

3.2 Graphs of Linear Equations

Learning Objectives

- Graph a linear function using an equation.
- Write equations and graph horizontal and vertical lines.
- Analyze graphs of linear functions and read conversion graphs.

Graph a Linear Equation

At the end of Lesson 4.1 we looked at graphing a function from a rule. A rule is a way of writing the relationship between the two quantities we are graphing. In mathematics, we tend to use the words **formula** and **equation** to describe what we get when we express relationships algebraically. Interpreting and graphing these equations is an important skill that you will use frequently in math.

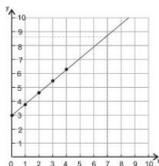
Example 1

A taxi fare costs more the further you travel. Taxis usually charge a fee on top of the per-mile charge to cover hire of the vehicle. In this case, the taxi charges \$3 as a set fee and \$0.80 per mile traveled. Here is the equation linking the cost in dollars (y) to hire a taxi and the distance traveled in miles (x).

$$y = 0.8x + 3$$

Graph the equation and use your graph to estimate the cost of a seven mile taxi ride.

We will start by making a table of values. We will take a few values for x 0, 1, 2, 3, 4, find the corresponding y values and then plot them. Since the question asks us to find the cost for a seven mile journey, we will choose a scale that will accommodate this.



x	y
0	3
1	3.8
2	4.6
3	5.4
4	6.2

The graph is shown to the right. To find the cost of a seven mile journey we first locate $x = 7$ on the horizontal axis and draw a line up to our graph. Next we draw a horizontal line across to the y axis and read where it hits. It appears to hit around half way between $y = 8$ and $y = 9$. Let's say it is 8.5.

Solution

A seven mile taxi ride would cost approximately \$8.50 (\$8.60 exactly).

There are a few interesting points that you should notice about this graph and the formula that generated it.

- The graph is a straight line (this means that the equation is **linear**), although the function is **discrete** and will graph as a series of points.
- The graph crosses the y -axis at $y = 3$ (look at the equation you will see a $+3$ in there!). This is the base cost of the taxi.
- Every time we move **over** by one square we move **up** by 0.8 squares (look at the coefficient of x in the equation). This is the rate of charge of the taxi (cost per mile).
- If we move over by three squares, we move up by 3×0.8 squares.

Example 2

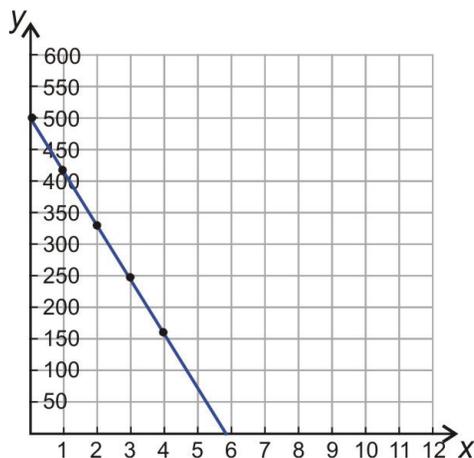
A small business has a debt of \$500000 incurred from start-up costs. It predicts that it can pay off the debt at a rate of \$85000 per year according to the following equation governing years in business (x) and debt measured in thousands of dollars(y).

$$y = -85x + 500$$

Graph the above equation and use your graph to predict when the debt will be fully paid.

First, we start with our table of values. We plug in x -values and calculate our corresponding y -values.

x	y
0	500
1	415
2	330
3	245
4	160



Then we plot our points and draw the line that goes through them.

Take note of the scale that has been chosen. There is no need to have any points above $y = 500$, but it is still wise to allow a little extra.

We need to determine how many years (the x value) that it takes the debt (y value) to reach zero. We know that it is greater than four (since at $x = 4$ the y value is still positive), so we need an x scale that goes well past $x = 4$. In this case the x value runs from 0 to 12, though there are plenty of other choices that would work well.

To read the time that the debt is paid off, we simply read the point where the line hits $y = 0$ (the x axis). It looks as if the line hits pretty close to $x = 6$. So the debt will definitely be paid off in six years.

Solution

The debt will be paid off in six years.

Graphs and Equations of Horizontal and Vertical Lines

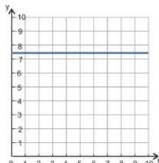
Example 3

"Mad-cabs" have an unusual offer going on. They are charging \$7.50 for a taxi ride of any length within the city limits. Graph the function that relates the cost of hiring the taxi (y) to the length of the journey in miles (x).

To proceed, the first thing we need is an **equation**. You can see from the problem that the cost of a journey does not depend on the length of the journey. It should come as no surprise that the equation then, does not have x in it. In fact, any value of x results in the same value of y (7.5). Here is the equation.

$$y = 7.5$$

The graph of this function is shown to the right. You can see that the graph $y = 7.5$ is simply a horizontal line.



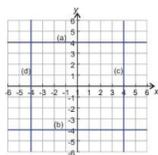
Any time you see an equation of the form $y = \text{constant}$ then the graph is a horizontal line that intercepts the y -axis at the value of the constant.

Similarly, when you see an equation of the form $x = \text{constant}$ then the graph is a vertical line that intercepts the x -axis at the value of the constant. Notice that this a relation, and not a function because each x value (theres only one in this case) corresponds to many (actually an infinite number) y values.

Example 4

Plot the following graphs.

- (a) $y = 4$
- (b) $y = -4$
- (c) $x = 4$
- (d) $x = -4$



- (a) $y = 4$ is a horizontal line that crosses the y -axis at 4
 (b) $y = -4$ is a horizontal line that crosses the y -axis at -4
 (c) $x = 4$ is a vertical line that crosses the x -axis at 4
 (d) $x = -4$ is a vertical line that crosses the x -axis at -4

Example 5

Find an equation for the x -axis and the y -axis.

