

1.5 The Distributive Property

Learning Objectives

- Apply the distributive property.
- Identify parts of an expression.
- Solve real-world problems using the distributive property.

Introduction

At the end of the school year, an elementary school teacher makes a little gift bag for each of his students. Each bag contains one class photograph, two party favors and five pieces of candy. The teacher will distribute the bags among his 28 students. How many of each item does the teacher need?

Apply the Distributive Property

When we have a problem like the one posed in the introduction, **The Distributive Property** can help us solve it. To begin, we can write an expression for the contents of each bag.

$$\text{Contents} = (\text{photo} + 2 \text{ favor} + 5 \text{ candy})$$

$$\text{Contents} = (p + 2f + 5c) \quad \text{We may even use single letter variables to write an expression.}$$

We know that the teacher has 28 students, therefore we can write the following expression for the number of items that the teacher will need.

$$\text{Items} = 28 \cdot (p + 2f + 5c) \quad \text{28 times the individual contents of each bag.}$$

We generally omit any multiplication signs that are not strictly necessary.

$$\text{Items} = 28(p + 2f + 5c)$$

The Distributive Property of Multiplication means that when faced with a term multiplying other terms inside parentheses, the outside term multiplies with each of the terms inside the parentheses.



$$28(p + 2f + 5c)$$

$$= 28(p + 2f + 5c) = 28(p) + 28(2f) + 28(5c) = 28p + 56f + 140c$$

So the teacher needs 28 class photos, 56 party favors and 140 pieces of candy.

The Distributive Property works when we have numbers inside the parentheses. You can see this by looking at a simple problem and considering the **Order of Operations**.

Example 1

Determine the value of $11(2+6)$ using both Order of Operations and the Distributive Property.

First, we consider the problem with the Order of Operations – PEMDAS dictates that we evaluate the amount inside the parentheses first.

Solution

$$11(2+6) = 11(8) = 88$$

Next we will use the Distributive Property. We multiply the 11 by each term inside the parentheses.

Solution

$$11(2+6) = 11(2) + 11(6) = 22 + 66 = 88$$

Example 2

Determine the value of $11(2-6)$ using both the Order of Operations and the Distributive Property.

First, we consider the Order of Operations and evaluate the amount inside the parentheses first.

Solution

$$11(2-6) = 11(-4) = -44$$

Next, the Distributive Property.

Solution

$$11(2-6) = 11(2) + 11(-6) = 22 - 66 = -44$$

Note When applying the Distributive Property you **MUST** take note of any **negative signs!**

Example 3

Use the Distributive Property to determine the following.

a) $11(2x+6)$

b) $7(3x-5)$

c) $\frac{2}{7}(3y^2-11)$

d) $\frac{2x}{7}\left(3y^2-\frac{11}{xy}\right)$

a) Simply multiply each term by 11.

Solution

$$11(2x+6) = 22x + 66$$

b) Note the negative sign on the second term.

Solution

$$7(3x - 5) = 21x - 35$$

$$c) \frac{2}{7}(3y^2 - 11) = \frac{2}{7}(3y^2) + \frac{2}{7}(-11) = \frac{6y^2}{7} - \frac{22}{7}$$

Solution

$$\frac{2}{7}(3y^2 - 11) = \frac{6y^2 - 22}{7}$$

$$d) \frac{2x}{7}\left(3y^2 - \frac{11}{xy}\right) = \frac{2x}{7}(3y^2) + \frac{2x}{7}\left(\frac{-11}{xy}\right) = \frac{6x^2y - 22x}{7xy}$$

Solution

$$\frac{2x}{7}\left(3y^2 - \frac{11}{xy}\right) = \frac{6xy^3 - 22}{7y}$$

Identify Expressions That Involve the Distributive Property

The Distributive Property often appears in expressions, and many times it does not involve parentheses as grouping symbols. In Lesson 1.2, we saw how the fraction bar acts as a grouping symbol. The following example involves using the Distributive Property with fractions.

Example 4

Simplify the following expressions.

a) $\frac{2x+8}{4}$

b) $\frac{9y-2}{3}$

c) $\frac{z+6}{2}$

Even though these expressions are not written in a form we usually associate with the Distributive Property, the fact that the numerator of fractions should be treated as if it were in parentheses makes this a problem that the Distributive Property can help us solve.

a) $\frac{2x+8}{4}$ can be re-written as $\frac{1}{4}(2x+8)$.

We can then proceed to distribute the $\frac{1}{4}$.

$$\frac{1}{4}(2x+8) = \frac{2x}{4} + \frac{8}{4} = \frac{\cancel{2}x}{\cancel{2} \cdot 2} + \frac{\cancel{4} \cdot 2}{\cancel{4}}$$

Solution

$$\frac{2x+8}{4} = \frac{x}{2} + 2$$

b) $\frac{9y-2}{3}$ can be re-written as $\frac{1}{3}(9y-2)$.

We can then proceed to distribute the $\frac{1}{3}$.

$$\frac{1}{3}(9y-2) = \frac{9y}{3} - \frac{2}{3} = \frac{3 \cdot 3y}{3} - \frac{2}{3}$$

Solution

$$\frac{9y-2}{3} = 3y - \frac{2}{3}$$

c) $\frac{z+6}{2}$ can be re-written as $\frac{1}{2}(z+6)$.

