4.6 Adding and Subtracting Mixed Fractions

In this section, we will learn how to add and subtract mixed fractions.

Adding Mixed Fractions

We can use tools we've already developed to add two or more mixed fractions.

You Try It!

EXAMPLE 1. Simplify: $2\frac{7}{8} + 1\frac{3}{4}$.

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

 $3\frac{2}{3} + 4\frac{1}{8}$

Simplify:

$$2\frac{7}{8} + 1\frac{3}{4} = \frac{23}{8} + \frac{7}{4}$$
 Change to improper fractions.
$$= \frac{23}{8} + \frac{7 \cdot 2}{4 \cdot 2}$$
 Equivalent fractions with LCD = 8.
$$= \frac{23}{8} + \frac{14}{8}$$
 Simplify numerators and denominators.
$$= \frac{37}{8}$$
 Add numerators over common denominator.

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 37 divided by 8 is 4, with a remainder of 5. Thus,

$$2\frac{7}{8} + 1\frac{3}{4} = 4\frac{5}{8}.$$

Answer: $7\frac{19}{24}$

Simplify:

You Try It!

EXAMPLE 2. Simplify: $3\frac{1}{4} + 2\frac{1}{3}$.

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$8\frac{1}{2} + 2\frac{2}{3}$$

$$\begin{array}{ll} 3\,\frac{1}{4}+2\,\frac{1}{3}=\frac{13}{4}+\frac{7}{3} & \text{Change to improper fractions.} \\ &=\frac{13\cdot3}{4\cdot3}+\frac{7\cdot4}{3\cdot4} & \text{Equivalent fractions with LCD}=12. \\ &=\frac{39}{12}+\frac{28}{12} & \text{Simplify numerators and denominators.} \\ &=\frac{67}{12} & \text{Add numerators over common denominator.} \end{array}$$

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Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 67 divided by 12 is 5, with a remainder of 7. Thus,

$$3\frac{1}{4} + 2\frac{1}{3} = 5\frac{7}{12}.$$

Answer: $11\frac{1}{6}$

Mixed Fraction Approach. There is another possible approach, based on the fact that a mixed fraction is a sum. Let's revisit Example 2.

You Try It!

Simplify:

EXAMPLE 3. Simplify: $3\frac{1}{4} + 2\frac{1}{3}$.

 $7\frac{2}{5} + 3\frac{1}{8}$

Solution. Use the commutative and associative properties to change the order of addition, make equivalent fractions with a common denominator, then add.

$$\begin{split} 3\,\frac{1}{4} + 2\,\frac{1}{3} &= \left(3 + \frac{1}{4}\right) + \left(2 + \frac{1}{3}\right) & \text{Mixed fractions as sums.} \\ &= (3+2) + \left(\frac{1}{4} + \frac{1}{3}\right) & \text{Reorder and regroup.} \\ &= 5 + \left(\frac{1\cdot 3}{4\cdot 3} + \frac{1\cdot 4}{3\cdot 4}\right) & \text{Add whole numbers: } 3+2=5. \\ &= \text{Equivalent fractions; LCD} = 12. \\ &= 5 + \left(\frac{3}{12} + \frac{4}{12}\right) & \text{Simplify numerators and denominators.} \\ &= 5 + \frac{7}{12} & \text{Add numerators over common denominator.} \end{split}$$

This result can be written in mixed fraction form. Thus,

$$3\frac{1}{4} + 2\frac{1}{3} = 5\frac{7}{12}$$
.

Answer: $10\frac{21}{40}$

Note that this solution is identical to the result found in Example 2.

Example 3 leads us to the following result.

Adding Mixed Fractions. To add two mixed fractions, add the whole number parts, then add the fractional parts.

Working in Vertical Format

When adding mixed fractions, many prefer to work in a vertical format. For example, here is how we would arrange the solution from Example 2 and Example 3 in vertical format. We create equivalent fractions, then add the whole number parts and fractional parts.

$$3\frac{1}{4} = 3\frac{1 \cdot 3}{4 \cdot 3} = 3\frac{3}{12}$$

$$+2\frac{1}{3} = +2\frac{1 \cdot 4}{3 \cdot 4} = +2\frac{4}{12}$$

$$5\frac{7}{12}$$

Note that the answer is identical to the answer found in Example 2 and Example 3. That is,

$$3\frac{1}{4} + 2\frac{1}{3} = 5\frac{7}{12}.$$

EXAMPLE 4. Sarah is making window curtains for two rooms in her house. The kitchen will require $5\frac{2}{3}$ yards of material and the dining room will require $6\frac{5}{8}$ yards of material. How much total material is required?

