

College Algebra

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Perface



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College Algebra



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OpenStax College

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PREFACE

Welcome to *College Algebra*, an OpenStax College resource. This textbook has been created with several goals in mind: accessibility, customization, and student engagement—all while encouraging students toward high levels of academic scholarship. Instructors and students alike will find that this textbook offers a strong foundation in college algebra in an accessible format.

About OpenStax College

OpenStax College is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks go through a rigorous editorial publishing process. Our texts are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of today's college courses. Unlike traditional textbooks, OpenStax College resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax College is working to improve access to higher education for all. OpenStax College is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations. OpenStax College textbooks are used at many colleges and universities around the world. Please go to <https://openstaxcollege.org/pages/adoptions> to see our rapidly expanding number of adoptions.

About OpenStax College's Resources

OpenStax College resources provide quality academic instruction. Three key features set our materials apart from others: they can be customized by instructors for each class, they are a "living" resource that grows online through contributions from educators, and they are available free or for minimal cost.

Customization

OpenStax College learning resources are designed to be customized for each course. Our textbooks provide a solid foundation on which instructors can build, and our resources are conceived and written with flexibility in mind. Instructors can select the sections most relevant to their curricula and create a textbook that speaks directly to the needs of their classes and student body. Teachers are encouraged to expand on existing examples by adding unique context via geographically localized applications and topical connections.

College Algebra can be easily customized using our online platform (<http://cnx.org/content/col11759/latest/>). Simply select the content most relevant to your current semester and create a textbook that speaks directly to the needs of your class. *College Algebra* is organized as a collection of sections that can be rearranged, modified, and enhanced through localized examples or to incorporate a specific theme to your course. This customization feature will ensure that your textbook truly reflects the goals of your course.

Curation

To broaden access and encourage community curation, *College Algebra* is “open source” licensed under a Creative Commons Attribution (CC-BY) license. The mathematics community is invited to submit feedback to enhance and strengthen the material and keep it current and relevant for today's students. Submit your suggestions to info@openstaxcollege.org, and check in on edition status, alternate versions, errata, and news on the StaxDash at <http://openstaxcollege.org>.

Cost

Our textbooks are available for free online, and in low-cost print and e-book editions.

About *College Algebra*

Written and reviewed by a team of highly experienced instructors, *College Algebra* provides a comprehensive and multi-layered exploration of algebraic principles. The text is suitable for a typical introductory algebra course, and was developed to be used flexibly. While the breadth of topics may go beyond what an instructor would cover, the modular approach and the richness of content ensures that the book meets the needs of a variety of programs.

College Algebra guides and supports students with differing levels of preparation and experience with mathematics. Ideas are presented as clearly as possible, and progress to more complex understandings with considerable reinforcement along

the way. A wealth of examples – usually several dozen per chapter – offer detailed, conceptual explanations, in order to build in students a strong, cumulative foundation in the material before asking them to apply what they’ve learned.

Coverage and Scope

In determining the concepts, skills, and topics to cover, we engaged dozens of highly experienced instructors with a range of student audiences. The resulting scope and sequence proceeds logically while allowing for a significant amount of flexibility in instruction.

Chapters 1 and 2 provide both a review and foundation for study of Functions that begins in Chapter 3. The authors recognize that while some institutions may find this material a prerequisite, other institutions have told us that they have a cohort that need the prerequisite skills built into the course.

Chapter 1: Prerequisites

Chapter 2: Equations and Inequalities

Chapters 3-6: The Algebraic Functions

Chapter 3: Functions

Chapter 4: Linear Functions

Chapter 5: Polynomial and Rational Functions

Chapter 6: Exponential and Logarithm Functions

Chapters 7-9: Further Study in College Algebra

Chapter 7: Systems of Equations and Inequalities

Chapter 8: Analytic Geometry

Chapter 9: Sequences, Probability, and Counting Theory

All chapters are broken down into multiple sections, the titles of which can be viewed in the Table of Contents.

