

4.3 Linear Equations in Standard Form

Learning Objectives

- Write equivalent equations in standard form.
- Find the slope and y -intercept from an equation in standard form.
- Write equations in standard form from a graph.
- Solve real-world problems using linear models in standard form.

Introduction

In this section, we are going to talk about the standard form for the equation of a straight line. The following linear equation is said to be in standard form.

$$ax + by = c$$

Here a , b and c are constants that have no factors in common and the constant a is a non-negative value. Notice that the b in the standard form is different than the b in the slope-intercept form. There are a few reasons why standard form is useful and we will talk about these in this section. The first reason is that standard form allows us to write equations for vertical lines which is not possible in slope-intercept form.

For example, let's find the equation of the line that passes through points $(2, 6)$ and $(2, 9)$.

Let's try the slope-intercept form $y = mx + b$

We need to find the slope $m = \frac{9-6}{2-2} = \frac{3}{0}$. The slope is undefined because we cannot divide by zero.

The point-slope form $y - y_0 = m(x - x_0)$ also needs the slope, so we cannot write an equation for this line in either the slope-intercept or the point-slope form.

Since we have two points in a plane, we know that a line passes through these two points, but how do we find the equation of that line? It turns out that this line has no y value in it. Notice that the value of x in both points is two for the different values of y , so we can say that it does not matter what y is because x will always equal two. Here is the equation in standard form.

$$1 \cdot x + 0 \cdot y = 2 \text{ or } x = 2$$

The line passing through point $(2, 6)$ and $(2, 9)$ is a **vertical line** passing through $x = 2$. Note that the equation of a horizontal line would have no x variable, since y would always be the same regardless of the value of x . For example, a **horizontal line** passing through point $(0, 5)$ has this equation in standard form.

$$0 \cdot x + 1 \cdot y = 5 \text{ or } y = 5$$

Write Equivalent Equations in Standard Form

So far you have learned how to write equations of lines in slope-intercept form and point-slope form. Now you will see how to rewrite equations in standard form.

Example 1

Rewrite the following equations in standard form.

a) $y = 5x - 7$

b) $y - 2 = -3(x + 3)$

c) $y = \frac{2}{3}x + \frac{1}{2}$

Solution

We need to rewrite the equations so that all the variables are on one side of the equation and the coefficient of x is not negative.

a) $y = 5x - 7$

Subtract y from both sides.

$$0 = 5x - y - 7$$

Add 7 to both sides.

$$7 = 5x - y$$

The equation in standard form is :

$$5x - y = 7$$

b) $y - 2 = -3(x + 3)$

Distribute the -3 on the right-hand-side.

$$y - 2 = -3x - 9$$

Add $3x$ to both sides.

$$y + 3x - 2 = -9$$

Add 2 to both sides.

$$y + 3x = -7$$

The equation in standard form is :

$$y + 3x = -7$$

c) $y = \frac{2}{3}x + \frac{1}{2}$

Find the common denominator for all terms in the equation. In this case, the common denominator equals 6.

Multiply all terms in the equation by 6.

$$6 \left(y = \frac{2}{3}x + \frac{1}{2} \right) \Rightarrow 6y = 4x + 3$$

Subtract $6y$ from both sides.

$$0 = 4x - 6y + 3$$

Subtract 3 from both sides.

$$-3 = 4x - 6y$$

The equation in standard form is :

$$4x - 6y = -3$$

Find the Slope and

The slope-intercept form and the point-slope form of the equation for a straight line both contain the slope of the equation explicitly, but the standard form does not. Since the slope is such an important feature of a line, it is useful to figure out how you would find the slope if you were given the equation of the line in standard form.

$$ax + by = c$$

Lets rewrite this equation in slope-intercept form by solving the equation for y .

Subtract ax from both sides.

$$by = -ax + c$$

Divide all terms by b .

$$y = -\frac{a}{b}x + \frac{c}{b}$$

If we compare with the slope-intercept form $y = mx + b$, we see that the slope, $m = -\frac{a}{b}$ and the y -intercept $= \frac{c}{b}$. Again, notice that the b in the standard form is different than the b in the slope-intercept form.

