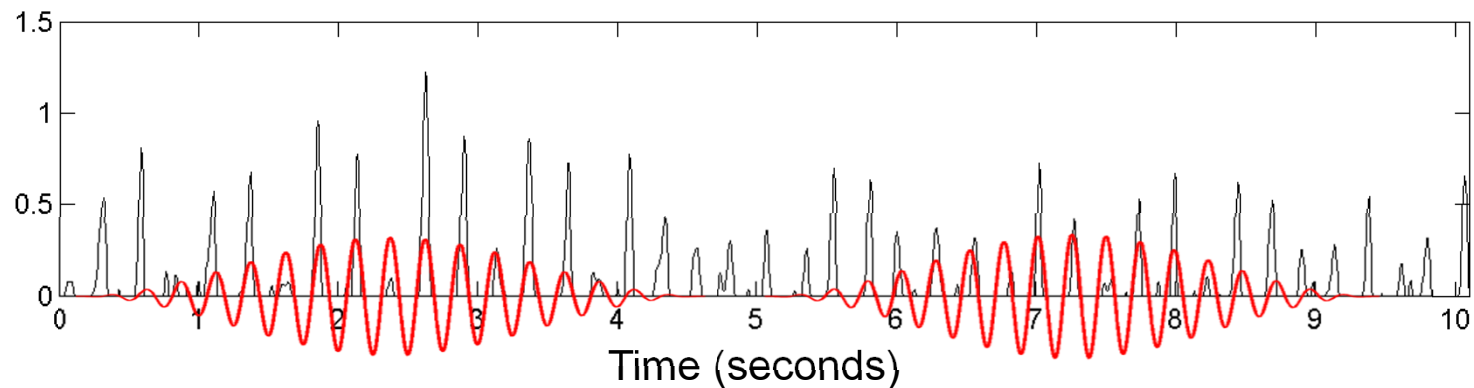
Optimizing local periodicity kernel



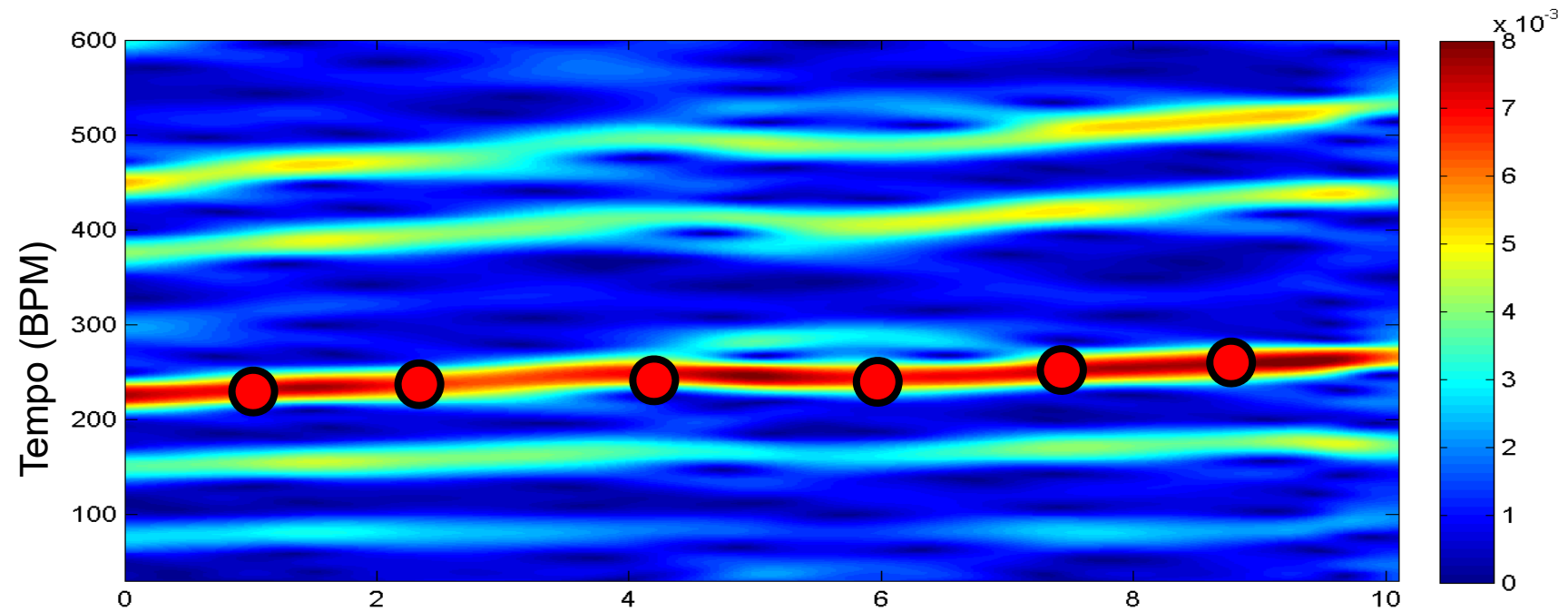
Local Pulse Tracking



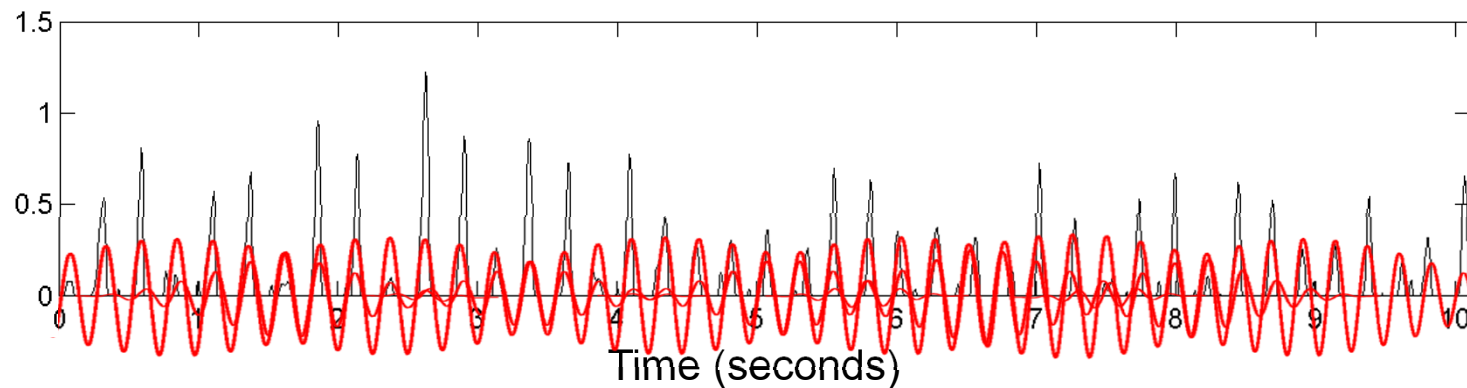
Optimizing local periodicity kernel



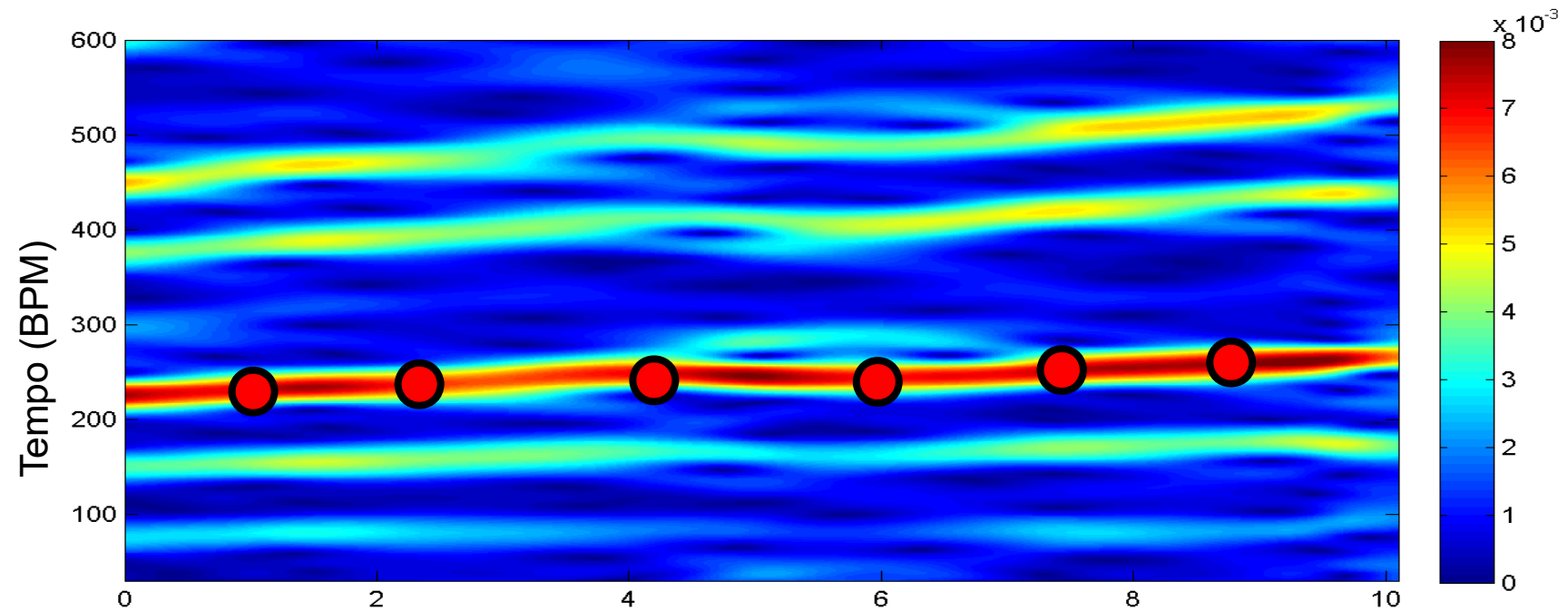
Local Pulse Tracking



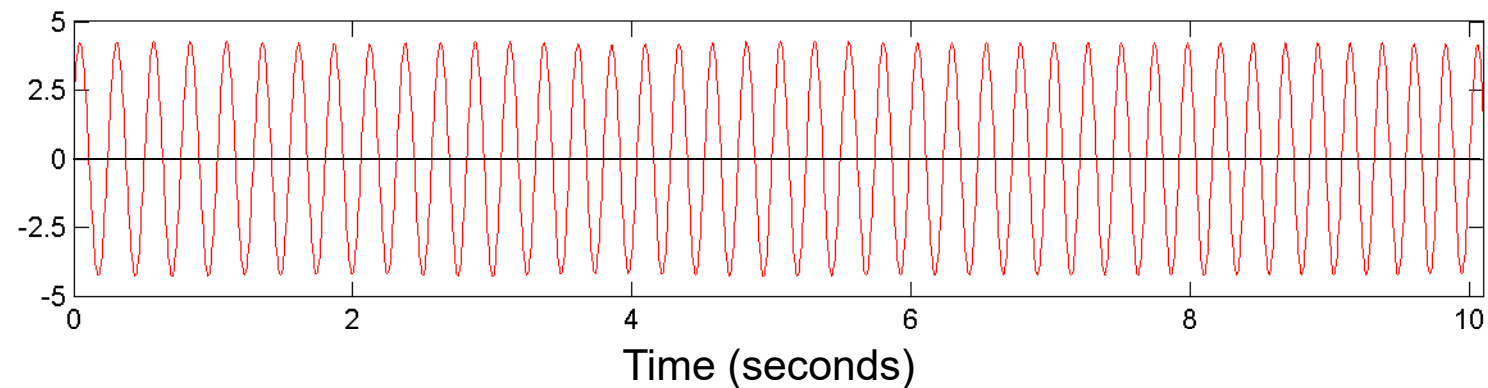
Optimizing local periodicity kernel