Development Overview

Openstax *College Algebra* is the product of a collaborative effort by a group of dedicated authors, editors, and instructors whose collective passion for this project has resulted in a text that is remarkably unified in purpose and voice. Special thanks is due to our Lead Author, Jay Abramson of Arizona State University, who provided the overall vision for the book and oversaw the development of each and every chapter, drawing up the initial blueprint, reading numerous drafts, and assimilating field reviews into actionable revision plans for our authors and editors.

The collective experience of our author team allowed us to pinpoint the subtopics, exceptions, and individual connections that give students the most trouble. And so the textbook is replete with well-designed features and highlights, which help students overcome these barriers. As the students read and practice, they are coached in methods of thinking through problems and internalizing mathematical processes.

For example, narrative text is often followed with the “How To” feature, which summarizes the presentation into a series of distinct steps. This approach addresses varying learning styles, and models for students an important learning skill for future studies. Furthermore, the extensive graphical representations immediately connect concepts with visuals.

Accuracy of the Content

We understand that precision and accuracy are imperatives in mathematics, and undertook a dedicated accuracy program led by experienced faculty.

1. Each chapter’s manuscript underwent rounds of review and revision by a panel of active instructors.
2. Then, prior to publication, a separate team of experts checked all text, examples, and graphics for mathematical accuracy; multiple reviewers were assigned to each chapter to minimize the chances of any error escaping notice.
3. A third team of experts was responsible for the accuracy of the Answer Key, dutifully re-working every solution to eradicate any lingering errors. Finally, the editorial team conducted a multi-round post-production review to ensure the integrity of the content in its final form.

The Solutions Manual, which was written and developed after the Student Edition, has also been rigorously checked for accuracy following a process similar to that described above. Incidentally, the act of writing out solutions step-by-step served as yet another round of validation for the Answer Key in the back of the Student Edition. In spite of the efforts described above, we acknowledge the possibility that—as with any textbook—some errata may have been missed. We encourage users to report errors via our [Errata \(https://openstaxcollege.org/errata\)](https://openstaxcollege.org/errata) page.

Pedagogical Foundations and Features

Learning Objectives

Each chapter is divided into multiple sections (or modules), each of which is organized around a set of learning objectives. The learning objectives are listed explicitly at the beginning of each section, and are the focal point of every instructional element

Narrative text

Narrative text is used to introduce key concepts, terms, and definitions, to provide real-world context, and to provide transitions between topics and examples. Throughout this book, we rely on a few basic conventions to highlight the most important ideas:

Key terms are boldfaced, typically when first introduced and/or when formally defined

Key concepts and definitions are called out in a blue box for easy reference.

Key equations, formulas, theorems, identities, etc. are assigned a number, which appears near the right margin. Occasionally the text may refer back to an equation or formula by its number.

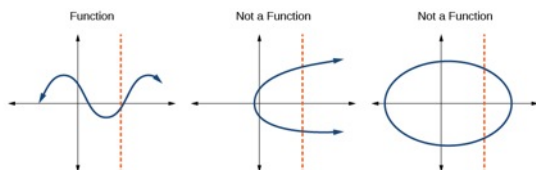
Examples

Each learning objective is supported by one or more worked examples, which demonstrate the problem-solving approaches that students must master. The multiple Examples model different approaches to the same type of problem, or introduce similar problems of increasing complexity.

All Examples follow a simple two- or three-part format. The question clearly lays out a mathematical problem to solve. The Solution walks through the steps, usually providing context for the approach --in other words, why the instructor is solving the problem in a specific manner. Finally, the Analysis (for select examples) reflects on the broader implications of the Solution just shown. Examples are followed by a “Try It,” question, as explained below.

Figures

Openstax *College Algebra* contains many figures and illustrations, the vast majority of which are graphs and diagrams. Art throughout the text adheres to a clear, understated style, drawing the eye to the most important information in each figure while minimizing visual distractions. Color contrast is employed with discretion to distinguish between the different functions or features of a graph.



Supporting Features

Four unobtrusive but important features, each marked by a distinctive icon, contribute to and check understanding.



A “Try It” exercise immediately follows an Example or a set of related Examples, providing the student with an immediate opportunity to solve a similar problem. In the Web View version of the text, students can click an Answer link directly below the question to check their understanding. In the PDF, answers to the Try-It exercises are located in the Answer Key.