Example 2

Find the slope and the y -intercept of the following equations written in standard form:

a) $3x + 5y = 6$

b) $2x - 3y = -8$

c) $x - 5y = 10$

Solution

The slope $m = -\frac{a}{b}$ and the y -intercept $= \frac{c}{b}$.

a) $3x + 5y = 6$ $m = -\frac{3}{5}$ and y -intercept $= \frac{6}{5}$

b) $2x - 3y = -8$ $m = \frac{2}{3}$ and y -intercept $= \frac{8}{3}$

c) $x - 5y = 10$ $m = \frac{1}{5}$ and y -intercept $= \frac{10}{-5} = -2$

Write Equations in Standard Form From a Graph

If we are given a graph of a straight line, it is fairly simple to write the equation in slope-intercept form by reading the slope and y -intercept from the graph. Lets now see how to write the equation of the line in standard form if we are given the graph of the line.

First, remember that to graph an equation from standard form we can use the cover-up method to find the intercepts of the line. For example, lets graph the line given by the equation $3x - 2y = 6$.

To find the x -intercept, cover up the y term (remember, x -intercept is where $y = 0$).

$$3x - \underbrace{2y}_{=0} = 6$$

$$3x = 6 \Rightarrow x = 2$$

The x -intercept is $(2, 0)$

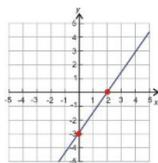
To find the y -intercept, cover up the x term (remember, y -intercept is where $x = 0$).

$$\underbrace{3x}_{=0} - 2y = 6$$

$$-2y = 6 \Rightarrow y = -3$$

The y -intercept is $(0, -3)$

We plot the intercepts and draw a line through them that extends in both directions.

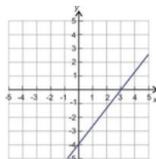


Now we want to apply this process in reverse. If we have the graph of the line, we want to write the equation of the line in standard form.

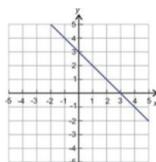
Example 3

Find the equation of the line and write in standard form.

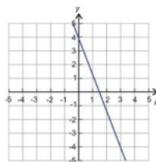
a)



b)



c)



Solution

a) We see that the x -intercept is $(3, 0) \Rightarrow x = 3$ and the y -intercept is $(0, -4) \Rightarrow y = -4$.

We saw that in standard form $ax + by = c$,

if we cover up the y term, we get $ax = c$

if we cover up the x term, we get $by = c$

We need to find the numbers that when multiplied with the intercepts give the same answer in both cases. In this case, we see that multiplying $x = 3$ by 4 and multiplying $y = -4$ by -3 gives the same result.

$$(x = 3) \times 4 \Rightarrow 4x = 12 \text{ and } (y = -4) \times (-3) \Rightarrow -3y = 12$$

Therefore, $a = 4$, $b = -3$ and $c = 12$ and the standard form is:

$$4x - 3y = 12$$

b) We see that the x -intercept is $(3, 0) \Rightarrow x = 3$ and the y -intercept is $(0, 3) \Rightarrow y = 3$.

The values of the intercept equations are already the same, so $a = 1$, $b = 1$ and $c = 3$. The standard form is:

$$x + y = 3$$

c) We see that the x -intercept is $(\frac{3}{2}, 0) \Rightarrow x = \frac{3}{2}$ and the y -intercept is $(0, 4) \Rightarrow y = 4$.

Lets multiply the x -intercept equation by 2 $\Rightarrow 2x = 3$

Then we see we can multiply the x -intercept again by 4 and the y -intercept by 3.

$$\Rightarrow 8x = 12 \text{ and } 3y = 12$$

The standard form is $8x + 3y = 12$.

Solve Real-World Problems Using Linear Models in Standard Form

Here are two examples of real-world problems where the standard form of the equation is useful.

Example 4



Nimitha buys fruit at her local farmers market. This Saturday, oranges cost \$2per pound and cherries cost \$3per pound. She has \$12to spend on fruit. Write an equation in standard form that describes this situation. If she buys 4 pounds of oranges, how many pounds of cherries can she buy?

Solution

Lets define our variables

x = pounds of oranges

y =pounds of cherries

The equation that describes this situation is: $2x + 3y = 12$

If she buys 4 pounds of oranges, we plug $x = 4$ in the equation and solve for y .

$$2(4) + 3y = 12 \Rightarrow 3y = 12 - 8 \Rightarrow 3y = 4 \Rightarrow y = \frac{4}{3}$$

Nimitha can buy $1\frac{1}{3}$ pounds of cherries.