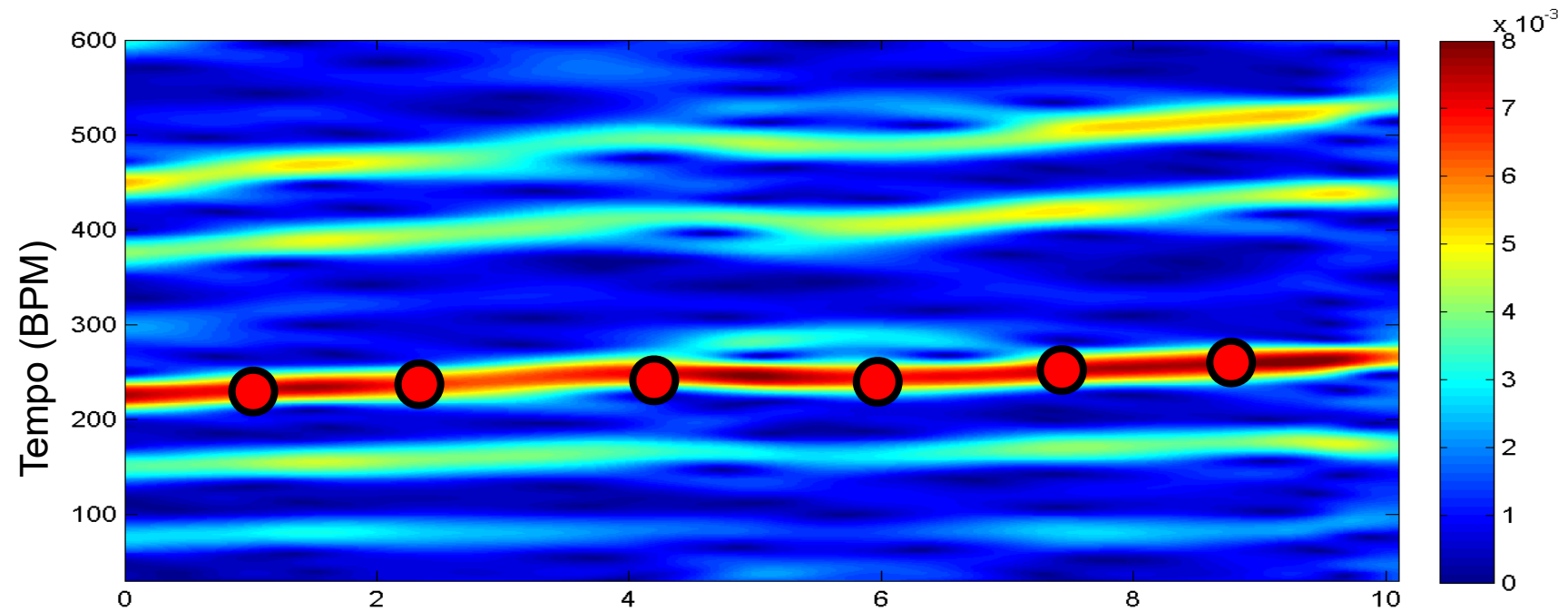
Local Pulse Tracking



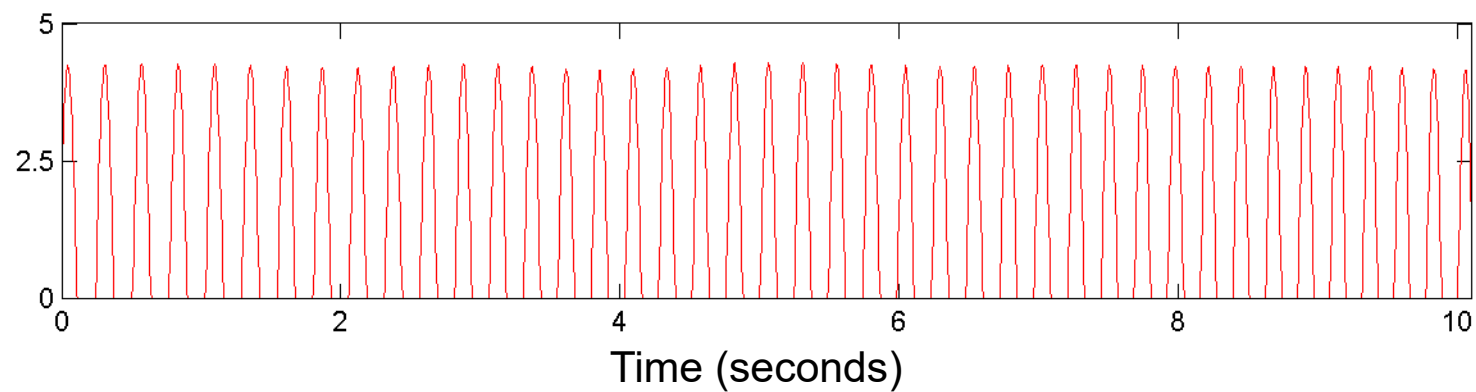
Accumulation of kernels



Local Pulse Tracking

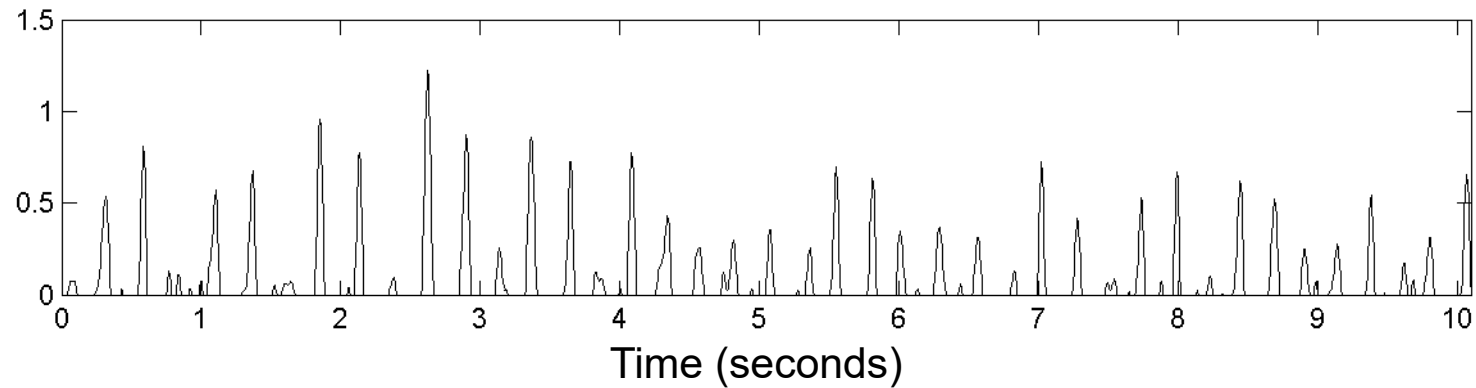


Halfwave rectification

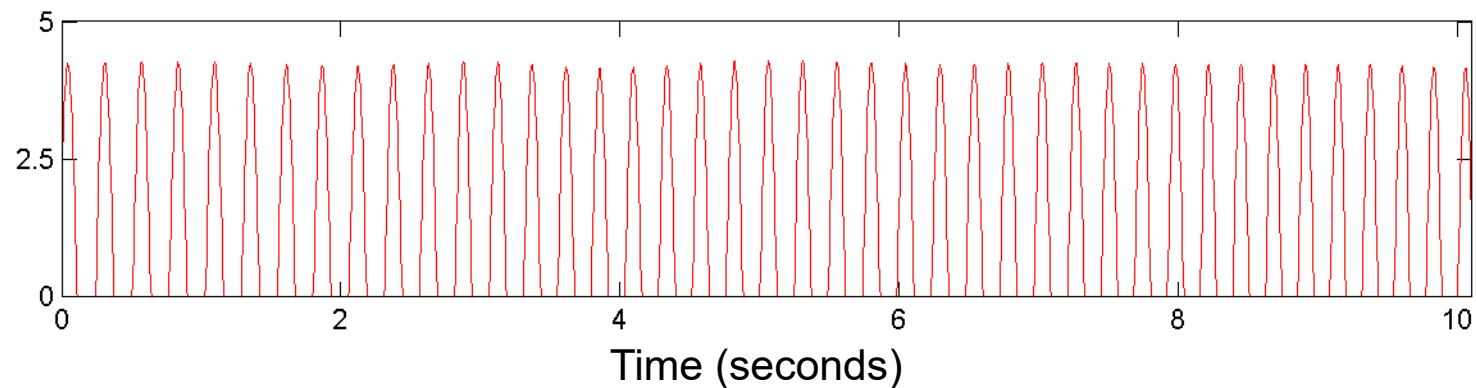


Local Pulse Tracking

Novelty Curve



Predominant Local Pulse (PLP)