Solution. To find the total material required for the two rooms, we must add $5\frac{2}{3}$ and $6\frac{5}{8}$. Create equivalent fractions with a common denominator, then add whole number parts and fractional parts.

$$5\frac{2}{3} = 5\frac{2 \cdot 8}{3 \cdot 8} = 5\frac{16}{24}$$

$$+6\frac{5}{8} = +6\frac{5 \cdot 3}{8 \cdot 3} = +6\frac{15}{24}$$

$$11\frac{31}{24}$$

An answer that is part mixed fraction, part improper fraction, is **not** allowed. To finish, we need to change the improper fractional part to a mixed fraction, then add. 31 divided by 24 is 1, with a remainder of 7. That is, $31/24 = 1\frac{7}{24}$. Now we can add whole number parts and fractional parts.

$$11\frac{31}{24} = 11 + 1\frac{7}{24}$$
$$= 12\frac{7}{24}.$$

Thus, the total material required is $12\frac{7}{24}$ yards.

You Try It!

Jim is working on a project that requires two boards, the first cut to a length of $6\frac{1}{2}$ feet, the second cut to a length of $5\frac{7}{8}$ feet. How many total feet of board is required?

Answer: $12\frac{3}{8}$ feet

Subtracting Mixed Fractions

Let's look at some examples that subtract two mixed fractions.

You Try It!

Simplify:

$$5\frac{2}{3} - 3\frac{1}{5}$$

EXAMPLE 5. Simplify: $4\frac{5}{8} - 2\frac{1}{16}$.

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract.

$$4\frac{5}{8} - 2\frac{1}{16} = \frac{37}{8} - \frac{33}{16}$$
 Change to improper fractions.
$$= \frac{37 \cdot 2}{8 \cdot 2} - \frac{33}{16}$$
 Equivalent fractions with LCD = 16.
$$= \frac{74}{16} - \frac{33}{16}$$
 Simplify numerators and denominators.
$$= \frac{41}{16}$$
 Add numerators over common denominator.

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 41 divided by 16 is 2, with a remainder of 9. Thus,

$$4\frac{5}{8} - 2\frac{1}{16} = 2\frac{9}{16}.$$

Answer: $2\frac{7}{15}$

You Try It!

Simplify:

$$4\frac{7}{9} - 2\frac{3}{18}$$

EXAMPLE 6. Simplify: $5\frac{3}{4} - 2\frac{1}{3}$.

Solution. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract.

$$\begin{array}{ll} 5\,\frac{3}{4}-2\,\frac{1}{3}=\frac{23}{4}-\frac{7}{3} & \text{Change to improper fractions.} \\ &=\frac{23\cdot 3}{4\cdot 3}-\frac{7\cdot 4}{3\cdot 4} & \text{Equivalent fractions with LCD}=12. \\ &=\frac{69}{12}-\frac{28}{12} & \text{Simplify numerators and denominators.} \\ &=\frac{41}{12} & \text{Add numerators over common denominator.} \end{array}$$

Although this answer is perfectly acceptable, let's change the answer to a mixed fraction: 41 divided by 12 is 3, with a remainder of 5. Thus,

$$5\frac{3}{4} - 2\frac{1}{3} = 3\frac{5}{12}$$
.

Answer:
$$2\frac{11}{18}$$

Mixed Fraction Approach. There is another possible approach, based on the fact that a mixed fraction is a sum. Let's revisit Example 6.

You Try It!

EXAMPLE 7. Simplify: $5\frac{3}{4} - 2\frac{1}{3}$.

Simplify:

Solution. A mixed fraction is a sum.

 $8\frac{5}{6} - 4\frac{3}{8}$

$$5\frac{3}{4} - 2\frac{1}{3} = \left(5 + \frac{3}{4}\right) - \left(2 + \frac{1}{3}\right)$$

Distribute the negative sign.

$$= 5 + \frac{3}{4} - 2 - \frac{1}{3}$$

We could change the subtraction to adding the opposite, change the order of addition, then change the adding of opposites back to subtraction. However, it is much easier if we look at this last line as a request to add four numbers, two of which are positive and two of which are negative. Changing the order does not affect the answer.

$$= (5-2) + \left(\frac{3}{4} - \frac{1}{3}\right)$$

Note that we did not change the signs of any of the four numbers. We just changed the order. Subtract the whole number parts. Make equivalent fractions with a common denominator, then subtract the fractional parts.

$$= 3 + \left(\frac{3 \cdot 3}{4 \cdot 3} - \frac{1 \cdot 4}{3 \cdot 4}\right)$$
 Create equivalent fractions.

$$= 3 + \left(\frac{9}{12} - \frac{4}{12}\right)$$
 Simplify numerators and denominators.

$$= 3 + \frac{5}{12}$$
 Subtract fractional parts.

