

3.5 Graphs Using Slope-Intercept Form

Learning Objectives

- Identify the slope and y -intercept of equations and graphs.
- Graph an equation in slope-intercept form.
- Understand what happens when you change the slope or intercept of a line.
- Identify parallel lines from their equations.

Identify Slope and y -intercept

One of the most common ways of writing linear equations prior to graphing them is called **slope-intercept form**. We have actually seen several slope-intercept equations so far. They take the following form:

$y = mx + b$ where m is the slope and the point $(0, b)$ is the y -intercept.

We know that the y -intercept is the point at which the line passes through the y -axis. The slope is a measure of the steepness of the line. Hopefully, you can see that if we know **one point** on a line and the slope of that line, we know what the line is. Being able to quickly identify the y -intercept and slope will aid us in graphing linear functions.

Example 1

Identify the slope and y -intercept of the following equations.

a) $y = 3x + 2$

b) $y = 0.5x - 3$

c) $y = -7x$

d) $y = -4$

Solution

a)

$y = 3x + 2$ with $y = mx + b$

Comparing, we see that $m = 3$ and $b = 2$.

$y = 3x + 2$ has a **slope of 3** and a **y -intercept of $(0, 2)$**

b)

$y = 0.5x - 3$

has a **slope of 0.5** and a **y -intercept of $(0, -3)$** .

Note that the y -intercept is **negative**. The b term includes the sign of the operator in front of the number. Just remember that $y = 0.5x - 3$ is identical to $y = 0.5x + (-3)$ and is in the form $y = mx + b$.

c) **At first glance, this does not appear to fit the slope-intercept form. To illustrate how we deal with this, let us rewrite the equation.**

$$y = mx + b$$

. We now see that we get a slope of -7 and a y -intercept of $(0,0)$.

Note that the slope is negative. The $(0,0)$ intercept means that the line passes through origin.

d) Rewrite as $y = 0x - 4$, giving us a slope of 0 and an intercept of $(0, -4)$.

Remember:

- When $m < 0$ the slope is negative.

For example, $y = -3x + 2$ has a slope of -3 .

- When $b < 0$ the intercept is below the x axis.

For example, $y = 4x - 2$ has a y -intercept of $(0, -2)$.

- When $m = 0$ the slope is zero and we have a horizontal line.

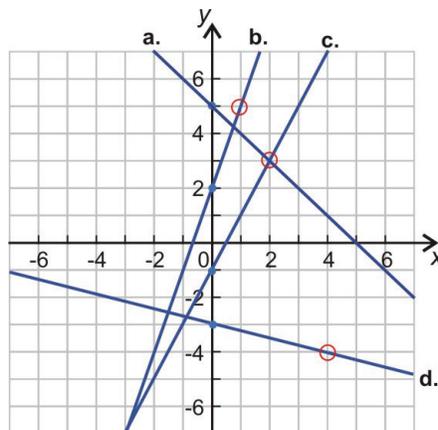
For example, $y = 3$ can be written as $y = 0x + 3$.

- When $b = 0$ the graph passes through the origin.

For example, $y = 4x$ can be written as $y = 4x + 0$.

Example 2

Identify the slope and y -intercept of the lines on the graph shown to the right.



The intercepts have been marked, as have a number of lattice points that lines pass through.

a. The y -intercept is $(0,5)$. The line also passes through $(2,3)$.

$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{-2}{2} = -1$$

b. The y -intercept is $(0,2)$. The line also passes through $(1,5)$.

$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$$

c. The y -intercept is $(0, -1)$. The line also passes through $(2, 3)$.

$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$$

d. The y -intercept is $(0, -3)$. The line also passes through $(4, -4)$.

$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{-1}{4} = \frac{-1}{4} \text{ or } -0.25$$

Graph an Equation in Slope-Intercept Form

Once we know the slope and intercept of a line it is easy to graph it. Just remember what slope means. Let's look back at this example from Lesson 4.1.

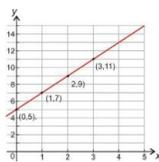
Example 3

Ahiga is trying to work out a trick that his friend showed him. His friend started by asking him to think of a number. Then double it. Then add five to what he got. Ahiga has written down a rule to describe the first part of the trick. He is using the letter x to stand for the number he thought of and the letter y to represent the result of applying the rule. His rule is:

$$y = 2x + 5$$

Help him visualize what is going on by graphing the function that this rule describes.