Local Pulse Tracking

Novelty Curve

- Indicates note onset candidates
- Extraction errors in particular for soft onsets
- Simple peak-picking problematic



Predominant Local Pulse (PLP)

- Periodicity enhancement of novelty curve
- Accumulation introduces error robustness
- Locality of kernels handles tempo variations



Local Pulse Tracking

- Local tempo at time t : $\tau_t \in \Theta$ $\Theta = [60:240]$ BPM

- Phase $\varphi_t := \frac{1}{2\pi} \arccos \left(\frac{\text{Re}(\mathcal{T}(t, \tau_t))}{|\mathcal{T}(t, \tau_t)|} \right)$

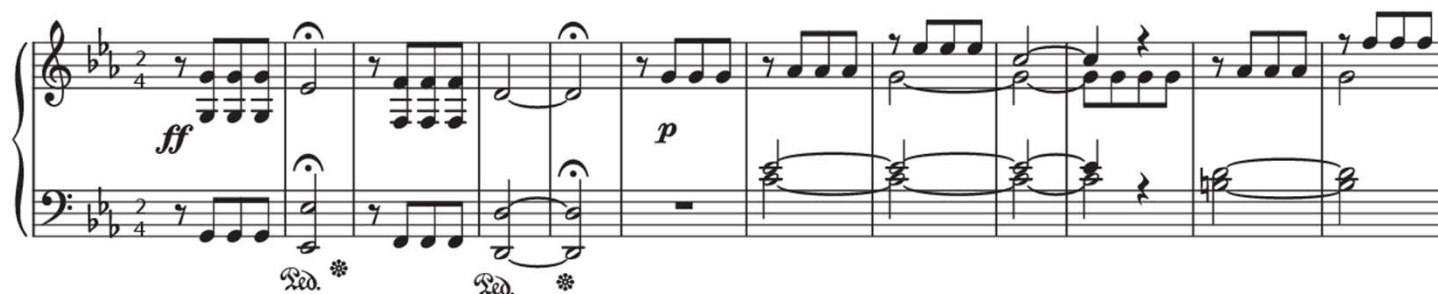
- Sinusoidal kernel $\kappa_t : \mathbb{Z} \rightarrow \mathbb{R}$

$$\kappa_t(n) := W(n - t) \cos(2\pi(\tau_t/60 \cdot n - \varphi_t)) \quad n \in \mathbb{Z}$$

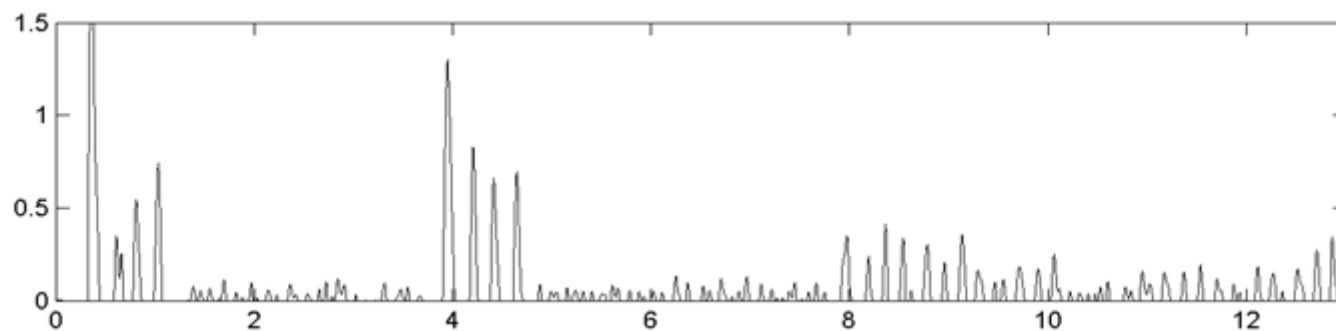
- Periodicity curve $\Gamma : [1 : T] \rightarrow \mathbb{R}_{\geq 0}$

$$\Gamma(n) = \left| \sum_{t \in [1:T]} \kappa_t(n) \right|_{\geq 0} \quad n \in [1 : T]$$

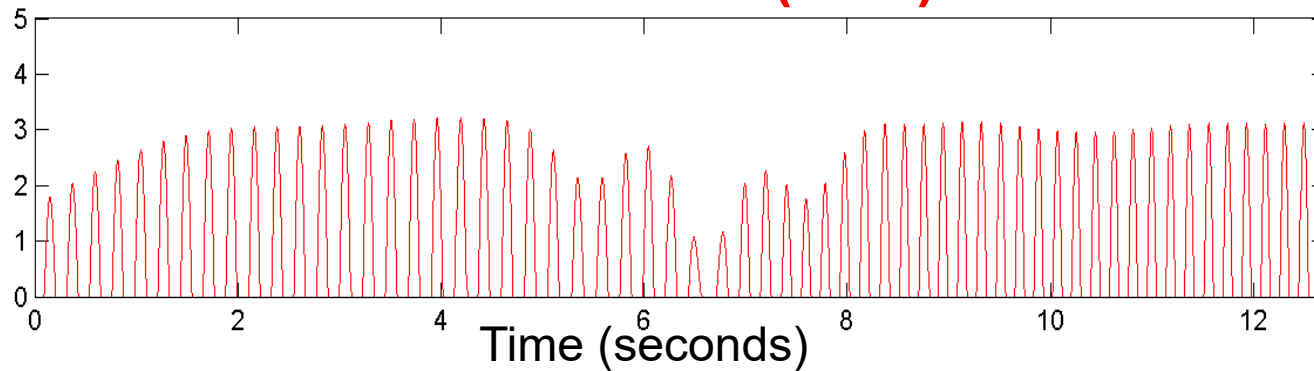
Local Pulse Tracking



Novelty Curve

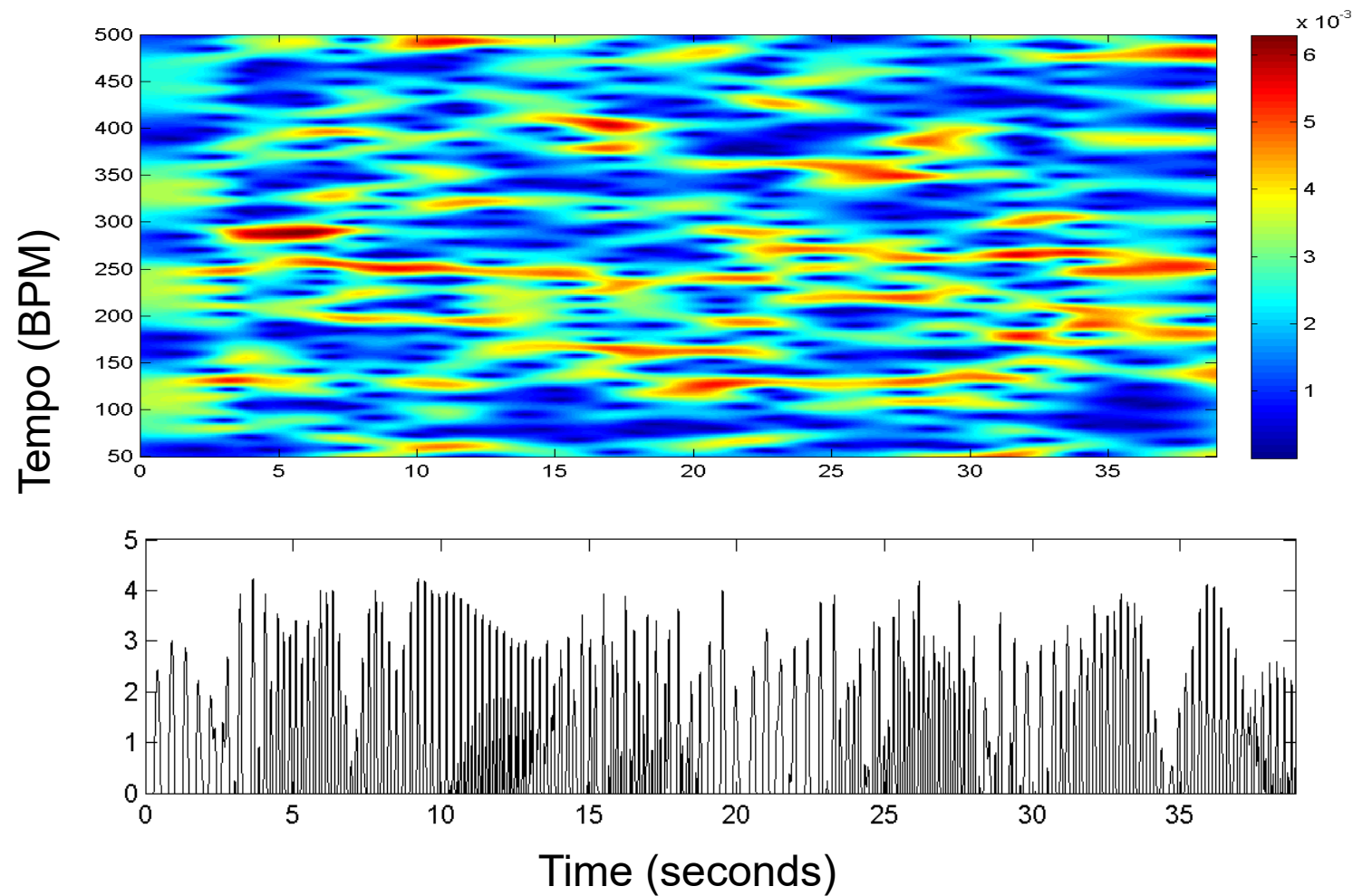


Predominant Local Pulse (PLP)



