

6.1 Solving One-Step Inequalities

Learning Objectives

- Write and graph inequalities in one variable on a number line.
- Solve an inequality using addition.
- Solve an inequality using subtraction.
- Solve an inequality using multiplication.
- Solve an inequality using division.
- Multiply or divide an inequality by a negative number.

Introduction

Inequalities are similar to equations in that they show a relationship between two expressions. We solve and graph inequalities in a similar way to equations. However, there are some differences that we will talk about in this chapter. The main difference is that for linear inequalities the answer is an interval of values whereas for a linear equation the answer is most often just one value.

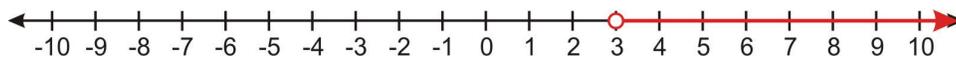
When writing inequalities we use the following symbols

$>$	is greater than
\geq	is greater than or equal to
$<$	is less than
\leq	is less than or equal to

Write and Graph Inequalities in One Variable on a Number Line

Let's start with the simple inequality $x > 3$

We read this inequality as “ x is greater than 3”. The solution is the set of all real numbers that are greater than three. We often represent the solution set of an inequality by a number line graph.



Consider another simple inequality $x \leq 4$

We read this inequality as “ x is less than or equal to 4”. The solution is the set of all real numbers that equal four or less than four. We graph this solution set on the number line.



In a graph, we use an empty circle for the endpoint of a strict inequality ($x > 3$) and a filled circle if the equal sign is included ($x \leq 4$).

Example 1

Graph the following inequalities on the number line.

a) $x < -3$

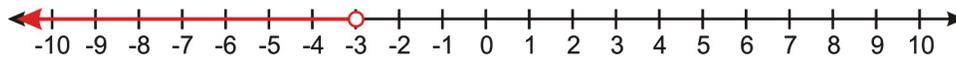
b) $x \geq 6$

c) $x > 0$

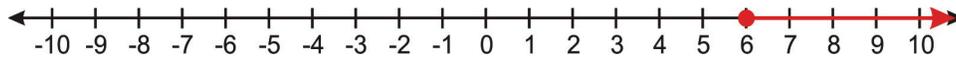
d) $x \leq 8$

Solution

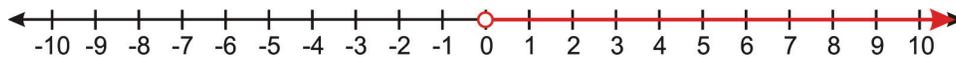
a) The inequality $x < -3$ represents all real numbers that are less than -3 . The number -3 is not included in the solution and that is represented by an open circle on the graph.



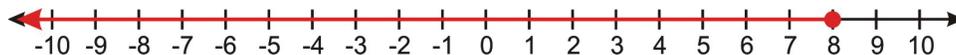
b) The inequality $x \geq 6$ represents all real numbers that are greater than or equal to six. The number six is included in the solution and that is represented by a closed circle on the graph.



c) The inequality $x > 0$ represents all real numbers that are greater than zero. The number zero is not included in the solution and that is represented by an open circle on the graph.

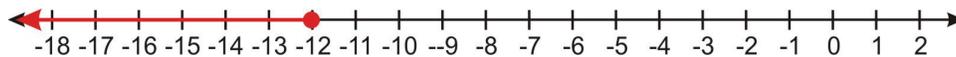


d) The inequality $x \leq 8$ represents all real numbers that are less than or equal to eight. The number eight is included in the solution and that is represented by a closed circle on the graph.

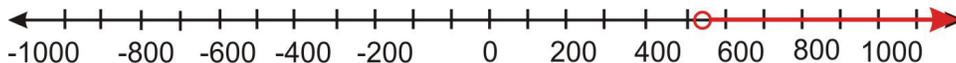
**Example 2**

Write the inequality that is represented by each graph.

a)



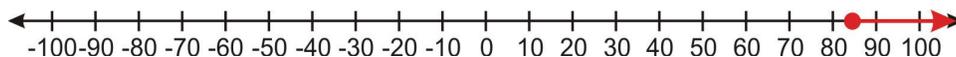
b)



c)



d)

**Solution:**

$$d) x \geq c) x < 65 \quad b) x > 540 \quad a) x \leq -12$$

Inequalities appear everywhere in real life. Here are some simple examples of real-world applications.