In that example, we constructed the following table of values.



x	y
0	$2 \cdot 0 + 5 = 0 + 5 = 5$
1	$2 \cdot 1 + 5 = 2 + 5 = 7$
2	$2 \cdot 2 + 5 = 4 + 5 = 9$
3	$2 \cdot 3 + 5 = 6 + 5 = 11$

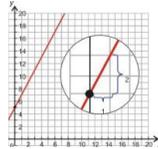
The first entry gave us our y intercept $(0, 5)$. The other points helped us graph the line.

We can now use our equation for slope, and two of the given points.

Slope between $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (3, 11)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 5}{3 - 0} = \frac{6}{3} = 2$$

Thus confirming that the slope, $m = 2$.



An easier way to graph this function is the slope-intercept method. We can now do this quickly, by identifying the intercept and the slope.

$$y = 2x + 5$$

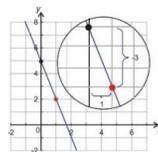
↑ slope = 2 ↓ y-intercept = 5

Look at the graph we drew, the line intersects the y -axis at 5, and every time we move to the right by one unit, we move up by two units.

So what about plotting a function with a negative slope? Just remember that a negative slope means the function decreases as we increase x .

Example 4

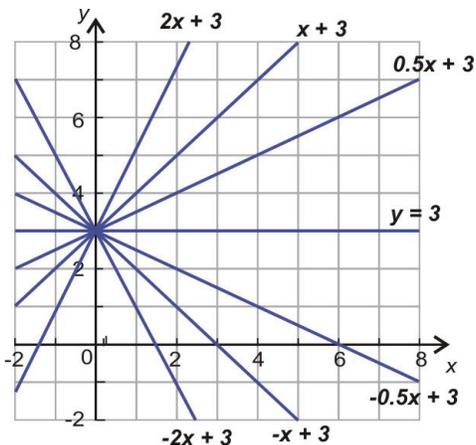
Graph the following function. $y = -3x + 5$



- Identify y -intercept $b = 5$
- Plot intercept $(0, 5)$
- Identify slope $m = -3$
- Draw a line through the intercept that has a slope of -3 .

To do this last part remember that slope = $\frac{\text{rise}}{\text{run}}$ so for every unit we move to the right the function increases by -3 (in other words, for every square we move right, the function comes **down** by 3).

Changing the Slope of a Line



Look at the graph on the right. It shows a number of lines with different slopes, but all with the same y -intercept $(0, 3)$.

You can see all the positive slopes increase as we move from left to right while all functions with negative slopes fall as we move from left to right.

Notice that the higher the value of the slope, the steeper the graph.

The graph of $y = 2x + 3$ appears as the mirror image of $y = -2x + 3$. The two slopes are equal but opposite.

Fractional Slopes and Rise Over Run

Look at the graph of $y = 0.5x + 3$. As we increase the x value by 1, the y value increases by 0.5. If we increase the x value by 2, then the y value increases by 1. In fact, if you express any slope as a fraction, you can determine how to plot the graph by looking at the numerator for the *rise* (keep any negative sign included in this term) and the denominator for the *run*.

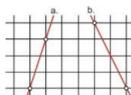
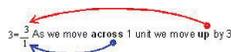
Example 5

Find integer values for the *rise* and *run* of following slopes then graph lines with corresponding slopes.

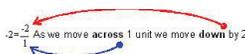
- a. $m = 3$
- b. $m = -2$
- c. $m = 0.75$
- d. $m = -0.375$

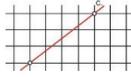
Solution:

a.

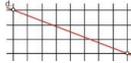


b.





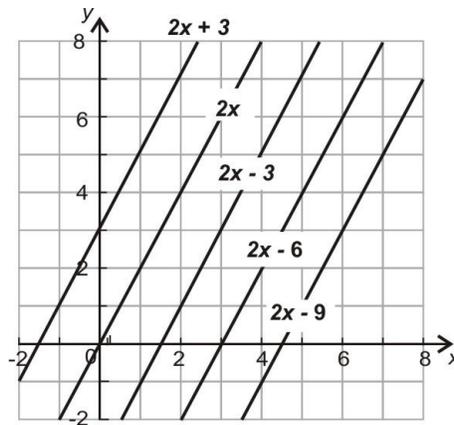
c.



d.



Changing the Intercept of a Line



When we take an equation (such as $y = 2x$) and change the y intercept (leaving the slope intact) we see the following pattern in the graph on the right.

Notice that changing the intercept simply translates the graph up or down. Take a point on the graph of $y = 2x$, such as $(1, 2)$. The corresponding point on $y = 2x + 3$ would be $(1, 4)$. Similarly the corresponding point on the $y = 2x - 3$ line would be $(1, -1)$.

Will These Lines Ever Cross?