Local Pulse Tracking

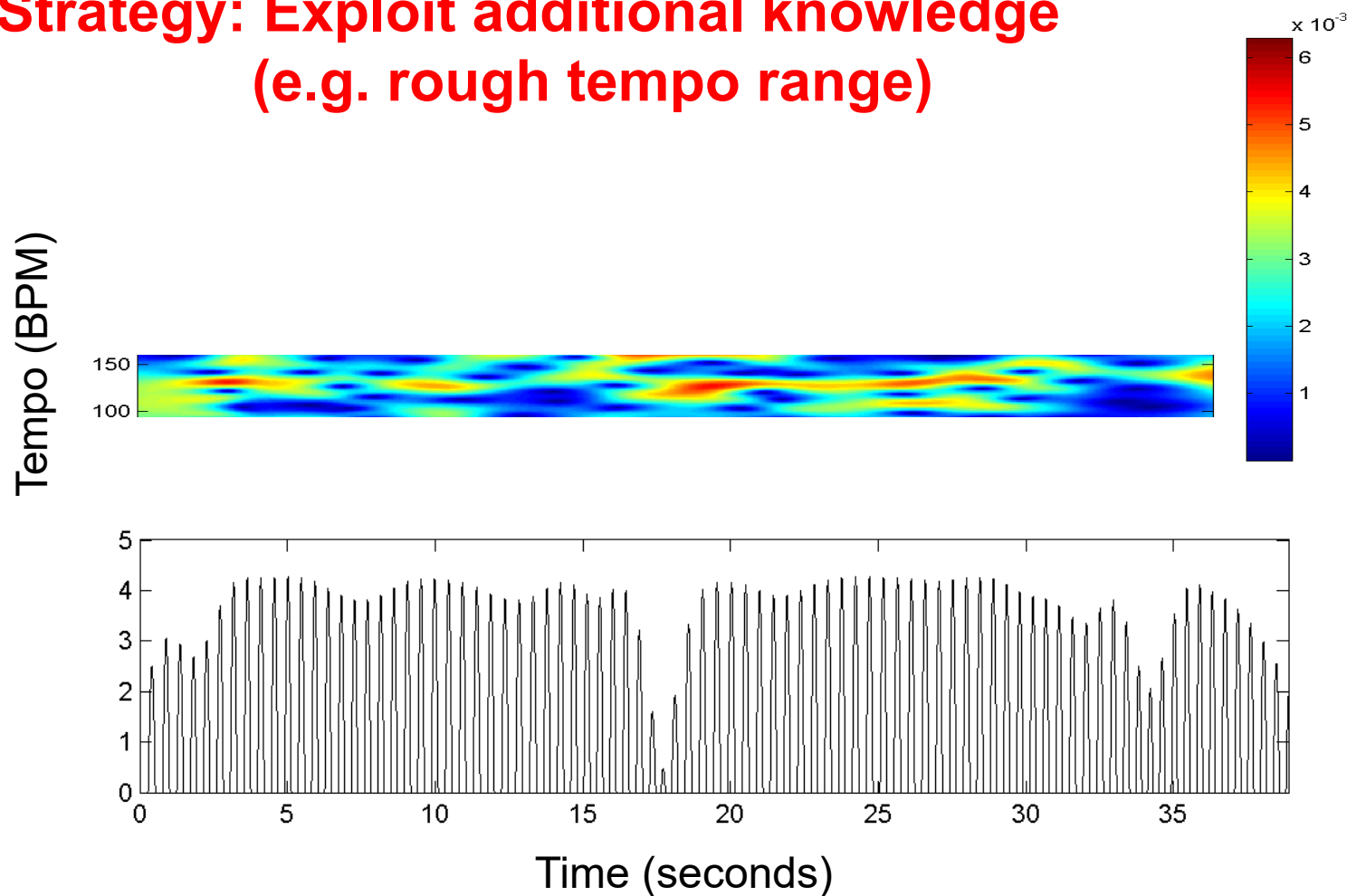
Borodin – String Quartet No. 2



Local Pulse Tracking

Borodin – String Quartet No. 2

**Strategy: Exploit additional knowledge
(e.g. rough tempo range)**



Pulse Levels

Piano Etude Op. 100 No. 2 by Burgmüller



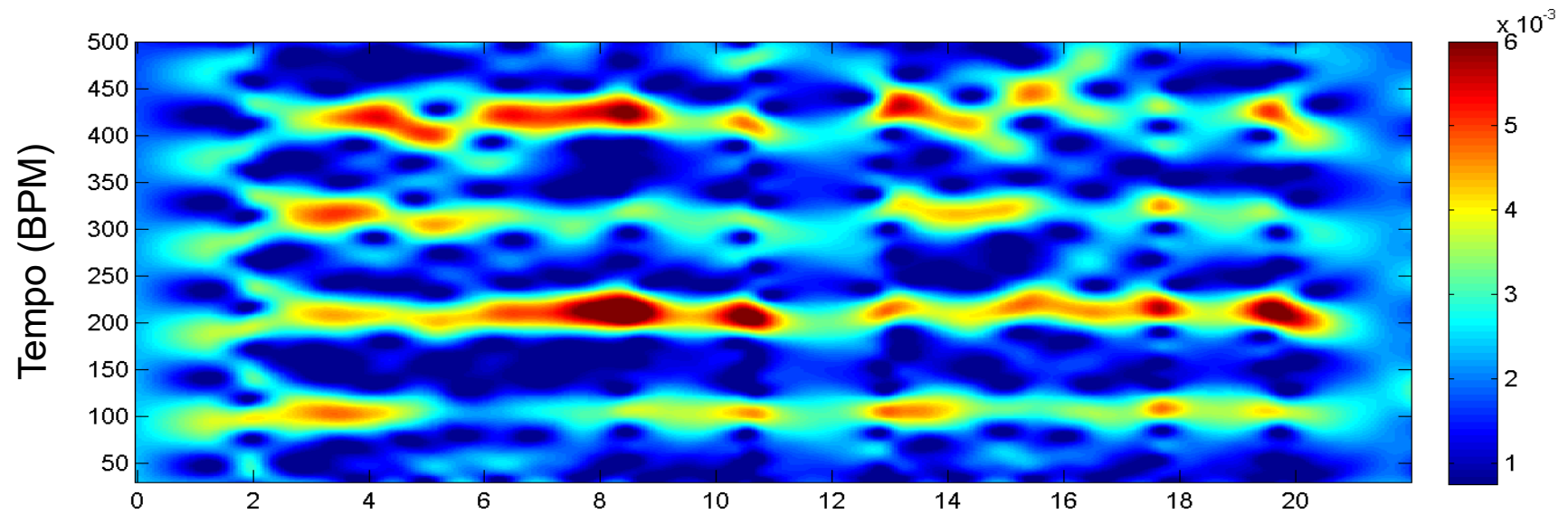
1/4 ● ● ● ●

1/8 • • • • • • • • • • • • • • • •

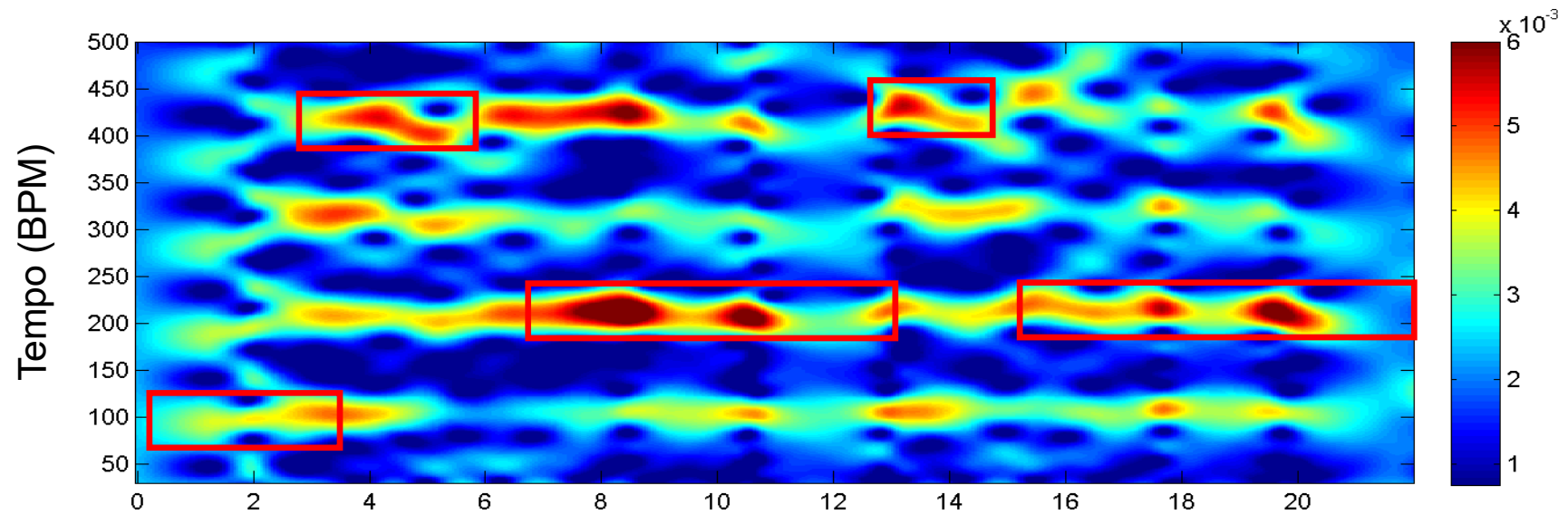
1/16

What is the pulse level: Measure – Tactus – Tatum?

