

## 2.6 Equations with Variables on Both Sides

As you may now notice, equations come in all sizes and styles. There are single-step, double-step, and multi-step equations. In this lesson, you will learn how to solve equations with a variable appearing on each side of the equation. The process you need to solve this type of equation is similar to solving a multi-step equation. The procedure is repeated here.

### Procedure to Solve Equations:

1. Remove any parentheses by using the Distributive Property or the Multiplication Property of Equality.
2. Simplify each side of the equation by combining like terms.
3. Isolate the  $ax$  term. Use the Addition Property of Equality to get the variable on one side of the equal sign and the numerical values on the other.
4. Isolate the variable. Use the Multiplication Property of Equality to get the variable alone on one side of the equation.
5. Check your solution.



Karen and Sarah have bank accounts. Karen has a starting balance of \$125.00 and is depositing \$20 each week. Sarah has a starting balance of \$43 and is depositing \$37 each week. When will the girls have the same amount of money?

To solve this problem, you could use the “guess and check” method. You are looking for a particular week in which the bank accounts are equal. This could take a long time! You could also translate the sentence into an equation. The number of weeks is unknown so this is our variable, call it  $w$ . Now translate this situation into an algebraic equation:

$$125 + 20w = 43 + 37w$$

This is a situation in which the variable  $w$  appears on both sides of the equation. To begin to solve for the unknown, we must use the Addition Property of Equality to gather the variables on one side of the equation.

**Example 1:** Determine when Sarah and Karen will have the same amount of money.

**Solution:** Using the Addition Property of Equality, move the variables to one side of the equation:

$$125 + 20w - 20w = 43 + 37w - 20w$$

Simplify:  $125 = 43 + 17w$

Solve using the steps from Lesson 3.3.

$$125 - 43 = 43 - 43 + 17w$$

$$82 = 17w$$

$$82 \div 17 = 17w \div 17$$

$$w \approx 4.82$$

It will take about 4.8 weeks for Sarah and Karen to have equal amounts of money.

**Example 2:** Solve for  $h$ :  $3(h + 1) = 11h - 23$ .

**Solution:** First you must remove the parentheses by using the Distributive Property.

$$3h + 3 = 11h - 23$$

Gather the variables on one side.

$$3h - 3h + 3 = 11h - 3h - 23$$

Simplify.

$$3 = 8h - 23$$

Solve using the steps from Lesson 3.3.

$$3 + 23 = 8h - 23 + 23$$

$$26 = 8h$$

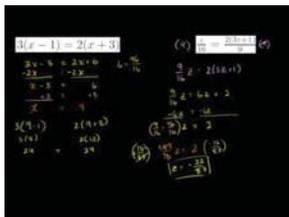
$$26 \div 8 = 8h \div 8$$

$$h = \frac{13}{4} = 3.25$$

**Multimedia Link:** Watch this video: [for further information on how to solve an equation with a variable on each side of the equation.](#)

## Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. [CK-12 Basic Algebra: Equations with Variables on Both Sides \(9:28\)](#)




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**MEDIA**

Click image to the left or use the URL below.

URL: <http://www.ck12.org/flx/render/embeddedobject/68>

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In 1–13, solve the equation.

- $3(x-1) = 2(x+3)$
- $7(x+20) = x+5$
- $9(x-2) = 3x+3$
- $2\left(a - \frac{1}{3}\right) = \frac{2}{5}\left(a + \frac{2}{3}\right)$
- $\frac{2}{7}\left(t + \frac{2}{3}\right) = \frac{1}{5}\left(t - \frac{2}{3}\right)$
- $\frac{1}{7}\left(v + \frac{1}{4}\right) = 2\left(\frac{3v}{2} - \frac{5}{2}\right)$
- $\frac{v-4}{11} = \frac{2}{5} \cdot \frac{2y+1}{3}$
- $\frac{z}{16} = \frac{2(3z+1)}{9}$
- $\frac{q}{16} + \frac{q}{6} = \frac{(3q+1)}{9} + \frac{3}{2}$
- $21 + 3b = 6 - 6(1 - 4b)$
- $-2x + 8 = 8(1 - 4x)$
- $3(-5v - 4) = -6v - 39$
- $-5(5k + 7) = 25 + 5k$
- Manoj and Tamar are arguing about how a number trick they heard goes. Tamar tells Andrew to think of a number, multiply it by five, and subtract three from the result. Then Manoj tells Andrew to think of a number, add five, and multiply the result by three. Andrew says that whichever way he does the trick he gets the same answer. What was Andrew's number?
- I have enough money to buy five regular priced CDs and have \$6 left over. However, all CDs are on sale today for \$4 less than usual. If I borrow \$2, I can afford nine of them. How much are CDs on sale for today?
- Jaime has a bank account with a balance of \$412 and is saving \$18 each week. George has a bank account with a balance of \$874 and is spending \$44 dollars each week. When will the two have the same amount of money?
- Cell phone plan A charges \$75.00 each month and \$0.05 per text. Cell phone plan B charges \$109 dollars and \$0.00 per text.
  - At how many texts will the two plans charge the same?
  - Suppose you plan to text 3,000 times per month. Which plan should you choose? Why?
- To rent a dunk tank, Modern Rental charges \$150 per day. To rent the same tank, Budgetwise charges \$7.75 per hour.
  - When will the two companies charge the same?
  - You will need the tank for a 24-hour fund raise-a-thon. Which company should you choose?

### Mixed Review

- Solve for  $t$ :  $-12 + t = -20$ .
- Solve for  $r$ :  $3r - 7r = 32$ .
- Solve for  $e$ :  $35 = 5(e + 2)$ .
- 25 more than four times a number is 13. What is the number?
- Find the opposite of  $9\frac{1}{5}$ . Write your answer as an improper fraction.
- Evaluate  $(|b| - a) - (|d| - a)$ . Let  $a = 4$ ,  $b = -6$ , and  $d = 5$ .
- Give an example of an integer that is not a counting number.

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**Quick Quiz**

1. Determine the inverse of addition.
2. Solve for  $w$ :  $-4w = 16$ .
3. Write an equation to represent the situation and solve. *Shauna ran the 400 meter dash in 56.7 seconds, 0.98 seconds less than her previous time. What was her previous time?*
4. Solve for  $b$ :  $\frac{1}{2}b + 5 = 9$ .
5. Solve for  $q$ :  $3q + 5 - 4q = 19$ .