Thus,

$$5\frac{3}{4} - 2\frac{1}{3} = 3\frac{5}{12}.$$

Note that this is exactly the same answer as that found in Example 6.

Answer: $4\frac{11}{24}$

In Example 6, we see that we handle subtraction of mixed fractions in exactly the same manner that we handle addition of mixed fractions.

Subtracting Mixed Fractions. To subtract two mixed fractions, subtract their whole number parts, then subtract their fractional parts.

Working in Vertical Format

When subtracting mixed fractions, many prefer to work in a vertical format. For example, here is how we would arrange the solution from Example 6 and Example 7 in vertical format. We create equivalent fractions, then subtract the whole number parts and fractional parts.

$$5\frac{3}{4} = 5\frac{3 \cdot 3}{4 \cdot 3} = 5\frac{9}{12}$$

$$-2\frac{1}{3} = -2\frac{1 \cdot 4}{3 \cdot 4} = -2\frac{4}{12}$$

$$3\frac{5}{12}$$

Note that the answer is identical to the answer found in Example 6 and Example 7. That is,

$$5\frac{3}{4} - 2\frac{1}{3} = 3\frac{5}{12}.$$

Borrowing in Vertical Format

Consider the following example.

You Try It!

Simplify:

$$7\frac{1}{14} - 2\frac{5}{21}$$

EXAMPLE 8. Simplify: $8\frac{1}{4} - 5\frac{5}{6}$.

Solution. Create equivalent fractions with a common denominator.

$$8\frac{1}{4} = 8\frac{1 \cdot 3}{4 \cdot 3} = 8\frac{3}{12}$$

$$-5\frac{5}{6} = -5\frac{5 \cdot 2}{6 \cdot 2} = -5\frac{10}{12}$$

You can see the difficulty. On the far right, we cannot subtract 10/12 from 3/12. The fix is to borrow 1 from 8 in the form of 12/12 and add it to the 3/12.

$$8\frac{3}{12} = 7 + \frac{12}{12} + \frac{3}{12} = 7\frac{15}{12}$$

$$-5\frac{10}{12} = -5\frac{10}{12} = -5\frac{10}{12}$$

$$2\frac{5}{12}$$

Now we can subtract. Hence, $8\frac{1}{4} - 5\frac{5}{6} = 2\frac{5}{12}$.

Answer: $4\frac{5}{6}$

You Try It!

EXAMPLE 9. Jim has a metal rod of length 10 inches. He cuts a length from the metal rod measuring $2\frac{7}{8}$ inches. What is the length of the remaining piece?

Solution. To find the length of the remaining piece, we must subtract $2\frac{7}{8}$ from 10. There is no fractional part on the first number. To remedy this absence, we borrow 1 from 10 in the form of 8/8. Then we can subtract.

Sarah has a length of curtain material that measures 12 feet. She cuts a length of $6\frac{2}{3}$ feet from her curtain material. What is the length of the remaining piece?

$$\begin{array}{rcl}
10 & = & 9 + \frac{8}{8} & = & 9\frac{8}{8} \\
-2\frac{7}{8} & = & -2\frac{7}{8} & = & -2\frac{7}{8} \\
\hline
& & & & & & \\
7\frac{1}{8}
\end{array}$$

Hence, the length of the remaining piece of the metal rod is $7\frac{1}{8}$ inches.

Answer: $5\frac{1}{3}$ feet

Exercises

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In Exercises 1-24, add or subtract the mixed fractions, as indicated, by first converting each mixed fraction to an improper fraction. Express your answer as a mixed fraction.