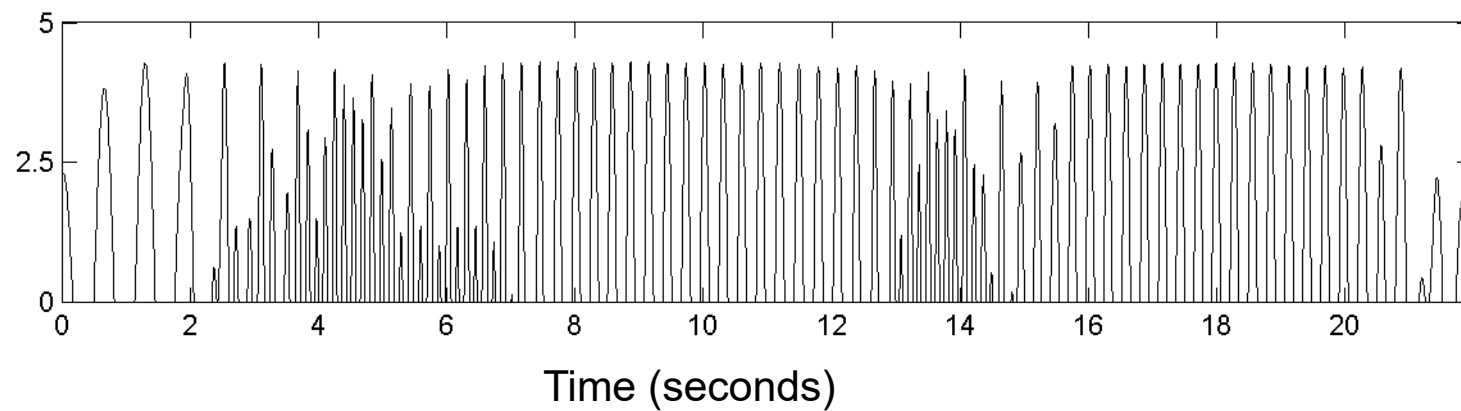
Pulse Levels



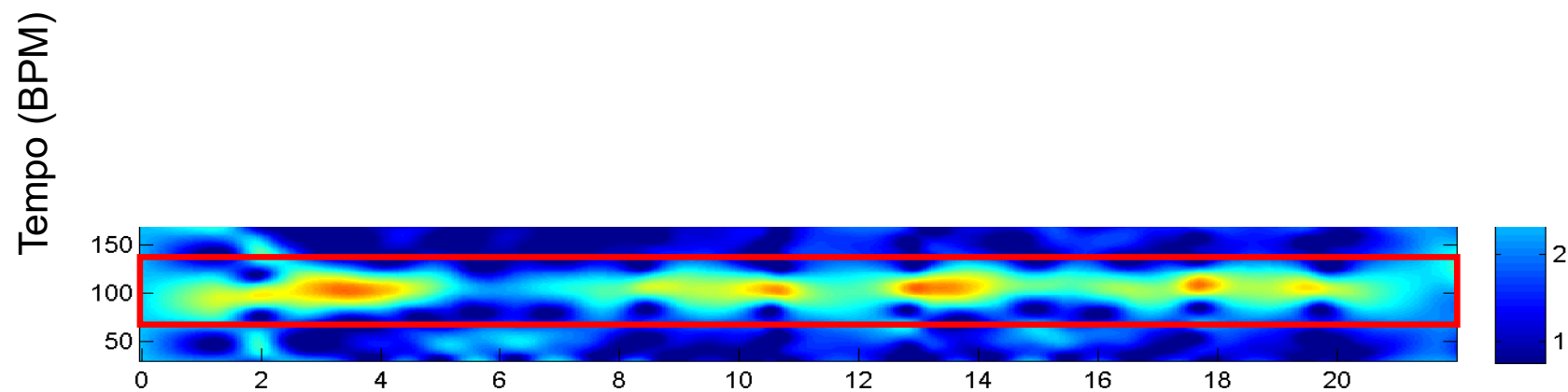
Pulse Levels



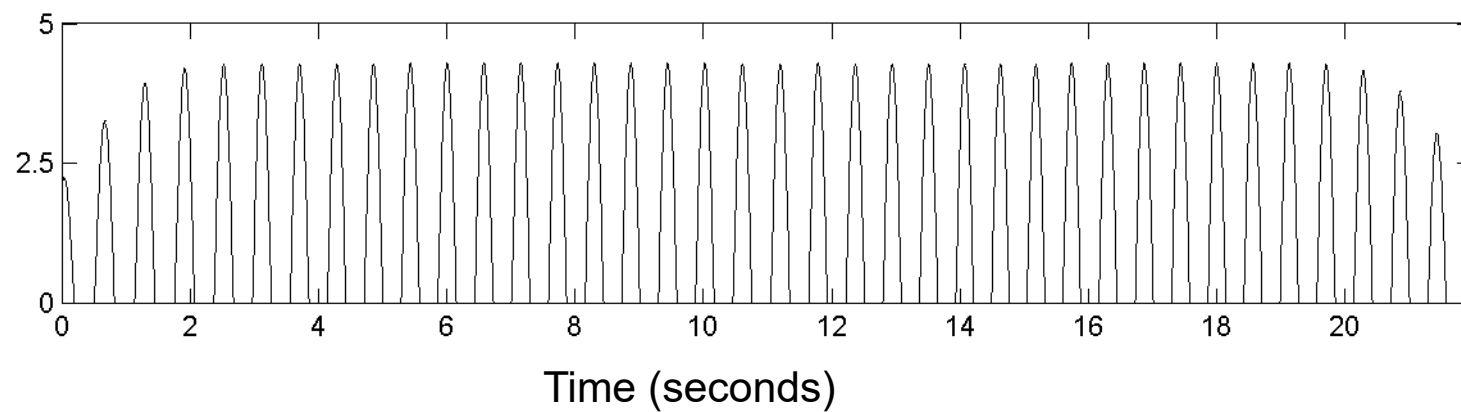
Switching of predominant pulse level



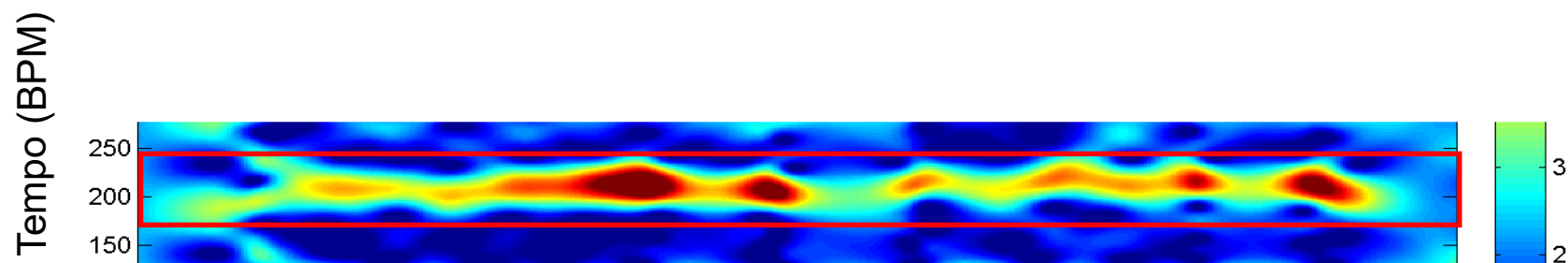
Pulse Levels



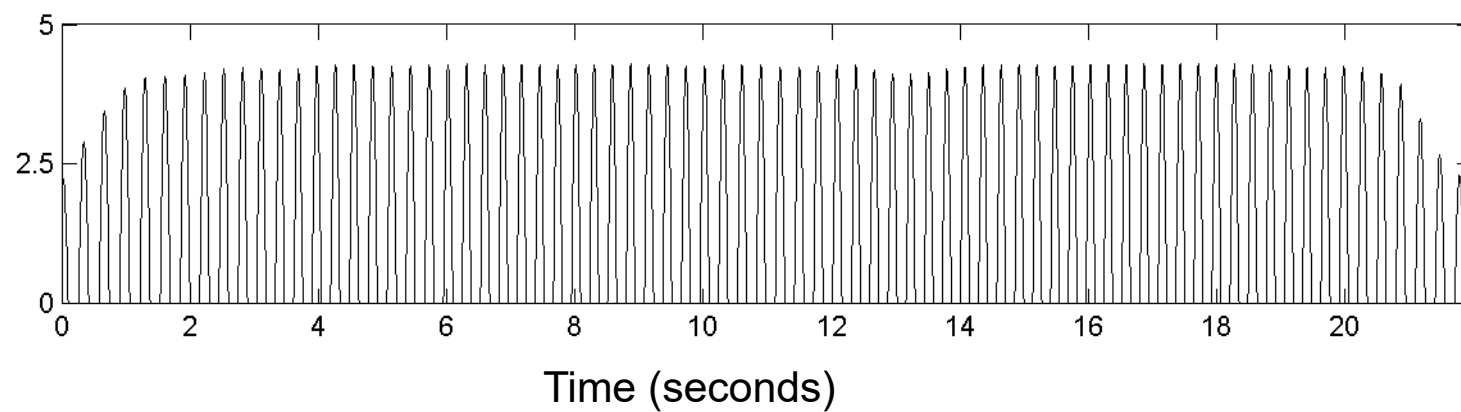
1/4 note pulse level



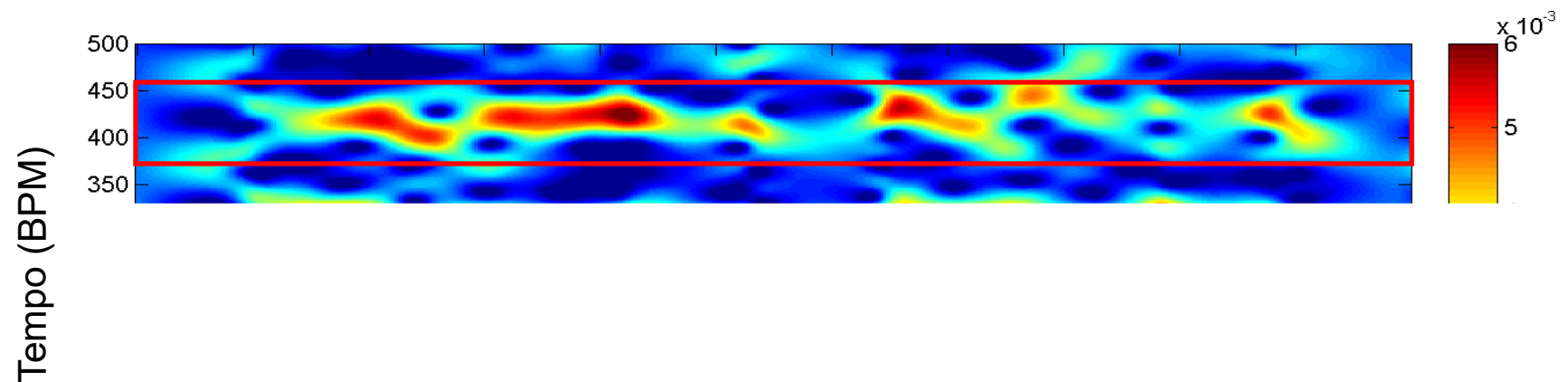
Pulse Levels



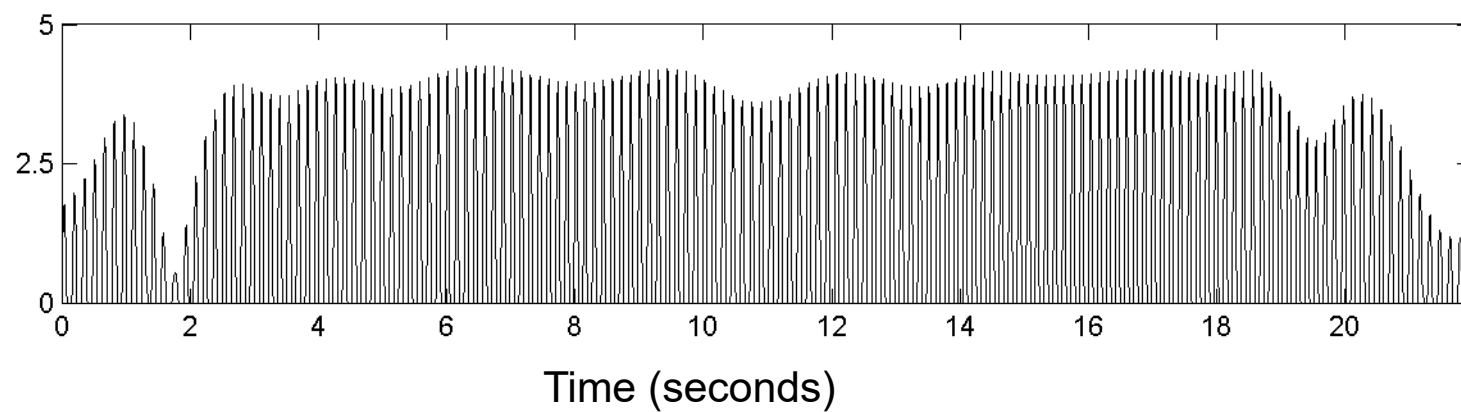
1/8 note pulse level



Pulse Levels

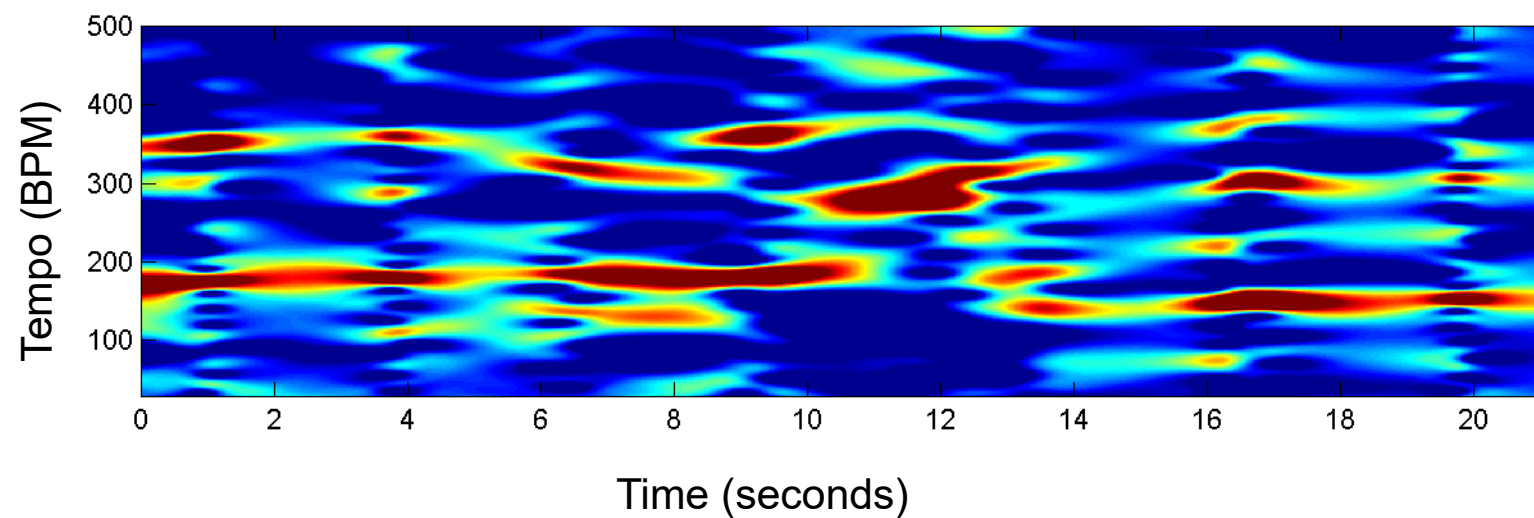


