
2.3 One-Step Equations

It's Easier than You Think

You have been solving equations since the beginning of this textbook, although you may not have recognized it. For example, in Lesson 1.4, you determined the answer to the pizza problem below.

\$20.00 was one-quarter of the money spent on pizza.

$\frac{1}{4}m = 20.00$ What divided by 4 equals 20.00?

The solution is 80. So, the amount of money spent on pizza was \$80.00.

By working through this question mentally, you were applying mathematical rules and solving for the variable m .

Definition: To **solve** an equation means to write an equivalent equation that has the variable by itself on one side. This is also known as **isolating the variable**.

In order to begin solving equations, you must understand three basic concepts of algebra: inverse operations, equivalent equations, and the Addition Property of Equality.

Inverse Operations and Equivalent Equations

In Lesson 1.2, you learned how to simplify an expression using the Order of Operations: **P**arentheses, **E**xponents, **M**ultiplication and **D**ivision completed in order from left to right, and **A**ddition and **S**ubtraction (also completed from left to right). Each of these operations has an **inverse**. Inverse operations “undo” each other when combined.

For example, the inverse of addition is subtraction. The inverse of an exponent is a root.

Example 1: Determine the inverse of division.

Solution: To undo dividing something, you would multiply.

By applying the same inverse operations to each side of an equation, you create an **equivalent equation**.

Definition: **Equivalent equations** are two or more equations having the same solution.

The Addition Property of Equality

Just like Spanish, chemistry, or even music, mathematics has a set of rules you must follow in order to be successful. These rules are called properties, theorems, or axioms. They have been proven or agreed upon years ago, so you can apply them to many different situations.

For example, the **Addition Property of Equality** allows you to apply the same operation to each side of the equation, or “what you do to one side of an equation you can do to the other.”

The Addition Property of Equality

For all real numbers a , b , and c :

If $a = b$, then $a + c = b + c$.

Solving One-Step Equations Using Addition or Subtraction

Because subtraction can be considered “adding a negative,” the Addition Property of Equality also works if you need to subtract the same value from each side of an equation.

Example 2:

Solve for y : $16 = y - 11$.

Solution: When asked to solve for y , your goal is to write an equivalent equation with the variable y isolated on one side.

Write the original equation $16 = y - 11$.

Apply the Addition Property of Equality $16 + 11 = y - 11 + 11$

Simplify by adding like terms $27 = y$.

The solution is $y = 27$.

Example 3: *One method to weigh a horse is to load it into an empty trailer with a known weight and reweigh the trailer. A Shetland pony is loaded onto a trailer that weighs 2,200 pounds empty. The trailer is then reweighed. The new weight is 2,550 pounds. How much does the pony weigh?*

Solution: Choose a variable to represent the weight of the pony, say p .

Write an equation $2550 = 2200 + p$.

Apply the Addition Property of Equality $2550 - 2200 = 2200 + p - 2200$.

Simplify $350 = p$.

The Shetland pony weighs 350 pounds.

Equations that take one step to isolate the variable are called **one-step equations**. Such equations can also involve multiplication or division.

Solving One-Step Equations Using Multiplication or Division

The Multiplication Property of Equality

For all real numbers a , b , and c :

If $a = b$, then $a(c) = b(c)$.

Example 4:

Solve for k : $-8k = -96$.

Solution: Because $-8k = -8 \times k$, the inverse operation of multiplication is division. Therefore, we must cancel multiplication by applying the Multiplication Property of Equality.

Write the original equation $-8k = -96$.

Apply the Multiplication Property of Equality $-8k \div -8 = -96 \div -8$.

The solution is $k = 12$.

When working with fractions, you must remember: $\frac{a}{b} \times \frac{b}{a} = 1$. In other words, in order to cancel a fraction using

division, you really must multiply by its reciprocal.

Example 5: Solve $\frac{1}{8} \cdot x = 1.5$.

