

## Week 4 Assignment

$$\left[ \frac{dx}{dt} = -qx + V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x \right]$$

$$\frac{dy}{dt} = qu - qy - \frac{1}{\epsilon} V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x$$

$$\left[ \frac{dz}{dt} = qv - qz - \frac{1}{\delta} V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x \right]$$

$$z(y) = ?$$

$$(1) \frac{dx}{dt} = 0 = -qx + V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x$$

$$qx = V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x$$

$$q = V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right)$$

$$\frac{dy}{dt} = 0 = qu - qy - \frac{1}{\epsilon} V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x$$

replace w/  $q$ :

$$0 = qu - qy - \frac{1}{\epsilon} (q) x$$

$$\frac{dz}{dt} = 0 = qv - qz - \frac{1}{\delta} V \left( \frac{y}{k+y} \right) \left( \frac{z}{M+z} \right) x$$

replace w/  $q$ :

$$0 = qv - qz - \frac{1}{\delta} (q) x$$

$\frac{dy}{dt}$  and  $\frac{dz}{dt}$  have  $(qx)$  in common

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$$\frac{dy}{dt} = q_u - q_y - \frac{1}{\varepsilon} q_x = 0$$

$$\hookrightarrow \frac{1}{\varepsilon} q_x = q_u - q_y$$

$$q_x = \varepsilon(q_u - q_y)$$

$$\frac{dz}{dt} = 0 = q_v - q_z - \frac{1}{\delta} q_x$$

$$\hookrightarrow \frac{1}{\delta} q_x = q_v - q_z$$

$$q_x = \delta(q_v - q_z)$$

$$q_x = q_x$$

$$\varepsilon(q_u - q_y) = \delta(q_v - q_z)$$

$$\text{Want } z(y) =$$

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$$(q_v - q_z) = \frac{\varepsilon}{\delta} (q_u - q_y)$$

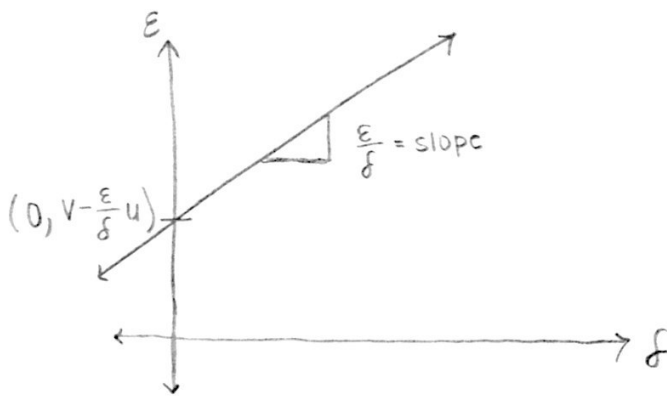
$$-q_z = \frac{\varepsilon}{\delta} (q_u - q_y) - q_v$$

$$z = \frac{1}{\delta} \frac{\varepsilon}{\delta} (q_u - q_y) + \left(\frac{1}{\delta}\right) q_v$$

$$z = -\frac{\varepsilon}{\delta} (u - y) + v$$

$$z = v - \frac{\varepsilon}{\delta} u + \frac{\varepsilon}{\delta} y$$

$$z = \frac{\varepsilon}{\delta} y + v - \frac{\varepsilon}{\delta} u$$



(2) The same as (1)

By setting the derivatives equal to zero, we can set each of the equations equal to each other and eliminate the  $x$  term so that the functions  $y$  and  $z$  are in terms of  $z$  and  $y$  (respectively) and the parameters.

(3) when  $\frac{dx}{dt}$ ,  $\frac{dy}{dt}$ , and  $\frac{dz}{dt}$  equal 0:

x in terms of parameters \*  
i. sub for y, z

$$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$$

$$\frac{dx}{dt} = -qx + v_{max} m n \left\{ \frac{y}{k+y}, \frac{z}{m+z} \right\} x = 0$$