1/16 note pulse level



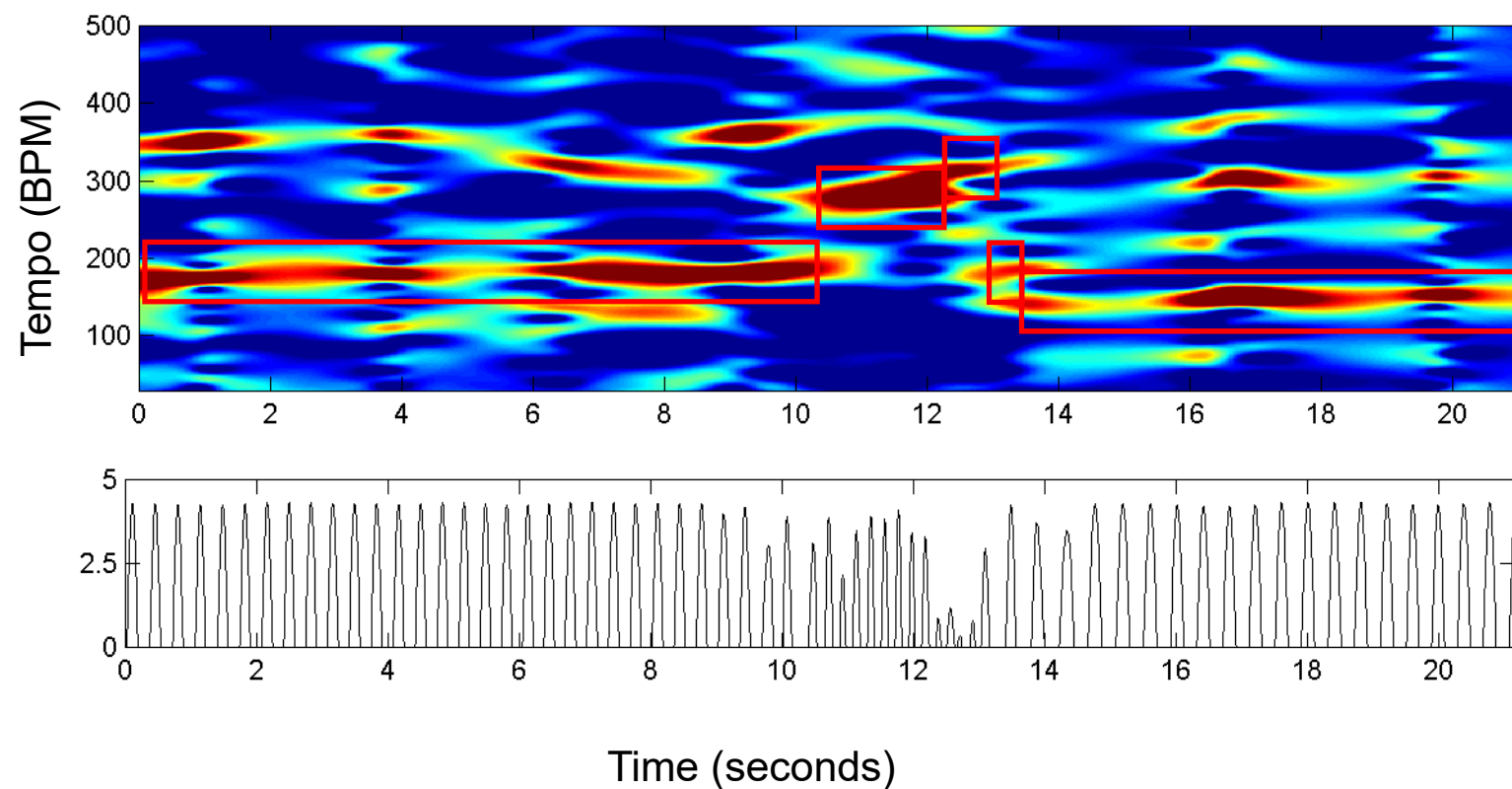
Local Pulse Tracking

Brahms Hungarian Dance No. 5



Local Pulse Tracking

Brahms Hungarian Dance No. 5

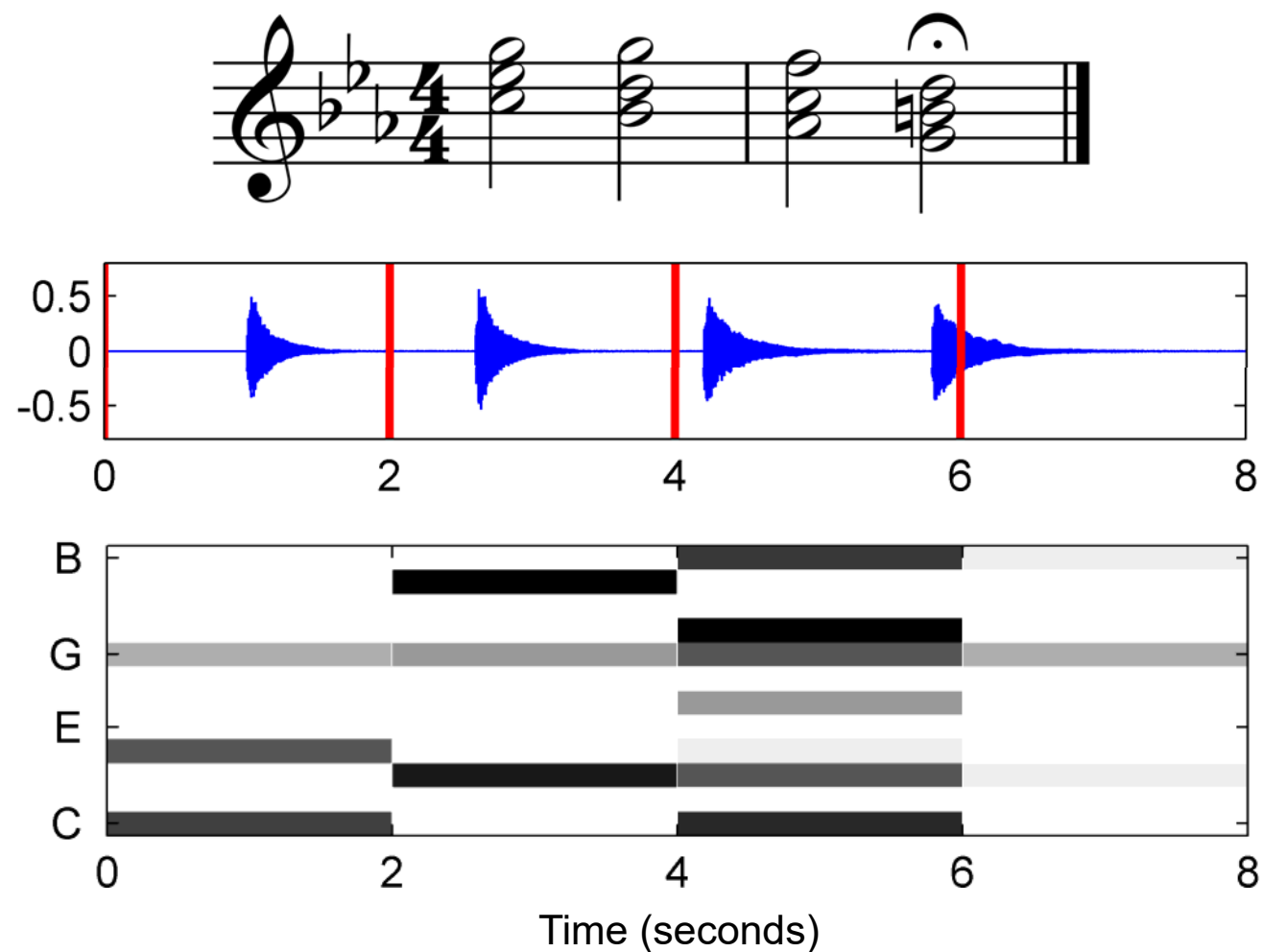


Applications

- Feature design
(beat-synchronous features, adaptive windowing)
- Digital DJ / audio editing
(mixing and blending of audio material)
- Music classification
- Music recommendation
- Performance analysis
(extraction of tempo curves)