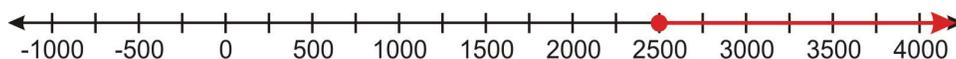
Example 3

Write each statement as an inequality and graph it on the number line.

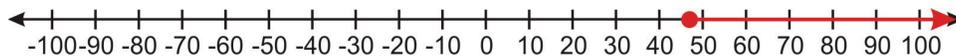
- You must maintain a balance of at least \$2500 in your checking account to get free checking.
- You must be at least 48 inches tall to ride the “Thunderbolt” Rollercoaster.
- You must be younger than 3 years old to get free admission at the San Diego Zoo.
- The speed limit on the interstate is 65 miles per hour.

Solution:

a) The inequality is written as $x \geq 2500$. The words “at least” imply that the value of \$2500 is included in the solution set.



b) The inequality is written as $x \geq 48$. The words “at least” imply that the value of 48 inches is included in the solution set.



c) The inequality is written as $x < 3$.



d) Speed limit means the highest allowable speed, so the inequality is written as $x \leq 65$.

**Solve an Inequality Using Addition**

To solve an inequality we must isolate the variable on one side of the inequality sign. To isolate the variable, we use the same basic techniques used in solving equations. For inequalities of this type:

$$x - a < b \text{ or } x - a > b$$

We isolate the x by adding the constant a to both sides of the inequality.

Example 4

Solve each inequality and graph the solution set.

$$a) x - 3 < 10$$

$$b) x - 1 > -10$$

c) $x - 1 \leq -5$

d) $x - 20 \geq 14$

Solution:

a)

To solve the inequality
Add 3 to both sides of the inequality.
Simplify

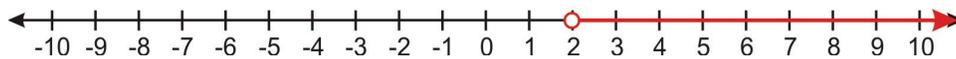
$$\begin{aligned}x - 3 &< 10 \\x - 3 + 3 &< 10 + 3 \\x &< 13\end{aligned}$$



b)

To solve the inequality
Add 12 to both sides of the inequality
Simplify

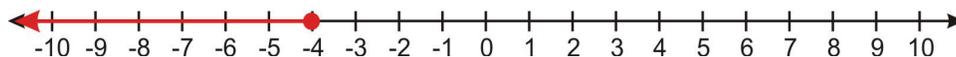
$$\begin{aligned}x - 1 &> -10 \\x - 12 + 12 &> -10 + 12 \\x &> 2\end{aligned}$$



c)

To solve the inequality
Add 1 to both sides of the inequality
Simplify to obtain

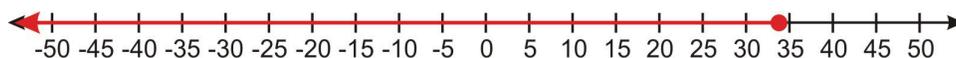
$$\begin{aligned}x - 1 &\leq -5 \\x - 1 + 1 &\leq -5 + 1 \\x &\leq -4\end{aligned}$$



d)

To solve the inequality
Add 20 to both sides of the inequality :
Simplify

$$\begin{aligned}x - 20 &\leq 14 \\x - 20 + 20 &\leq 14 + 20 \\x &\leq 34\end{aligned}$$

**Solve an Inequality Using Subtraction**

For inequalities of this type:

$x + 1 < b$ or $x + 1 > b$

We isolate the x by subtracting the constant a on both sides of the inequality.

Example 5

Solve each inequality and graph the solution set.

a) $x + 2 < 7$

b) $x + 8 \leq -7$

c) $x + 4 > 13$

d) $x + 5 \geq \frac{3}{4}$

Solution:

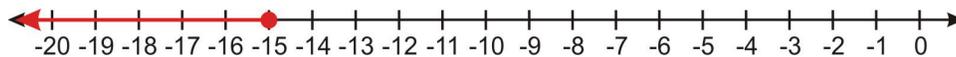
a)

To solve the inequality	$x + 2 < 7$
Subtract 2 on both sides of the inequality	$x + 2 - 2 < 7 - 2$
Simplify to obtain	$x < 5$



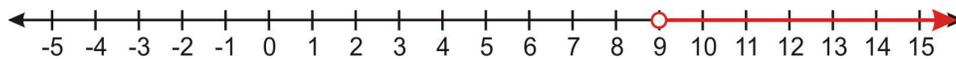
b)