We can then proceed to distribute the $\frac{1}{2}$.

$$\frac{1}{2}(z+6) = \frac{z}{2} + \frac{6}{2}$$

Solution

$$\frac{z+6}{2} = \frac{z}{2} + 3$$

Solve Real-World Problems Using the Distributive Property

The Distributive Property is one of the most common mathematical properties to be seen in everyday life. It crops up in business and in geometry. Anytime we have two or more groups of objects, the Distributive Property can help us solve for an unknown.



Example 5

An octagonal gazebo is to be built as shown right. Building code requires five foot long steel supports to be added along the base and four foot long steel supports to be added to the roof-line of the gazebo. What length of steel will be required to complete the project?

Each side will require two lengths, one of five and four feet respectively. There are eight sides, so here is our equation.

$$\text{Steel required} = 8(4 + 5) \text{ feet.}$$

We can use the distributive property to find the total amount of steel:

$$\text{Steel required} = 8 \times 4 + 8 \times 5 = 32 + 40 \text{ feet.}$$

Solution

A total of 72 feet of steel is required for the project.

**Example 6**

Each student on a field trip into a forest is to be given an emergency survival kit. The kit is to contain a flashlight, a first aid kit, and emergency food rations. Flashlights cost \$12 each, first aid kits are \$7 each and emergency food rations cost \$2 per day. There is \$500 available for the kits and 17 students to provide for. How many days worth of rations can be provided with each kit?

The unknown quantity in this problem is the number of days' rations. This will be x in our expression. Each kit will contain the following items.

- 1 · \$12 flashlight.
- 1 · \$7 first aid kit.
- x · \$2 daily rations.

The number of kits = 17, so the total cost is equal to the following equation.

$$\text{Total cost} = 17(12 + 7 + 2x)$$

We can use the Distributive Property on this expression.

$$17(12 + 7 + 2x) = 204 + 119 + 34x$$

We know that there is \$500 available to buy the kits. We can substitute the cost with the money available.

$$204 + 119 + 34x = 500$$

The sum of the numbers on the left equal to the money available

$$323 + 34x = 500$$

Subtract 323 from both sides

$$-323 - 323$$

$$34x = 177$$

Divide both sides by 34

$$x = 5.20588\dots$$

Since this represents the number of daily rations that can be bought, we must **round to the next lowest whole number**. We wouldn't have enough money to buy a sixth day of supplies.

Solution

Five days worth of emergency rations can be purchased for each survival kit.

Lesson Summary

- **Distributive Property** The multiplication of a number and the sum of two numbers is equal to the first number times the second number plus the first number times the third number.

Ex: $4 \times (6 + 3) = 4 \times 6 + 4 \times 3$

- When applying the Distributive Property you **MUST** take note of any **negative signs!**

Review Questions

- Use the Distributive Property to simplify the following expressions.
 - $(x + 4) - 2(x + 5)$
 - $\frac{1}{2}(4z + 6)$
 - $(4 + 5) - (5 + 2)$
 - $(x + 2 + 7)$
 - $y(x + 7)$
 - $13x(3y + z)$
- Use the Distributive Property to remove the parentheses from the following expressions.
 - $\frac{1}{2}(x - y) - 4$
 - $0.6(0.2x + 0.7)$
 - $6 + (x - 5) + 7$
 - $6 - (x - 5) + 7$
 - $4(m + 7) - 6(4 - m)$
 - $-5(y - 11) + 2y$
- Use the Distributive Property to simplify the following fractions.
 - $\frac{8x+12}{4}$
 - $\frac{9x+12}{3}$
 - $\frac{11x+12}{2}$
 - $\frac{3y+2}{6}$
 - $-\frac{6z-2}{3}$
 - $\frac{7-6p}{3}$
- A bookshelf has five shelves, and each shelf contains seven poetry books and eleven novels. How many of each type of book does the bookcase contain?
- Amar is making giant holiday cookies for his friends at school. He makes each cookie with 6 oz of cookie dough and decorates them with macadamia nuts. If Amar has 5 lbs of cookie dough (1 lb = 16 oz) and 60 macadamia nuts, calculate the following.
 - How many (**full**) cookies he can make?
 - How many macadamia nuts he can put on each cookie, if each is to be identical?

Review Answers

- $-x - 6$
 - $2z + 3$
 - 2
 - $x + 9$
 - $xy + 7y$

- (f) $39xy + 13xz$
2. (a) $\frac{x}{2} - \frac{y}{2} - 4$
(b) $0.12x + 0.42$
(c) $x + 8$
(d) $18 - x$
(e) $10m + 4$
(f) $55 - 3y$
3. (a) $2x + 3$
(b) $3x + 4$
(c) $\frac{11x}{2} + 6$
(d) $\frac{y}{2} + \frac{1}{3}$
(e) $\frac{2}{3} - 2z$
(f) $\frac{7}{3} - 2p$
4. The bookshelf contains 35 poetry books and 55 novels.
5. (a) Amar can make 13 cookies (2 oz leftover).
(b) Each cookie has 4 macadamia nuts (8 left over).