A Q&A may appear at any point in the narrative, but most often follows an Example. This feature pre-empts misconceptions by posing a commonly asked yes/no question, followed by a detailed answer and explanation.



The “Media” icon appears at the conclusion of each section, just prior to the Section Exercises. This icon marks a list of links to online video tutorials that reinforce the concepts and skills introduced in the section.

Disclaimer: While we have selected tutorials that closely align to our learning objectives, we did not produce these tutorials, nor were they specifically produced or tailored to accompany Openstax *College Algebra*. We are deeply grateful to James Sousa for compiling his incredibly robust and excellent library of video tutorials, which he has made available to the public under a CC-BY-SA license at <http://mathispower4u.yolasite.com/>. Most or all of the videos to which we link in our “Media” feature (plus many more) are found in the Algebra 2 video library at the above site.

Section Exercises

Each section of every chapter concludes with a well-rounded set of exercises that can be assigned as homework or used selectively for guided practice. With over 4600 exercises across the 9 chapters, instructors should have plenty to choose from^[1].

Section Exercises are organized by question type, and generally appear in the following order:

Verbal questions assess conceptual understanding of key terms and concepts.

Algebraic problems require students to apply algebraic manipulations demonstrated in the section.

Graphical problems assess students’ ability to interpret or produce a graph.

Numeric problems require the student perform calculations or computations.

Technology problems encourage exploration through use of a graphing utility, either to visualize or verify algebraic results or to solve problems via an alternative to the methods demonstrated in the section.

Extensions pose problems more challenging than the Examples demonstrated in the section. They require students to synthesize multiple learning objectives or apply critical thinking to solve complex problems.

Real-World Applications present realistic problem scenarios from fields such as physics, geology, biology, finance, and the social sciences.

Chapter Review Features

Each chapter concludes with a review of the most important takeaways, as well as additional practice problems that students can use to prepare for exams.

Key Terms provides a formal definition for each bold-faced term in the chapter.

Key Equations presents a compilation of formulas, theorems, and standard-form equations.

Key Concepts summarizes the most important ideas introduced in each section, linking back to the relevant Example(s) in case students need to review.

Chapter Review Exercises include 40-80 practice problems that recall the most important concepts from each section.

Practice Test includes 25-50 problems assessing the most important learning objectives from the chapter. Note that the practice test is not organized by section, and may be more heavily weighted toward cumulative objectives as opposed to the foundational objectives covered in the opening sections.

Answer Key includes the answers to all Try It exercises and every other exercise from the Section Exercises, Chapter Review Exercises, and Practice Test.

Ancillaries

OpenStax projects offer an array of ancillaries for students and instructors. Currently the following resources are available.

Instructor’s Solutions Manual

Student’s Solutions Manual

PowerPoint Slides

1. 4,649 total exercises. Includes Chapter Reviews and Practice Tests.

Please visit <http://openstaxcollege.org> to view an up-to-date list of the Learning Resources for this title and to find information on accessing these resources.

Online Homework



WebAssign is an independent online homework and assessment solution first launched at North Carolina State University in 1997. Today, WebAssign is an employee-owned benefit corporation and participates in the education of over a million students each year. WebAssign empowers faculty to deliver fully customizable assignments and high quality content to their students in an interactive online environment. WebAssign supports College Algebra with hundreds of problems covering every concept in the course, each containing algorithmically-generated values and links directly to the eBook providing a completely integrated online learning experience.

XYZ Homework

XYZ Homework provides powerful online instructional tools for faculty and students. Our unified learning environment combines online assessment with MathTV.com video lessons to reinforce the concepts taught in the classroom. Randomized questions provide unlimited practice and instant feedback with all the benefits of automatic grading.

About Our Team

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Jay Abramson has been teaching Precalculus for 28 years, the last 14 at Arizona State University, where he is the head lecturer in the mathematics department. His accomplishments at ASU include developing the university's first hybrid and online math courses as well as an extensive library of video lectures and tutorials. In addition, he has served as a contributing author for two of Pearson Education's math programs, NovaNet Precalculus and Trigonometry. Prior to coming to ASU, Jay taught at Texas State Technical College and Amarillo College. He received Teacher of the Year awards at both institutions.

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