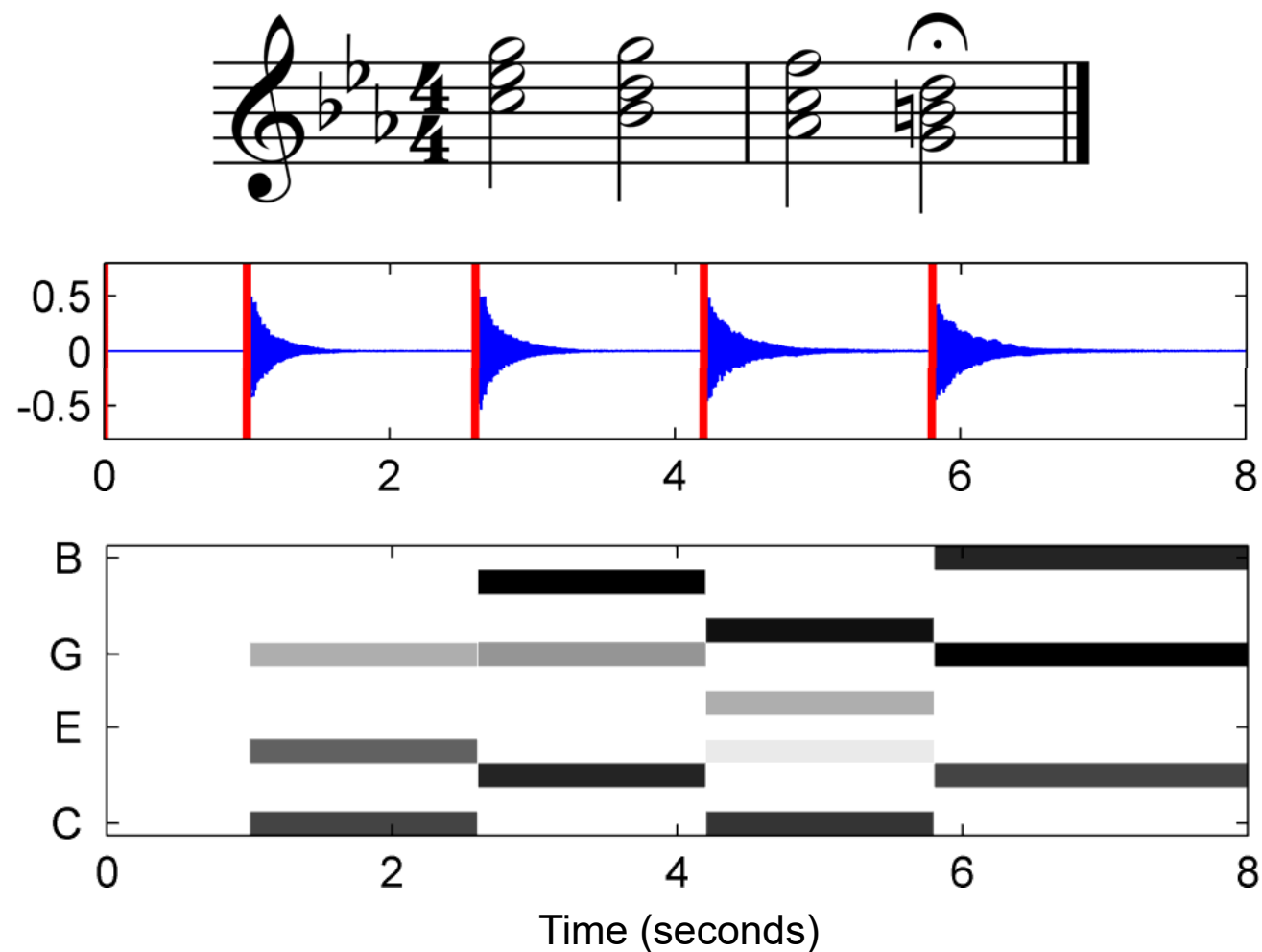
Application: Feature Design

Fixed window size



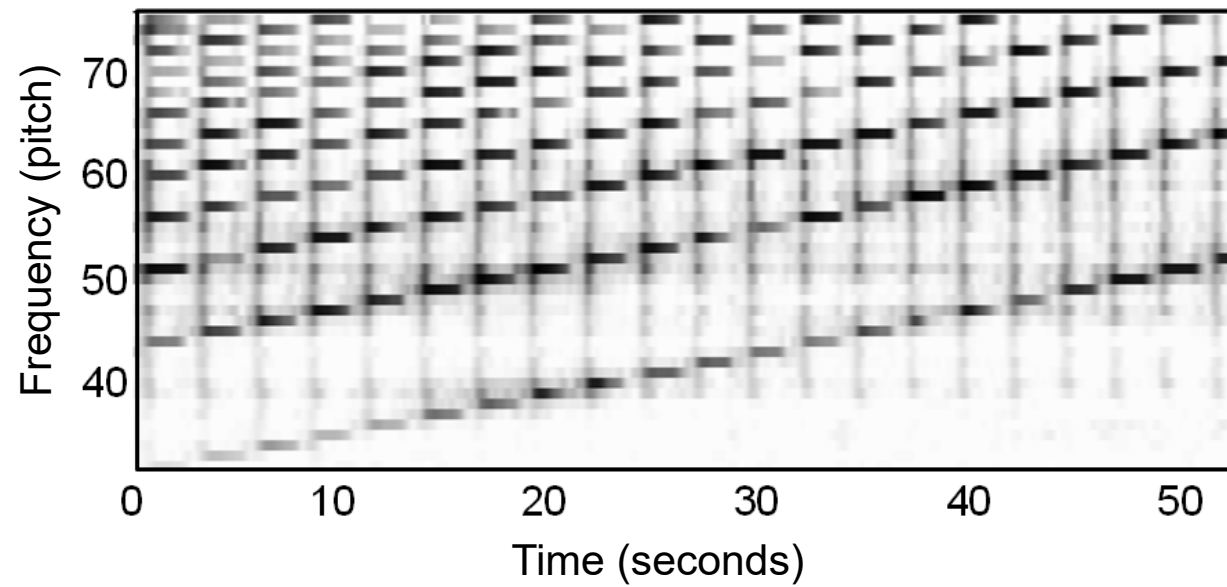
Application: Feature Design

Adaptive window size



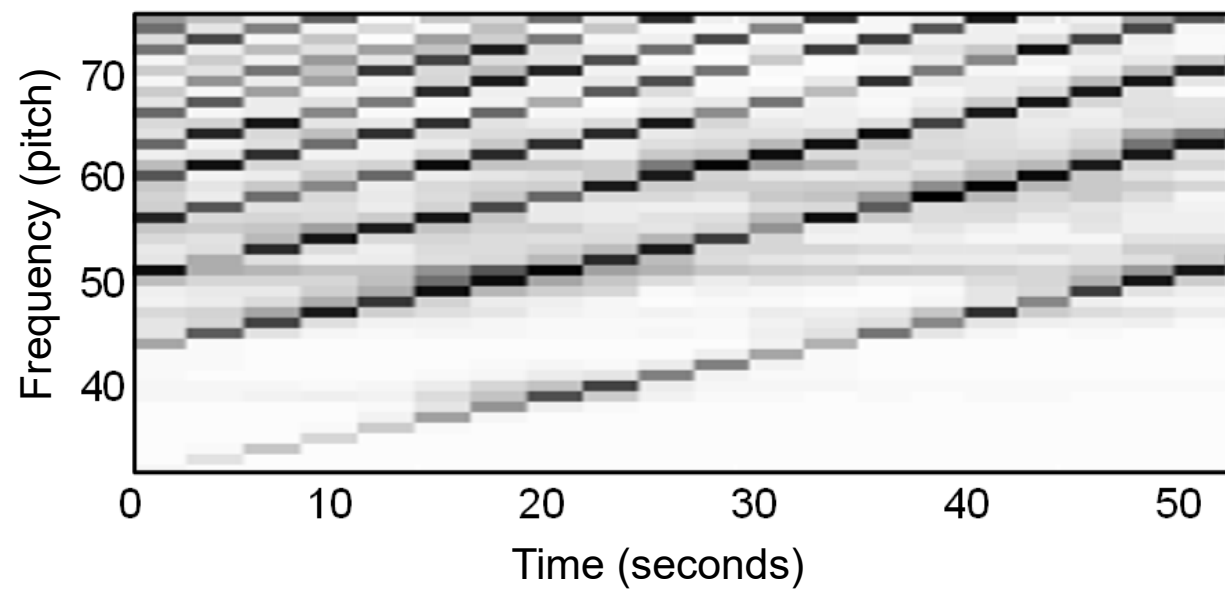
Application: Feature Design

Fixed window size



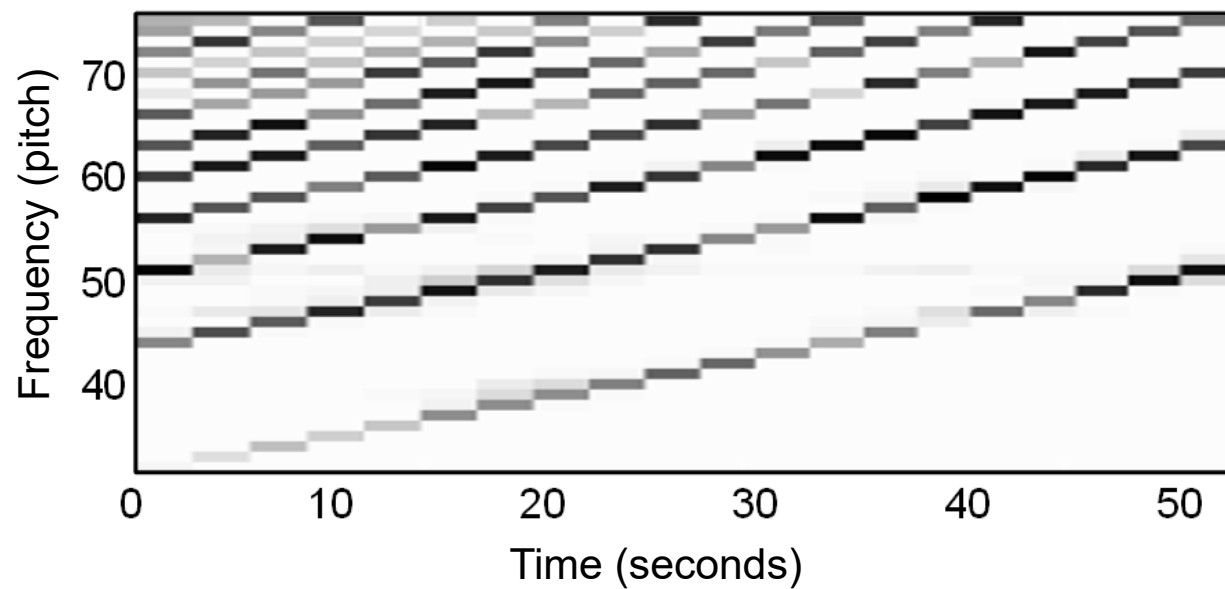
Application: Feature Design

Adaptive window size



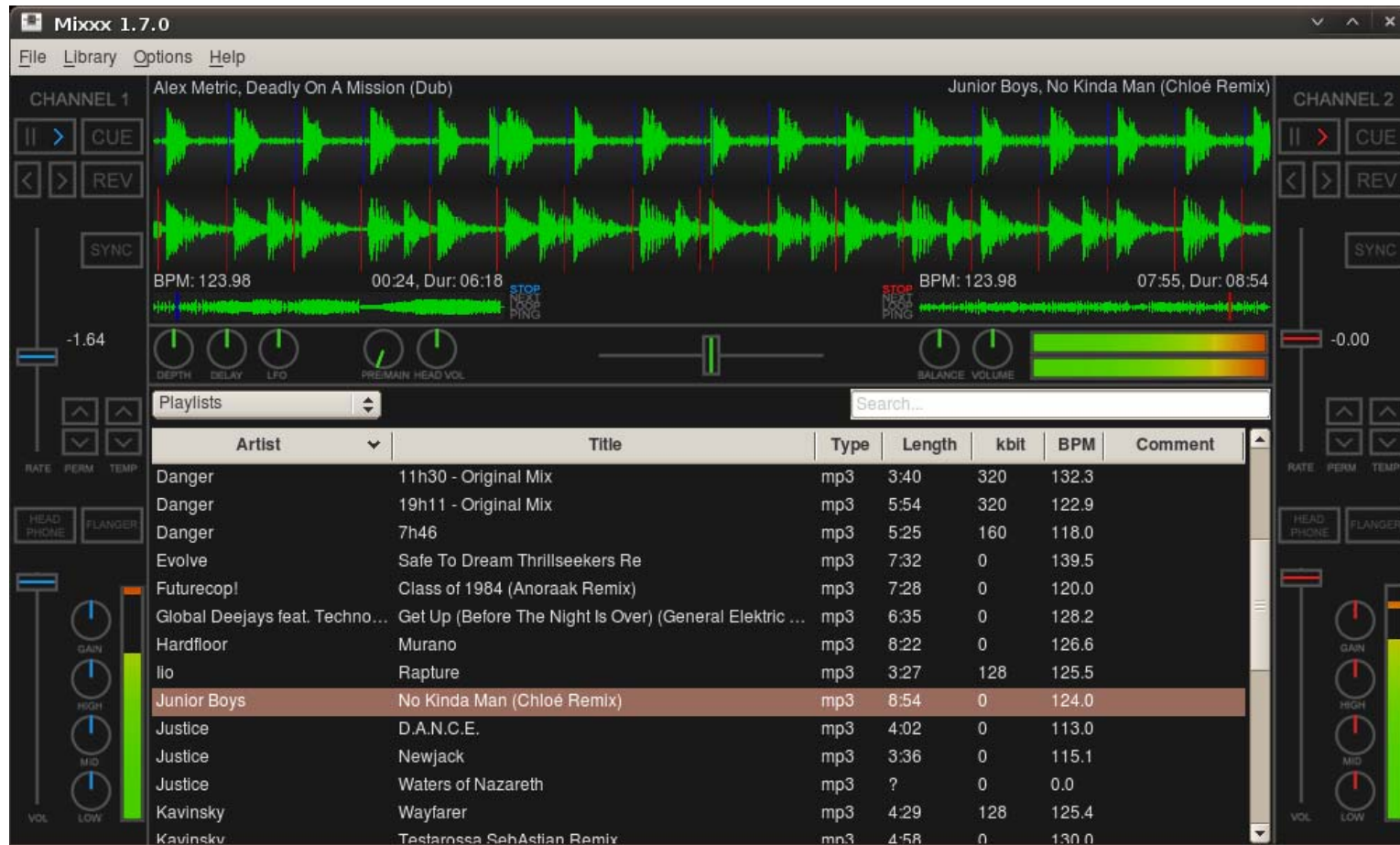
Application: Feature Design

Adaptive window size



Denoising by excluding boundary neighborhoods

Application: Audio Editing (Digital DJ)



Application: Beat-Synchronous Light Effects



Summary

1. Onset Detection

- Novelty curve (*something is changing*)
- Indicates note onset candidates
- Hard task for non-percussive instruments (strings)

2. Tempo Estimation

- Fourier tempogram
- Autocorrelation tempogram
- Musical knowledge (tempo range, continuity)

3. Beat tracking

- Find most likely beat positions
- Exploiting phase information from Fourier tempogram