

$$\textcircled{1} \quad 0 = q_{un} - q_n - y V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right)$$

$$q = r V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) \quad y = r u_c - r_n$$

$$0 = \left[r V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) u_n - r V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) n \right] - \left[(r u_c - r_n) V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) \right]$$

$$\left[r V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) u_n - r V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) n \right] = \left[(r u_c - r_n) V_{max} \left(\frac{n}{k_n + n} \right) \left(\frac{c}{k_c + c} \right) \right]$$

$$\left[r V_{max} u_n \left(\frac{nc}{k_n k_c + k_n c + k_c n + nc} \right) - r V_{max} n \left(\frac{nc}{k_n k_c + k_n c + k_c n + nc} \right) \right] = (r u_c - r_n) V_{max} \left(\frac{nc}{k_n k_c + k_n c + k_c n + nc} \right)$$

$$r V_{max} u_n - r V_{max} n = (r u_c - r_n) V_{max}$$

$$r V_{max} u_n - r V_{max} n = V_{max} r u_c - V_{max} r_n$$

$$\underline{r (V_{max} u_n - V_{max} n)} = \underline{r (V_{max} u_c - V_{max} n)}$$

$$V_{max} u_n - V_{max} n = V_{max} u_c - V_{max} n$$

$$V_{max} u_n - V_{max} n = V_{max} u_c - V_{max} n + V_{max} n$$

$$\frac{V_{max} u_n}{V_{max}} = \frac{V_{max} u_c}{V_{max}}$$

$$\boxed{u_n = u_c}$$