1.
$$9\frac{1}{4} + 9\frac{1}{2}$$

2.
$$2\frac{1}{3} + 9\frac{1}{2}$$

3.
$$6\frac{1}{2} - 1\frac{1}{3}$$

4.
$$5\frac{1}{3} - 1\frac{3}{4}$$

5.
$$9\frac{1}{2} + 7\frac{1}{4}$$

6.
$$1\frac{1}{3} + 9\frac{3}{4}$$

7.
$$5\frac{2}{3} + 4\frac{1}{2}$$

8.
$$1\frac{9}{16} + 2\frac{3}{4}$$

9.
$$3\frac{1}{3}-1\frac{1}{4}$$

10.
$$2\frac{1}{2} - 1\frac{1}{4}$$

11.
$$8\frac{1}{2} - 1\frac{1}{3}$$

12.
$$5\frac{1}{2}-1\frac{2}{3}$$

13.
$$4\frac{1}{2} - 1\frac{1}{8}$$

14.
$$2\frac{1}{2} - 1\frac{1}{3}$$

15.
$$4\frac{7}{8} + 1\frac{3}{4}$$

16.
$$1\frac{1}{8} + 5\frac{1}{2}$$

17.
$$2\frac{1}{3} - 1\frac{1}{4}$$

18.
$$5\frac{1}{3} - 1\frac{1}{4}$$

19.
$$9\frac{1}{2} - 1\frac{3}{4}$$

20.
$$5\frac{1}{2} - 1\frac{3}{16}$$

21.
$$4\frac{2}{3} + 1\frac{1}{4}$$

22.
$$1\frac{1}{4} + 1\frac{1}{3}$$

23.
$$9\frac{1}{2} + 3\frac{1}{8}$$

24.
$$1\frac{1}{4} + 1\frac{2}{3}$$

In Exercises 25-48, add or subtract the mixed fractions, as indicated, by using vertical format. Express your answer as a mixed fraction.

25.
$$3\frac{1}{2} + 3\frac{3}{4}$$

26.
$$1\frac{1}{2} + 2\frac{2}{3}$$

27.
$$1\frac{3}{8}+1\frac{1}{4}$$

28.
$$2\frac{1}{4} + 1\frac{2}{3}$$

29.
$$1\frac{7}{8} + 1\frac{1}{2}$$

30.
$$1\frac{3}{4} + 4\frac{1}{2}$$

31.
$$8\frac{1}{2} - 5\frac{2}{3}$$

32.
$$8\frac{1}{2} - 1\frac{2}{3}$$

33.
$$7\frac{1}{2} - 1\frac{3}{16}$$

34.
$$5\frac{1}{2} - 1\frac{1}{3}$$

35.
$$9\frac{1}{2} - 1\frac{1}{3}$$

36.
$$2\frac{1}{2} - 1\frac{3}{16}$$

37.
$$5\frac{1}{3} - 2\frac{1}{2}$$

38.
$$4\frac{1}{4} - 1\frac{1}{2}$$

39.
$$9\frac{1}{2} - 2\frac{2}{3}$$

40.
$$7\frac{1}{2} - 4\frac{2}{3}$$

41.
$$1\frac{1}{16} + 1\frac{3}{4}$$

42.
$$1\frac{1}{4} + 1\frac{1}{3}$$

43.
$$8\frac{1}{2} + 3\frac{2}{3}$$

44.
$$1\frac{2}{3} + 2\frac{1}{2}$$

45.
$$6\frac{1}{2} - 1\frac{3}{16}$$

46.
$$4\frac{1}{2} - 1\frac{1}{3}$$

47.
$$2\frac{2}{3}+1\frac{1}{4}$$

48.
$$1\frac{1}{2} + 1\frac{1}{16}$$

Answers

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1.
$$18\frac{3}{4}$$

3.
$$5\frac{1}{6}$$

5.
$$16\frac{3}{4}$$

7.
$$10\frac{1}{6}$$

9.
$$2\frac{1}{12}$$

11.
$$7\frac{1}{6}$$

13.
$$3\frac{3}{8}$$

15.
$$6\frac{5}{8}$$

- 17. $1\frac{1}{12}$
- **19.** $7\frac{3}{4}$
- **21.** $5\frac{11}{12}$
- **23.** $12\frac{5}{8}$
- **25.** $7\frac{1}{4}$
- **27.** $2\frac{5}{8}$
- **29.** $3\frac{3}{8}$
- **31.** $2\frac{5}{6}$

- **33.** $6\frac{5}{16}$
- **35.** $8\frac{1}{6}$
- **37.** $2\frac{5}{6}$
- **39.** $6\frac{5}{6}$

- **41.** $2\frac{13}{16}$
- **43.** $12\frac{1}{6}$
- **45.** $5\frac{5}{16}$
- **47.** $3\frac{11}{12}$