

8.4 Scientific Notation

Learning Objectives

- Write numbers in scientific notation.
- Evaluate expressions in scientific notation.
- Evaluate expressions in scientific notation using a graphing calculator.

Introduction Powers of 10

Consider the number six hundred and forty three thousand, two hundred and ninety seven. We write it as 643,297 and each digit's position has a value assigned to it. You may have seen a table like this before.

hundred-thousands	ten-thousands	thousands	hundreds	tens	units (ones)
6	4	3	2	9	7

We have seen that when we write an exponent above a number it means that we have to multiply a certain number of factors of that number together. We have also seen that a zero exponent always gives us one, and negative exponents make fractional answers. Look carefully at the table above. Do you notice that all the column headings are powers of ten? Here they are listed.

$$100,000 = 10^5$$

$$10,000 = 10^4$$

$$1,000 = 10^3$$

$$100 = 10^2$$

$$10 = 10^1$$

Even the units column is really just a power of ten. **Unit** means 1 and $1 = 10^0$.

If we divide 643,297 by 100,000 we get 6.43297. If we multiply this by 100,000 we get back to our original number. But we have just seen that 100,000 is the same as 10^5 , so if we multiply 6.43297 by 10^5 we should also get our original answer. In other words

$$6.43297 \times 10^5 = 643,297$$

So we have found a new way of writing numbers! What do you think happens when we continue the powers of ten? Past the units column down to zero we get into decimals, here the exponent becomes negative.

Writing Numbers Greater Than One in Scientific Notation

Scientific notation numbers are always written in the following form.

$$a \times 10^b$$

Where $1 \leq a < 10$ and b , the exponent, is an integer. This notation is especially useful for numbers that are either very small or very large. When we use scientific notation to write numbers, the exponent on the 10 determines the position of the decimal point.

Look at the following examples.

$$\begin{aligned} 1.07 \times 10^4 &= 10,700 \\ 1.07 \times 10^3 &= 1,070 \\ 1.07 \times 10^2 &= 107 \\ 1.07 \times 10^1 &= 10.7 \\ 1.07 \times 10^0 &= 1.07 \\ 1.07 \times 10^{-1} &= 0.107 \\ 1.07 \times 10^{-2} &= 0.0107 \\ 1.07 \times 10^{-3} &= 0.00107 \\ 1.07 \times 10^{-4} &= 0.000107 \end{aligned}$$

Look at the first term of the list and examine the position of the decimal point in both expressions.

$$1.07 \times 10^4 = 1.07 \times \underbrace{1000}_{4 \text{ zeros}} = \underbrace{10,700.0}_{4 \text{ decimal places difference}}$$

decimal point after 1st digit

So the exponent on the ten acts to move the decimal point over to the right. An exponent of 4 moves it 4 places and an exponent of 3 would move it 3 places.

$$1.07 \times 10^3 = \underbrace{1,070.0}_{3 \text{ decimal places difference}}$$

$$1.07 \times 10^2 = \underbrace{107.0}_{2 \text{ decimal places difference}}$$

Example 1

Write the following numbers in scientific notation.

- (a) 63
- (b) 9,654
- (c) 653,937,000

(d) 1,000,000,006

$$(a) 63 = 6.3 \times 10 = 6.3 \times 10^1$$

$$(b) 9,654 = 9.654 \times 1,000 = 9.654 \times 10^3$$

$$(c) 653,937,000 = 6.53937000 \times 100,000,000 = 6.53937 \times 10^8$$

$$(d) 1,000,000,006 = 1.000000006 \times 1,000,000,000 = 1.000000006 \times 10^9$$

Example 2

The Sun is approximately 93 million miles from Earth. Write this distance in scientific notation.

This time we will simply write out the number long-hand (with a decimal point) and count decimal places.

Solution

$$\underbrace{93,000,000.0}_{7 \text{ decimal places}} = 9.3 \times 10^7 \text{ miles}$$

A Note on Significant Figures

We often combine scientific notation with rounding numbers. If you look at Example 2, the distance you are given has been rounded. It is unlikely that the distance is **exactly** 93 million miles! Looking back at the numbers in Example 1, if we round the final two answers to 2 significant figures (2 s.f.) they become:

$$1(c) 6.5 \times 10^8$$

$$1(d) 1.0 \times 10^9$$

Note that the zero after the decimal point has been left in for Example 1(d) to indicate that the result has been rounded. It is important to know when it is OK to round and when it is not.