$$qx = v_{max} m n \left\{ \frac{y}{k+y}, \frac{z}{m+z} \right\} *$$

$$\frac{q}{v_{max}} = \min \left\{ \frac{y}{k+y}, \frac{z}{m+z} \right\}$$

$$\frac{y}{k+y} \leq \frac{z}{m+z} \text{ when } \downarrow$$

$$\frac{y}{k+y} \geq \frac{z}{m+z} \text{ when } \downarrow$$

$$y(m+z) \leq z(k+y)$$

$$y(m+z) \geq z(k+y)$$

$$yM \leq zK + zy - zy$$

$$yM \geq zK + yz - yz$$

$$\frac{yM}{K} \leq z$$

$$\frac{yM}{K} \geq z$$

$$\frac{q}{v_{max}} = \min \left\{ \frac{y}{k+y}, \frac{z}{m+z} \right\} = \begin{cases} \frac{y}{k+y}, & \frac{m}{K} y \leq z \\ \frac{z}{m+z}, & \frac{yM}{K} \leq z \end{cases}$$

Substitute z:

$$z = \frac{\epsilon}{\delta} y + v - \frac{\epsilon}{\delta} u$$

$$\frac{M}{K} y = \frac{\epsilon}{\delta} y + v - \frac{\epsilon}{\delta} u$$

$$\frac{M}{K} y - \frac{\epsilon}{\delta} y = v - \frac{\epsilon}{\delta} u$$

$$y \left( \frac{M}{K} - \frac{\epsilon}{\delta} \right) = v - \frac{\epsilon}{\delta} u$$

$$y = \frac{K(v\delta - u\epsilon)}{M\delta - K\epsilon}$$

$$y = \frac{\left( v - \frac{\epsilon}{\delta} u \right)}{\left( \frac{M}{K} - \frac{\epsilon}{\delta} \right)} = \frac{\left( \frac{\delta v - u\epsilon}{\delta} \right)}{\left( \frac{M\delta - K\epsilon}{K\delta} \right)} = \left( \frac{\delta v - u\epsilon}{\delta} \right) \left( \frac{K\delta}{M\delta - K\epsilon} \right)$$

(3) now find z.

$$z = \frac{\epsilon}{\delta} y + v - \frac{\epsilon}{\delta} u$$

$$\frac{M}{K} y = z \rightarrow y = \frac{K}{M} z$$

$$\frac{\epsilon}{\delta} y = z - v + \frac{\epsilon}{\delta} u$$

$$y = \frac{\delta}{\epsilon} \left( z - v + \frac{\epsilon}{\delta} u \right)$$

$$y = \frac{\delta}{\epsilon} z - \frac{\delta}{\epsilon} v + u$$

$$\frac{K}{M} z = \frac{\delta}{\epsilon} z - \frac{\delta}{\epsilon} v + u$$

$$\frac{K}{M} z - \frac{\delta}{\epsilon} z = u - \frac{\delta}{\epsilon} v$$

$$z \left( \frac{K}{M} - \frac{\delta}{\epsilon} \right) = u - \frac{\delta}{\epsilon} v$$

$$z \left( \frac{K\epsilon - \delta M}{M\epsilon} \right) = \frac{u\epsilon - \delta v}{\epsilon}$$

$$z = \left( \frac{u\epsilon - \delta v}{\epsilon} \right) \left( \frac{M\epsilon}{K\epsilon - \delta M} \right) \Rightarrow \boxed{z = \frac{M(u\epsilon - \delta v)}{K\epsilon - \delta M}}$$

$$(4) z = \frac{M(u\epsilon - \delta v)}{K\epsilon - \delta M} = \frac{\epsilon}{\delta} y + v - \frac{\epsilon}{\delta} u$$

$$\frac{\epsilon}{\delta} y = \frac{M(u\epsilon - \delta v)}{K\epsilon - \delta M} - v + \frac{\epsilon}{\delta} u$$

$$y = \frac{\delta}{\epsilon} \left( \frac{M(u\epsilon - \delta v)}{K\epsilon - \delta M} - v + \frac{\epsilon}{\delta} u \right)$$

now sub in:

$$qx = \epsilon(qu - qy)$$

$$x = \epsilon(u - y)$$

$$x = \epsilon u - \epsilon y$$

$$x = \epsilon u - \frac{\delta M(u\epsilon - \delta v)}{K\epsilon - \delta M} + \delta v - \epsilon u$$

$$\boxed{x = \delta v - \frac{\delta M(u\epsilon - \delta v)}{K\epsilon - \delta M}}$$

(5)

