

$$\textcircled{1} \frac{dn}{dt} = qu_n - qn - yV_{\max} \left(\frac{n}{K_n + n} \right) \left(\frac{c}{K_c + c} \right)$$

$$\textcircled{2} \frac{dc}{dt} = qu_c - qc - yV_{\max} \left(\frac{n}{K_n + n} \right) \left(\frac{c}{K_c + c} \right)$$

$$\textcircled{3} \frac{dy}{dt} = ryV_{\max} \left(\frac{n}{K_n + n} \right) \left(\frac{c}{K_c + c} \right) - qy$$

$$\rightarrow 0 = y \left[rV \left(\frac{n}{K_n + n} \right) \left(\frac{c}{K_c + c} \right) - q \right]$$

$$q = rV \left(\frac{n}{K_n + n} \right) \left(\frac{c}{K_c + c} \right)$$

$$\times \frac{q}{rV} = \left(\frac{n}{K_n + n} \right) \left(\frac{c}{K_c + c} \right)$$

↳ plug into eq ①

$$\frac{dn}{dt} = qu_n - qn - yV \left(\frac{q}{rV} \right)$$

$$0 = qu_n - qn - \underline{qy}$$

$$\frac{qy}{r} = qu_n - qn$$

$$\frac{y}{r} = u_n - n$$

$$y = r(u_n - n)$$

$$n = u_n - \frac{y}{r}$$

$$c = u_c - \frac{y}{r}$$

↳ plug into eq ②

$$\frac{dc}{dt} = qu_c - qc - yV \left(\frac{q}{rV} \right)$$

$$0 = qu_c - qc - \underline{qy}$$

$$\frac{qy}{r} = qu_c - qc$$

$$\frac{y}{r} = u_c - c$$

$$y = r(u_c - c)$$