Writing Numbers Less Than One in Scientific Notation

We have seen how we can use scientific notation to express large numbers, but it is equally good at expressing extremely small numbers. Consider the following example.

Example 3

The time taken for a light beam to cross a football pitch is 0.0000004 seconds. Express this time in scientific notation.

We will proceed in a similar way as before.

$$0.0000004 = 4 \times 0.0000001 = 4 \times \frac{1}{10,000,000} = 4 \times \frac{1}{10^7} = 4 \times 10^{-7}$$

So...

$$4 \times 10^{-7} = 0.0000004$$

↑ decimal position ↓ 7 decimal places difference

Just as a positive exponent on the ten moves the decimal point that many places to the right, a negative exponent moves the decimal place that many places to the left.

Example 4

Express the following numbers in scientific notation.

- (a) 0.003
 (b) 0.000056
 (c) 0.00005007
 (d) 0.00000000000954

Lets use the method of counting how many places we would move the decimal point before it is after the first non-zero number. This will give us the value for our negative exponent.

- (a) $\underbrace{0.003}_{3 \text{ decimal places}} = 3 \times 10^{-3}$
 (b) $\underbrace{0.000056}_{5 \text{ decimal places}} = 5.6 \times 10^{-5}$
 (c) $\underbrace{0.00005007}_{5 \text{ decimal places}} = 5.007 \times 10^{-5}$
 (d) $\underbrace{0.00000000000954}_{12 \text{ decimal places}} = 9.54 \times 10^{-12}$

Evaluating Expressions in Scientific Notation

When we are faced with products and quotients involving scientific notation, we need to remember the rules for exponents that we learned earlier. It is relatively straightforward to work with scientific notation problems if you remember to deal with all the powers of 10 together. The following examples illustrate this.

Example 5

Evaluate the following expressions and write your answer in scientific notation.

- (a) $(3.2 \times 10^6) \cdot (8.7 \times 10^{11})$
 (b) $(5.2 \times 10^{-4}) \cdot (3.8 \times 10^{-19})$
 (c) $(1.7 \times 10^6) \cdot (2.7 \times 10^{-11})$

The key to evaluating expressions involving scientific notation is to keep the powers of 10 together and deal with them separately. Remember that when we use scientific notation, the leading number **must be between 1 and 10**. We need to move the decimal point over one place to the left. See how this adds 1 to the exponent on the 10.

(a)

$$\begin{aligned} (3.2 \times 10^6) \cdot (8.7 \times 10^{11}) &= \underbrace{3.2 \times 8.7}_{27.84} \times \underbrace{10^6 \times 10^{11}}_{10^{17}} \\ (3.2 \times 10^6) \cdot (8.7 \times 10^{11}) &= 2.784 \times 10^1 \times 10^{17} \end{aligned}$$

Solution

$$(3.2 \times 10^6) \cdot (8.7 \times 10^{11}) = 2.784 \times 10^{18}$$

(b)

$$\begin{aligned}
 (5.2 \times 10^{-4}) \cdot (3.8 \times 10^{-19}) &= \underbrace{5.2 \times 3.8}_{19.76} \times \underbrace{10^{-4} \times 10^{-19}}_{10^{-23}} \\
 &= 1.976 \times 10^1 \times 10^{-23}
 \end{aligned}$$

Solution

$$(5.2 \times 10^{-4}) \cdot (3.8 \times 10^{-19}) = 1.976 \times 10^{-22}$$

(c)

$$(1.7 \times 10^6) \cdot (2.7 \times 10^{-11}) = \underbrace{1.7 \times 2.7}_{4.59} \times \underbrace{10^6 \times 10^{-11}}_{10^{-5}}$$

Solution

$$(1.7 \times 10^6) \cdot (2.7 \times 10^{-11}) = 4.59 \times 10^{-5}$$

Example 6

Evaluate the following expressions. Round to 3 significant figures and write your answer in scientific notation.