To solve the inequality	$x + 8 \leq -7$
Subtract 8 on both sides of the inequality	$x + 8 - 8 \leq -7 - 8$
Simplify to obtain :	$x \leq -15$



c)

To solve the inequality	$x + 4 > 13$
Subtract 4 on both sides of the inequality	$x + 4 - 4 > 13 - 4$
Simplify	$x > 9$



d)

To solve the inequality	$x + 5 \geq \frac{3}{4}$
Subtract 5 on both sides of the inequality	$x + 5 - 5 \geq -\frac{3}{4} - 5$
Simplify to obtain :	$x \geq -5\frac{3}{4}$



Solve an Inequality Using Multiplication

Consider the problem

$$\frac{x}{5} \leq 3$$

To find the solution we multiply both sides by 5.

$$5 \cdot \frac{x}{5} \leq 3 \cdot 5$$

We obtain

$$x \leq 15$$

The answer of an inequality can be expressed in four different ways:

1. **Inequality notation** The answer is simply expressed as $x < 15$.
2. **Set notation** The answer is $\{x | x < 15\}$. You read this as “the set of all values of x , such that x is a real number less than 15”.
3. **Interval notation** uses brackets to indicate the range of values in the solution.

The interval notation solution for our problem is $(-\infty, 15)$. Interval notation also uses the concept of **infinity** ∞ and **negative infinity** $-\infty$. Round or **open brackets** “(” and “)” indicate that the number next to the bracket is not included in the solution set. Square or **closed brackets** “[” and “]” indicate that the number next to the bracket is included in the solution set.

1. **Solution graph** shows the solution on the real number line. A closed circle on a number indicates that the number is included in the solution set. While an open circle indicates that the number is not included in the set. For our example, the solution graph is drawn here.



Example 6

- a) $[-4, 6]$ says that the solutions is all numbers between -4 and 6 **including** -4 and 6 .

FIGURE 6.1



- b) $(8, 24)$ says that the solution is all numbers between 8 and 24 but **does not include** the numbers 8 and 24.

FIGURE 6.2



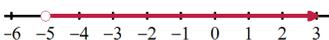
- c) $[3, 12)$ says that the solution is all numbers between 3 and 12, **including** 3 but **not including** 12.

FIGURE 6.3



- d) $(-5, \infty)$ says that the solution is all numbers greater that -5 , **not including** -5 .

FIGURE 6.4



e) $(-\infty, \infty)$ says that the solution is all real numbers.

FIGURE 6.5



Solving an Inequality Using Division

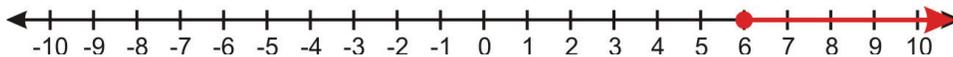
We solve the inequality $2x \geq 12$ by dividing both sides by 2. Then we simplify

Original problem.	$2x \geq 12$
Divide both sides by 2.	$\frac{2x}{2} \geq \frac{12}{2}$
Simplify	$x \geq 6$

Let's write the solution in the four different notations you just learned:

Inequality notation	$x \geq 6$
Set notation	$\{x x \geq 6\}$
Interval notation	$[6, \infty)$

Solution graph



Multiplying and Dividing an Inequality by a Negative Number

We solve an inequality in a similar way to solving a regular equation. We can add or subtract numbers on both sides of the inequality. We can also multiply or divide **positive** numbers on both sides of an inequality without changing the solution.

Something different happens if we multiply or divide by **negative** numbers. **In this case, the inequality sign changes direction.**

For example, to solve $-3x < 9$

We divide both sides by $[-3]$. The inequality sign changes from $<$ to $>$ because we divide by a negative number. $\frac{-3x}{-3} > \frac{9}{-3}$

$$x > -3$$

We can explain why this happens with a simple example. We know that two is less than three, so we can write the inequality.

$$2 < 3$$

If we multiply both numbers by -1 we get -2 and -3 , but we know that -2 is greater than -3 .

$$-2 > -3$$

You see that multiplying both sides of the inequality by a negative number caused the inequality sign to change direction. This also occurs if we divide by a negative number.