The variable x is being multiplied by one-eighth. Instead of dividing two fractions, we multiply by the reciprocal of $\frac{1}{8}$, which is 8.

$$\begin{aligned} 8 \left(\frac{1}{8} \cdot x \right) &= 8(1.5) \\ x &= 12 \end{aligned}$$

Solving Real-World Problems Using Equations

As was mentioned in the chapter opener, many careers base their work on manipulating linear equations. Consider the botanist studying bamboo as a renewable resource. She knows bamboo can grow up to 60 centimeters per day. If the specimen she measured was 1 meter tall, how long would it take to reach 5 meters in height? By writing and solving this equation, she will know exactly how long it should take for the bamboo to reach the desired height.

Example 6: In good weather, tomato seeds can grow into plants and bear ripe fruit in as few as 19 weeks. Lorna planted her seeds 11 weeks ago. How long must she wait before her tomatoes are ready to be picked?

Solution: The variable in question is the number of weeks until the tomatoes are ready. Call this variable w .

Write an equation $w + 11 = 19$.

Solve for w by using the Addition Property of Equality.

$$\begin{aligned} w + 11 - 11 &= 19 - 11 \\ w &= 8 \end{aligned}$$



It will take as few as 8 weeks for the plant to bear ripe fruit.

Example 7: In 2004, Takeru Kobayashi of Nagano, Japan, ate $53\frac{1}{2}$ hot dogs in 12 minutes. He broke his previous world record, set in 2002, by three more hot dogs. Calculate:

a) How many minutes it took him to eat one hot dog.

b) How many hot dogs he ate per minute.

c) What his old record was.

Solution:

a) Write an equation, letting m represent the number of minutes to eat one hot dog. Then, $53.5m = 12$

Applying the Multiplication Property of Equality,

$$\frac{53.5m}{53.5} = \frac{12}{53.5}$$

$$m = 0.224 \text{ minutes}$$

It took approximately 0.224 minutes or 13.44 seconds to eat one hot dog.

Questions b) and c) are left for you to complete in the exercises.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. [CK-12 Basic Algebra: One-Step Equations](#) (12:30)



MEDIA

Click image to the left or use the URL below.

URL: <http://www.ck12.org/flx/render/embeddedobject/65>

Solve for the given variable.

1. $x + 11 = 7$
2. $x - 1.1 = 3.2$
3. $7x = 21$
4. $4x = 1$
5. $\frac{5x}{12} = \frac{2}{3}$
6. $x + \frac{5}{2} = \frac{2}{3}$
7. $x - \frac{5}{6} = \frac{3}{8}$
8. $0.01x = 11$
9. $q - 13 = -13$
10. $z + 1.1 = 3.0001$
11. $21s = 3$
12. $t + \frac{1}{2} = \frac{1}{3}$
13. $\frac{7f}{11} = \frac{7}{11}$
14. $\frac{3}{4} = -\frac{1}{2} \cdot y$
15. $6r = \frac{3}{8}$
16. $\frac{9b}{16} = \frac{3}{8}$

17. Peter is collecting tokens on breakfast cereal packets in order to get a model boat. In eight weeks he has collected 10 tokens. He needs 25 tokens for the boat. Write an equation and determine the following information.
- How many more tokens he needs to collect, n .
 - How many tokens he collects per week, w .
 - How many more weeks remain until he can send off for his boat, r .
18. Juan has baked a cake and wants to sell it in his bakery. He is going to cut it into 12 slices and sell them individually. He wants to sell it for three times the cost of making it. The ingredients cost him \$8.50, and he allowed \$1.25 to cover the cost of electricity to bake it. Write equations that describe the following statements.
- The amount of money that he sells the cake for (u).
 - The amount of money he charges for each slice (c).
 - The total profit he makes on the cake (w).
19. Solve the remaining two questions regarding Takeru Kobayashi in Example 7.

Mixed Review

20. Simplify $\sqrt{48}$.
21. Classify 6.23 according to the real number chart.
22. Reduce $\frac{118}{4}$.
23. Graph the following ordered pairs: $\{(2, -2), (4, -1), (5, -5), (3, -2)\}$.
24. Define *evaluate*.
25. Underline the math verb in this sentence: The difference between m and n is 16.
26. What property is illustrated here? $4(a + 11.2) = 4(a) + 4(11.2)$