(a) $(3.2 \times 10^6) \div (8.7 \times 10^{11})$

(b) $(5.2 \times 10^{-4}) \div (3.8 \times 10^{-19})$

(c) $(1.7 \times 10^6) \div (2.7 \times 10^{-11})$

It will be easier if we convert to fractions and THEN separate out the powers of 10.

(a)

$$\begin{aligned}
 (3.2 \times 10^6) \div (8.7 \times 10^{11}) &= \frac{3.2 \times 10^6}{8.7 \times 10^{11}} \\
 &= \frac{3.2}{8.7} \times \frac{10^6}{10^{11}} \\
 &= 0.368 \times 10^{(6-11)} \\
 &= 3.68 \times 10^{-1} \times 10^{-5}
 \end{aligned}$$

Next we separate the powers of 10.

Evaluate each fraction (round to 3 s.f.):

Remember how to write scientific notation!

Solution

$$(3.2 \times 10^6) \div (8.7 \times 10^{11}) = 3.86 \times 10^{-6} \text{ (rounded to 3 significant figures)}$$

(b)

$$\begin{aligned}
 (5.2 \times 10^{-4}) \div (3.8 \times 10^{-19}) &= \frac{5.2 \times 10^{-4}}{3.8 \times 10^{-19}} \\
 &= \frac{5.2}{3.8} \times \frac{10^{-4}}{10^{-19}} \\
 &= 1.37 \times 10{((-4) - (-19))} \\
 &= 1.37 \times 10^{15}
 \end{aligned}$$

Separate the powers of 10.

Evaluate each fraction (round to 3 s.f.).

Solution

$$(5.2 \times 10^{-4}) \div (3.8 \times 10^{-19}) = 1.37 \times 10^{15} \text{ (rounded to 3 significant figures)}$$

(c)

$$\begin{aligned} (1.7 \times 10^6) \div (2.7 \times 10^{-11}) &= \frac{1.7 \times 10^6}{2.7 \times 10^{-11}} \\ &= \frac{1.7}{2.7} \times \frac{10^6}{10^{-11}} \\ &= 0.630 \times 10^{6-(-11)} \\ &= 6.30 \times 10^{-1} \times 10^{17} \end{aligned}$$

Next we separate the powers of 10.

Evaluate each fraction (round to 3 s.f.).

Remember how to write scientific notation!

Solution

$$(1.7 \times 10^6) \div (2.7 \times 10^{-11}) = 6.30 \times 10^{16} \text{ (rounded to 3 significant figures)}$$

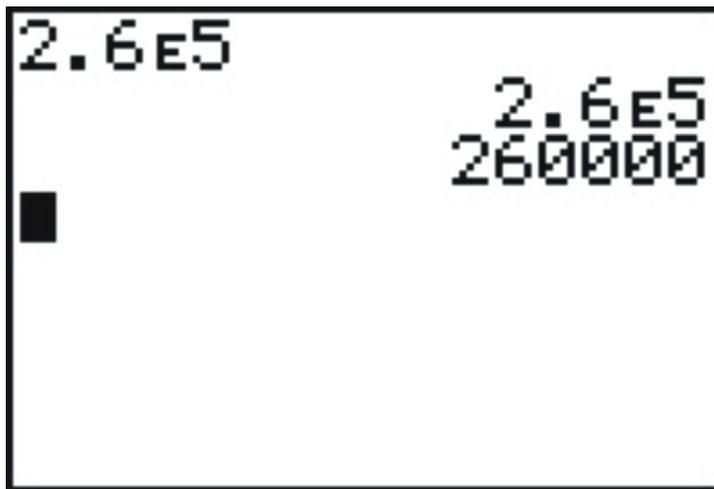
Note that the final zero has been left in to indicate that the result has been rounded.

Evaluate Expressions in Scientific Notation Using a Graphing Calculator

All scientific and graphing calculators have the ability to use scientific notation. It is extremely useful to know how to use this function.

To insert a number in scientific notation, use the **[EE]** button. This is **[2nd] [,]** on some TI models.

For example to enter 2.6×10^5 enter 2.6**[EE]** 5.