Example 5

Jethro skateboards part of the way to school and walks for the rest of the way. He can skateboard at 7miles per hour and he can walk at 3miles per hour. The distance to school is 6miles. Write an equation in standard form that describes this situation. If Jethro skateboards for $\frac{1}{2}$ an hour, how long does he need to walk to get to school?

Solution

Lets define our variables.

x = hours Jethro skateboards

y = hours Jethro walks

The equation that describes this situation is $7x + 3y = 6$

If Jethro skateboards $\frac{1}{2}$ an hour, we plug $x = 0.5$ in the equation and solve for y .

$$7(0.5) + 3y = 6 \Rightarrow 3y = 6 - 3.5 \Rightarrow 3y = 2.5 \Rightarrow y = \frac{5}{6}$$

Jethro must walk $\frac{5}{6}$ of an hour.

Lesson Summary

- A linear equation in the form $ax + by = c$ is said to be in **standard form**. Where a , b and c are constants (b is different than the y -intercept b) and a is non-negative.
- Given an equation in standard form, $ax + by = c$, the **slope**, $a = -\frac{a}{b}$, and the **y -intercept** $= \frac{c}{b}$.
- The **cover-up method** is useful for graphing an equation in standard form. To find the y -intercept, cover up the x term and solve the remaining equation for y . Likewise, to find the x -intercept, cover up the y term and solve the remaining equation for x .

Review Questions

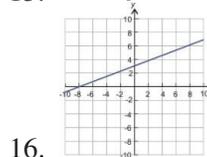
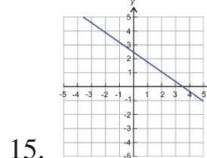
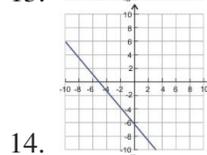
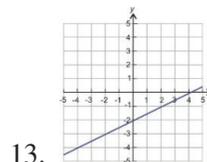
Rewrite the following equations in standard form.

1. $y = 3x - 8$
2. $y - 7 = -5(x - 12)$
3. $2y = 6x + 9$
4. $y = \frac{9}{4}x + \frac{1}{4}$
5. $y + \frac{3}{5} = \frac{2}{3}(x - 2)$
6. $3y + 5 = 4(x - 9)$

Find the slope and y -intercept of the following lines.

7. $5x - 2y = 15$
8. $3x + 6y = 25$
9. $x - 8y = 12$
10. $3x - 7y = 20$
11. $9x - 9y = 4$
12. $6x + y = 3$

Find the equation of each line and write it in standard form.



17. Andrew has two part time jobs. One pays \$6 per hour and the other pays \$10 per hour. He wants to make \$366 per week. Write an equation in standard form that describes this situation. If he is only allowed to work 15 hour per week at the \$10 per hour job, how many hours does he need to work per week in his \$6 per hour job in order to achieve his goal?
18. Anne invests money in two accounts. One account returns 5% annual interest and the other returns 7% annual interest. In order not to incur a tax penalty, she can make no more than \$400 in interest per year. Write an equation in standard form that describes this problem. If she invests \$5000 in the 5% interest account, how much money does she need to invest in the other account?

Review Answers

1. $3x - y = 8$
2. $5x + y = 67$
3. $6x - 2y = -9$
4. $9x - 4y = -1$
5. $10x - 15y = 29$
6. $4x - 3y = 41$
7. $m = (5/2), b = -15/2$
8. $m = -(1/2), b = 25/6$
9. $m = (1/8), b = -3/2$
10. $m = (3/7), b = -20/7$
11. $m = 1, b = -4/9$
12. $m = -6, b = 3$
13. $x - 2y = 4$
14. $6x + 5y = -30$
15. $10x + 14y = 35$
16. $3x - 8y = -24$
17. $x =$ number of hours per week worked at \$6 per hour job $y =$ number of hours per week worked at \$10 per hour job Equation $6x + 10y = 366$ Answer 36 hours
18. $x =$ amount of money invested at 5% annual interest $y =$ amount of money invested at 7% annual interest Equation $5x + 7y = 40000$ Answer \$2142.86