Look at the axes on any of the graphs from previous examples. We have already said that they intersect at the origin (the point where $x = 0$ and $y = 0$). The following definition could easily work for each axis.

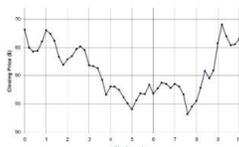
x -axis: A horizontal line crossing the y -axis at zero.

y -axis: A vertical line crossing the x -axis at zero.

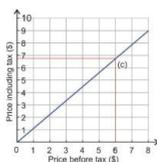
So using example 3 as our guide, we could define the x -axis as the line $y = 0$ and the y -axis as the line $x = 0$.

Analyze Graphs of Linear Functions

We often use line graphs to represent relationships between two linked quantities. It is a useful skill to be able to interpret the information that graphs convey. For example, the chart below shows a fluctuating stock price over ten weeks. You can read that the index closed the first week at about \$68, and at the end of the third week it was at about \$62. You may also see that in the first five weeks it lost about 20% of its value and that it made about 20% gain between weeks seven and ten. Notice that this relationship is discrete, although the dots are connected for ease of interpretation.



Analyzing line graphs is a part of life whether you are trying to decide to buy stock, figure out if your blog readership is increasing, or predict the temperature from a weather report. Many of these graphs are very complicated, so for now we will start off with some simple linear conversion graphs. Algebra starts with basic relationships and builds to the complicated tasks, like reading the graph above. In this section, we will look at reading information from simple linear conversion graphs.

**Example 6**

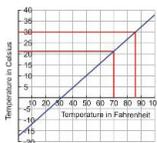
The graph shown at the right shows a chart for converting marked prices in a downtown store into prices that include sales tax. Use the graph to determine the cost inclusive of sales tax for a \$6.00 pen in the store.

To find the relevant price with tax we find the correct pre-tax price on the x -axis. This is the point $x = 6$.

Draw the line $x = 6$ up until it meets the function, then draw a horizontal line to the y -axis. This line hits at $y \approx 6.75$ (about three fourths of the way from $y = 6$ to $y = 7$).

Solution

The approximate cost including tax is \$6.75



Example 7

The chart for converting temperature from Fahrenheit to Celsius is shown to the right. Use the graph to convert the following:

1. 70° Fahrenheit to Celsius
2. 0° Fahrenheit to Celsius
3. 30° Celsius to Fahrenheit
4. 0° Celsius to Fahrenheit

1. To find 70° Fahrenheit we look along the Fahrenheit-axis (in other words the x -axis) and draw the line $x = 70$ up to the function. We then draw a horizontal line to the Celsius-axis (y -axis). The horizontal line hits the axis at a little over 20 (21 or 22).

Solution

70° Fahrenheit is approximately equivalent to 21° Celsius

2. To find 0° Fahrenheit, we are actually looking at the y -axis. Don't forget that this axis is simply the line $x = 0$. We just look to see where the line hits the y -axis. It hits just below the half way point between -15 and -20 .

Solution: 0° Fahrenheit is approximately equivalent to -18° Celsius .

3. To find 30° Celsius, we look up the Celsius-axis and draw the line $y = 30$ along to the function. When this horizontal line hits the function, draw a line straight down to the Fahrenheit-axis. The line hits the axis at approximately 85.

Solution

30° Celsius is approximately equivalent to 85° Fahrenheit.

4. To find 0° Celsius we are looking at the Fahrenheit-axis (the line $y = 0$). We just look to see where the function hits the x -axis. It hits just right of 30.

Solution

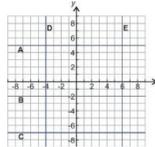
0° Celsius is equivalent to 32° Fahrenheit.

Lesson Summary

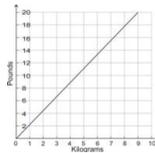
- Equations with the variables y and x can be graphed by making a chart of values that fit the equation and then plotting the values on a coordinate plane. This graph is simply another representation of the equation and can be analyzed to solve problems.
- Horizontal lines are defined by the equation $y = \text{constant}$ and vertical lines are defined by the equation $x = \text{constant}$.
- Be aware that although we graph the function as a line to make it easier to interpret, the function may actually be discrete.

Review Questions

- Make a table of values for the following equations and then graph them.
 - $y = 2x + 7$
 - $y = 0.7x - 4$
 - $y = 6 - 1.25x$
- "Think of a number. Triple it, and then subtract seven from your answer"**. Make a table of values and plot the function that represents this sentence.

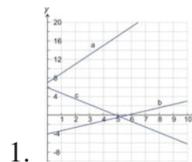


- Write the equations for the five (A through E) lines plotted in the graph to the right.
- At the Airport, you can change your money from dollars into Euros. The service costs \$5, and for every additional dollar you get 0.7 Euros. Make a table for this and plot the function on a graph. Use your graph to determine how many Euros you would get if you give the office \$50.
- The graph to below shows a conversion chart for converting between weight in kilograms to weight in pounds. Use it to convert the following measurements.



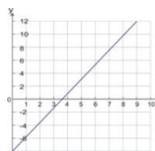
- 4 kilograms into weight in pounds
- 9 kilograms into weight in pounds
- 12 pounds into weight in kilograms
- 17 pounds into weight in kilograms

Review Answers

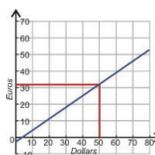


1.

2. $y = 3x - 7$



- $Ay = 5By = -2Cy = -7Dx = -4Ex = 6$
- $y = 0.7(x - 5)$



5.
 - (a) 9 lb
 - (b) 20 lb
 - (c) 5.5 kg
 - (d) 7.75 kg