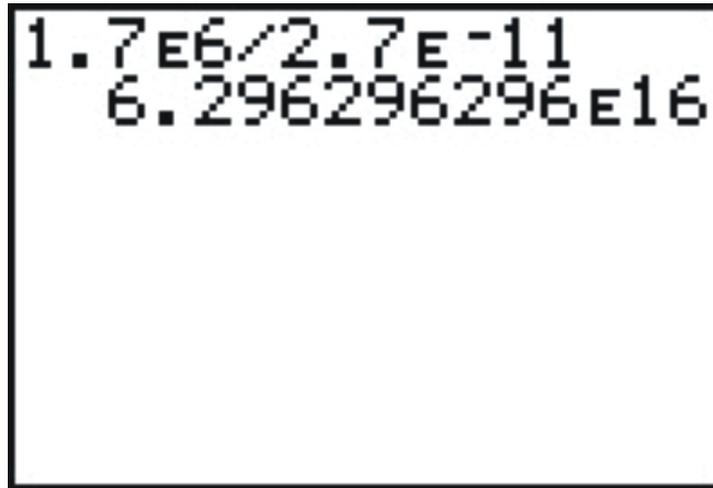
When you hit **[ENTER]** the calculator displays 2.6E5 if its set in **Scientific** mode OR it displays 260000 if its set in **Normal** mode.

(To change the mode, press the Mode key)

Example 7

Evaluate $(1.7 \times 10^6) \div (2.7 \times 10^{-11})$ using a graphing calculator.

[ENTER] 1.7 EE 6 \div 2.7 EE -11 and press **[ENTER]**



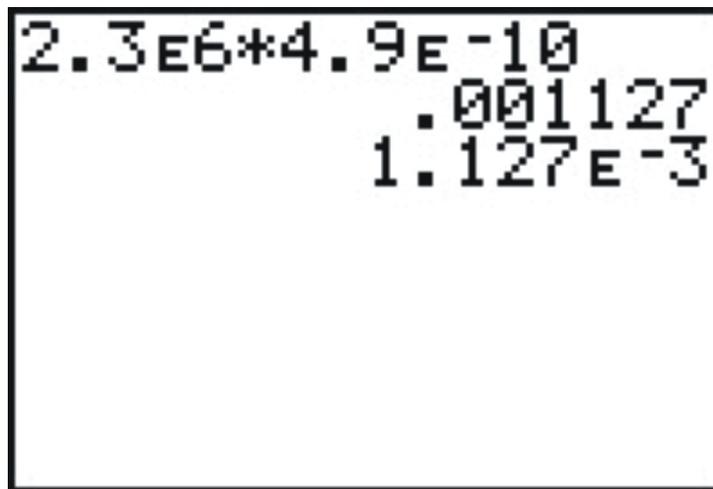
The calculator displays $6.296296296E16$ whether it is in Normal mode or Scientific mode. This is the case because the number is so big that it does not fit inside the screen in Normal mode.

Solution

$$(1.7 \times 10^6) \div (2.7 \times 10^{-11}) = 6.\bar{3} \times 10^{16}$$

Example 8

Evaluate $(2.3 \times 10^6) \times (4.9 \times 10^{-10})$ using a graphing calculator.



[ENTER] 2.3 EE 6 \times 4.9 EE -10 and press [ENTER]

The calculator displays .001127 in Normal mode or $1.127E - 3$ in Scientific mode.

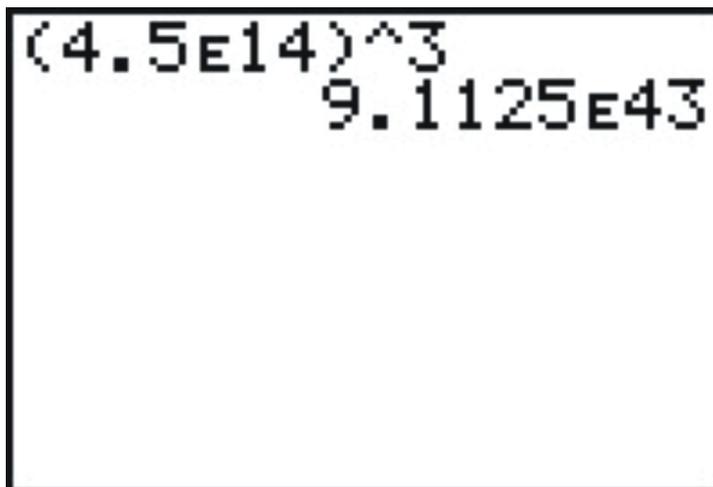
Solution

$$(2.3 \times 10^6) \times (4.9 \times 10^{-10}) = 1.127 \times 10^{-3}$$

Example 9

Evaluate $(4.5 \times 10^{14})^3$ using a graphing calculator.