To answer that question, let us take two of the equations $y = 2x$ and $y = 2x + 3$ and solve for values of x and y that satisfy both equations. This will give us the (x, y) coordinates of the point of intersection.

$$2x = 2x + 3$$

$$0 = 0 + 3$$

or

Subtract $2x$ from both sides.

$$0 = 3 \text{ This statement is FALSE!}$$

When we get a false statement like this, it means that there are **no** (x, y) values that satisfy both equations simultaneously. The lines will **never** cross, and so they **must be parallel**.

Identify Parallel Lines

In the previous section, when we changed the intercept but left the slope the same, the new line was parallel to the original line. This would be true whatever the slope of the original line, as changing the intercept on a $y = mx + b$ graph does nothing to the slope. This idea can be summed up as follows.

Any two lines with identical slopes are parallel.

Lesson Summary

- A common form of a line (linear equation) is **slope-intercept form**:

$$y = mx + b \text{ where } m \text{ is the slope and the point } (0, b) \text{ is the } y\text{-intercept}$$

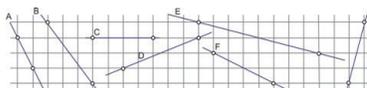
- Graphing a line in slope-intercept form is a matter of first plotting the y -intercept $(0, b)$, then plotting more points by moving a step to the right (adding 1 to x) and moving the value of the slope vertically (adding m to y) before plotting each subsequent point.
- Any two lines with identical slopes are **parallel**.

Review Questions

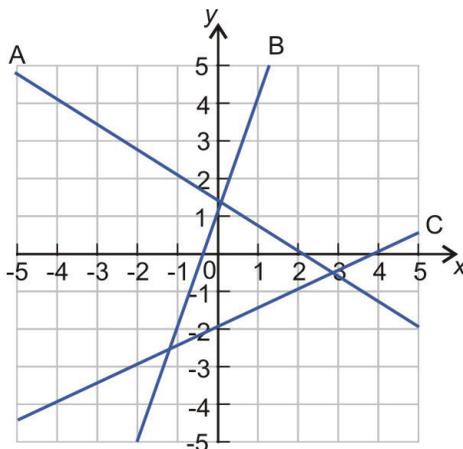
- Identify the slope and y -intercept for the following equations.

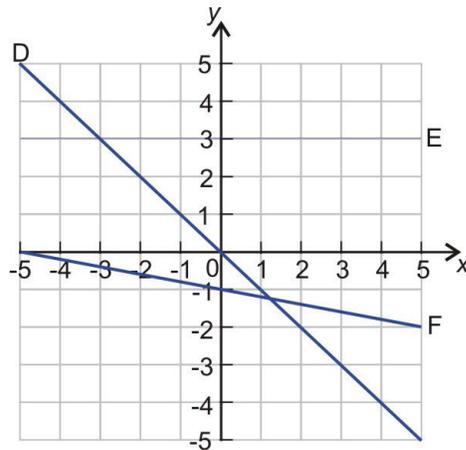
- $y = 2x + 5$
- $y = -0.2x + 7$
- $y = x$
- $y = 3.75$

- Identify the slope of the following lines.



- Identify the slope and y -intercept for the following functions.





4. Plot the following functions on a graph.

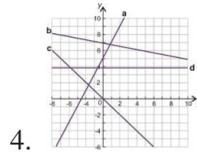
- (a) $y = 2x + 5$
- (b) $y = -0.2x + 7$
- (c) $y = -x$
- (d) $y = 3.75$

5. Which two of the following lines are parallel?

- (a) $y = 2x + 5$
- (b) $y = -0.2x + 7$
- (c) $y = -x$
- (d) $y = 3.75$
- (e) $y = -\frac{1}{3}x - 11$
- (f) $y = -5x + 5$
- (g) $y = -3x + 11$
- (h) $y = 3x + 3.5$

Review Answers

1. (a) $m = 2, (0, 5)$
 (b) $m = -0.2, (0, 7)$
 (c) $m = 1, (0, 0)$
 (d) $m = 0, (0, 3.75)$
2. (a) A. $m = -2$
 (b) B. $m = -\frac{4}{3}$
 (c) C. $m = 0$
 (d) D. $m = \frac{2}{5}$
 (e) E. $m = -0.25$
 (f) F. $m = -0.5$
 (g) G. $m = 4$
3. (a) A. $y = -\frac{2}{3}x + 1.5$
 (b) B: $y = 3x + 1$
 (c) C: $y = 0.5x - 2$
 (d) D: $y = -x$
 (e) E: $y = 3$
 (f) F: $y = -0.2x - 1$



5. b and e