Example 7

Solve each inequality. Give the solution in inequality notation and interval notation.

a) $4x < 24$

b) $-9x \geq -\frac{3}{5}$

c) $-5x \leq 21$

d) $12 > -30$

Solution:

a)

Original problem.	$4x < 24$
Divide both sides by 4.	$\frac{4x}{4} < \frac{24}{4}$
Simplify	$x < 6$ or $(-\infty, 6)$ Answer

b)

Original problem :	$-9x \frac{-3}{5}$
Divide both sides by -9 .	$\frac{-9}{-9} \leq \frac{-3}{5} \cdot \frac{1}{-9}$ Direction of the inequality is changed
Simplify.	$x \geq \frac{1}{15}$ or $\left[\frac{1}{15}, \infty\right)$ Answer

Original problem :	$-5x \leq 21$
Divide both sides by -5 .	$\frac{-5x}{-5} \geq \frac{21}{-5}$ Direction of the inequality is changed
Simplify.	$x \geq -\frac{21}{5}$ or $\left[-\frac{21}{5}, \infty\right)$ Answer

d)

Original problem	$12x > -30$
Divide both sides by 12.	$\frac{12x}{12} > \frac{-30}{12}$
Simplify.	$x > -\frac{5}{2}$ or $\left(-\frac{5}{2}, \infty\right)$ Answer

Example 8

Solve each inequality. Give the solution in inequality notation and solution graph.

a) $\frac{x}{2} > 40$

b) $\frac{x}{-3} \leq -12$

c) $\frac{x}{25} < \frac{3}{2}$

d) $\frac{x}{-7} \geq 9$

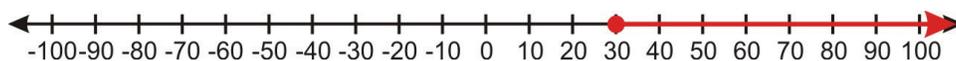
Solution

a)

Original problem	$\frac{x}{2} > 40$
Multiply both sides by 2.	$2 \cdot \frac{x}{2} > 40 \cdot 2$ Direction of inequality is NOT changed
Simplify.	$x > -80$ Answer

b)

Original problem	$\frac{x}{-3} \leq -12$
Multiply both sides by -3 .	$-3 \cdot \frac{x}{-3} \geq -12 \cdot (-3)$ Direction of inequality is changed
Simplify.	$x \geq 36$ Answer



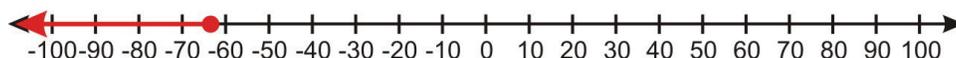
c)

Original problem	$\frac{x}{25} < \frac{3}{2}$
Multiply both sides by 25.	$25 \cdot \frac{x}{25} < \frac{3}{2} \cdot 25$ direction of inequality is NOT changed
Simplify.	$x < \frac{75}{2}$ or $x < 37.5$ Answer



d)

Original problem	$\frac{x}{-7} \geq 9$
Multiply both sides by -7 .	$-7 \cdot \frac{x}{-7} \leq 9 \cdot (-7)$ Direction of inequality is changed
Simplify.	$x \leq -63$ Answer



Lesson Summary

- The answer to an **inequality** is often an **interval of values**. Common **inequalities** are:
- $>$ is greater than
- \geq is greater than or equal to
- $<$ is less than
- \leq is less than or equal to
- Solving inequalities with **addition** and **subtraction** works just like solving an equation. To solve, we isolate the variable on one side of the equation.
- There are four ways to represent an inequality:

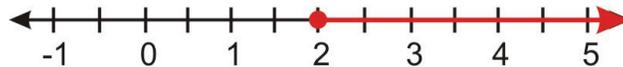
1. **Equation notation** $x \geq 2$

2. **Set notation** $x \geq 2$

3. **Interval notation** $[2, \infty)$

Closed brackets “[” and “]” mean inclusive, parentheses “(” and “)” mean exclusive.

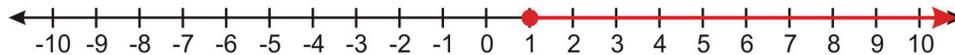
4. **Solution graph**



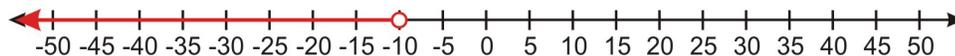
- When multiplying or dividing both sides of an inequality by a negative number, you need to **reverse the inequality**.