[ENTER] (4.5EE14)³ and press [ENTER].



The calculator displays 9.1125E43

Solution

$$(4.5 \times 10^{14})^3 = 9.1125 \times 10^{43}$$

Solve Real-World Problems Using Scientific Notation

Example 10

The mass of a single lithium atom is approximately one percent of one millionth of one billionth of one billionth of one kilogram. Express this mass in scientific notation.

We know that percent means we divide by 100, and so our calculation for the mass (in kg) is

$$\frac{1}{100} \times \frac{1}{1,000,000} \times \frac{1}{1,000,000,000} \times \frac{1}{1,000,000,000} = 10^{-2} \times 10^{-6} \times 10^{-9} \times 10^{-9} \times 10^{-9}$$

Next, we use the product of powers rule we learned earlier in the chapter.

$$10^{-2} \times 10^{-6} \times 10^{-9} \times 10^{-9} = 10^{((-2)+(-6)+(-9)+(-9))} = 10^{-26} \text{ kg.}$$

Solution

The mass of one lithium atom is approximately 1×10^{-26} kg .

Example 11

You could fit about 3million E. coli bacteria on the head of a pin. If the size of the pin head in question is $1.2 \times 10^{-5} \text{ m}^2$, calculate the area taken up by one E. coli bacterium. Express your answer in scientific notation.

Since we need our answer in scientific notation it makes sense to convert 3 million to that format first:

$$3,000,000 = 3 \times 10^6$$

Next, we need an expression involving our unknown. The area taken by one bacterium. Call this A.

$$3 \times 10^6 \cdot A = 1.2 \times 10^{-5}$$

Since 3 million of them make up the area of the pin-head.

Isolate A:

$$A = \frac{1}{3 \times 10^6} \cdot 1.2 \times 10^{-5}$$

Rearranging the terms gives

$$A = \frac{1.2}{3} \cdot \frac{1}{10^6} \times 10^{-5}$$

Then using the definition of a negative exponent

$$A = \frac{1.2}{3} \times 10^{-6} \times 10^{-5}$$

Evaluate combine exponents using the product rule.

$$A = 0.4 \times 10^{-11}$$

We cannot, however, leave our answer like this.

Solution

The area of one bacterium $A = 4.0 \times 10^{-12} \text{ m}^2$

Notice that we had to move the decimal point over one place to the right, subtracting 1 from the exponent on the 10.

Review Questions

- Write the numerical value of the following.
 - 3.102×10^2
 - 7.4×10^4
 - 1.75×10^{-3}
 - 2.9×10^{-5}
 - 9.99×10^{-9}
- Write the following numbers in scientific notation.
 - 120,000
 - 1,765,244
 - 12
 - 0.00281
 - 0.000000027
- The moon is approximately a sphere with radius $r = 1.08 \times 10^3$ miles. Use the formula Surface Area = $4\pi r^2$ to determine the surface area of the moon, in square miles. Express your answer in scientific notation, rounded to 2 significant figures.
- The charge on one electron is approximately 1.60×10^{-19} coulombs. One **Faraday** is equal to the total charge on 6.02×10^{23} electrons. What, in coulombs, is the charge on one Faraday?
- Proxima Centauri, the next closest star to our Sun is approximately 2.5×10^{13} miles away. If light from Proxima Centauri takes 3.7×10^4 hours to reach us from there, calculate the speed of light in miles per hour. Express your answer in scientific notation, rounded to 2 significant figures.

Review Answers

- 310.2
 - 74,000
 - 0.00175
 - 0.000029

- (e) 0.00000000999
2. (a) 1.2×10^5
(b) 1.765224×10^{10}
(c) 1.2×10^1
(d) 2.81×10^{-3}
(e) 2.7×10^{-8}
3. 1.5×10^7 miles²
4. 96,320 or 9.632×10^4
5. 6.8×10^8 miles per hour