Review Questions 1

1. Write the inequality represented by the graph.



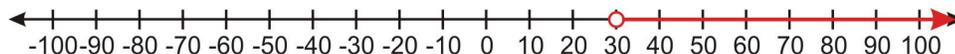
2. Write the inequality represented by the graph.



3. Write the inequality represented by the graph.



4. Write the inequality represented by the graph.



Graph each inequality on the number line.

5. $x < -35$
6. $x > -17$
7. $x \geq 20$
8. $x \leq 3$

Solve each inequality and graph the solution on the number line.

9. $x - 5 < 35$
10. $x + 15 \geq -60$
11. $x - 2 \leq 1$
12. $x - 8 > -20$
13. $x + 11 > 13$
14. $x + 65 < 100$
15. $x - 32 \leq 0$
16. $x + 68 \geq 75$

Review Questions 2

Solve each inequality. Give the solution in inequality notation and solution graph.

1. $3x \leq 6$
2. $\frac{x}{5} > -\frac{3}{10}$
3. $-10x > 250$
4. $\frac{x}{-7} \geq -5$

Solve each inequality. Give the solution in inequality notation and interval notation.

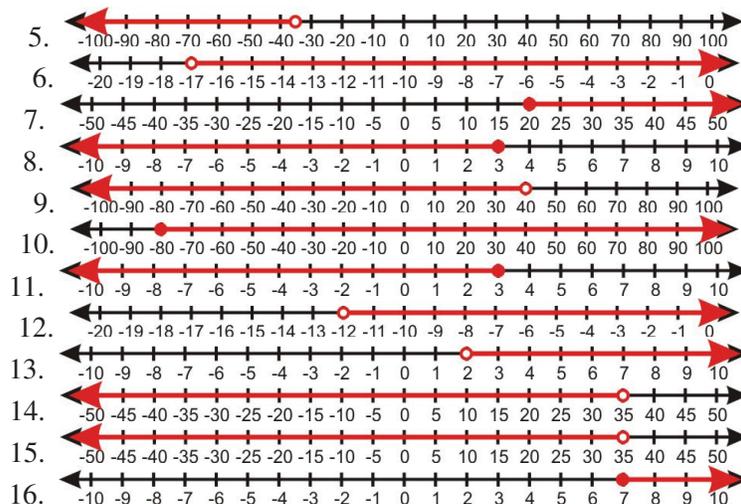
5. $9x > -\frac{3}{4}$
6. $-\frac{x}{15} > \leq 5$
7. $620x > 2400$
8. $\frac{x}{20} \geq -\frac{7}{40}$

Solve each inequality. Give the solution in inequality notation and set notation.

9. $-0.5x \leq 7.5$
10. $75x \geq 125$
11. $\frac{x}{-3} > -\frac{10}{9}$
12. $\frac{x}{-15} < 8$

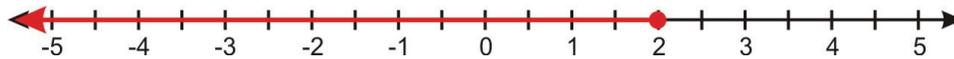
Review Answers 1

1. $x \geq 1$
2. $x < -10$
3. $x \leq -10$
4. $x > 30$

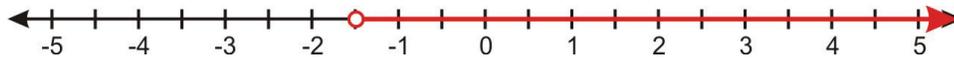


Review Answers 2

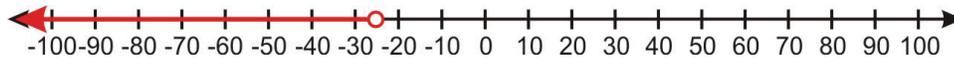
1. $x \leq 2$ or



2. $x > -\frac{3}{2}$ or



3. $x < -25$ or



4. $x \leq 35$ or



5. $x > -\frac{1}{12}$ or $(-\frac{1}{12}, \infty)$

6. $x \geq -75$ or $[-75, \infty)$

7. $x < 3.9$ or $(-\infty, 3.9)$

8. $x \geq -\frac{7}{2}$ or $[-\frac{7}{2}, \infty)$

9. $x \geq -15$ or x is a real number $| x \geq -15$

10. $x \geq \frac{5}{3}$ or x is a real number $| x \geq \frac{5}{3}$

11. $x < -\frac{10}{3}$ or x is a real number $| x < -\frac{10}{3}$

12. $x > -120$ or x is a real number $| x > -120$