

$$\textcircled{2} \quad \frac{dy}{dt} = yR \frac{c_1}{K_1 + c_1} \frac{c_2}{K_2 + c_2} - ay = 0$$

$$ay = yR \frac{c_1}{K_1 + c_1} \frac{c_2}{K_2 + c_2}$$

$$\boxed{\frac{a}{R} = \frac{c_1}{K_1 + c_1} \frac{c_2}{K_2 + c_2}}$$

$$\textcircled{2} \frac{dc_1}{dt} = qu_1 - qc_1 - yV_1 \frac{c_1}{K_1 + c_1} \frac{c_2}{K_2 + c_2} = 0$$

$$qu_1 - qc_1 = yV_1 \frac{c_1}{K_1 + c_1} \frac{c_2}{K_2 + c_2}$$

$$q(u_1 - c_1) = yV_1 \frac{q}{R}$$

$$Rq(u_1 - c_1) = yV_1 q$$

$$R(u_1 - c_1) = yV_1$$

$$y^* = \frac{R}{V_1} (u_1 - c_1^*)$$

$$\frac{dc_2}{dt} = qu_2 - qc_2 - yV_2 \frac{c_1}{K_1 + c_1} \frac{c_2}{K_2 + c_2} = 0$$

$$qu_2 - qc_2 = yV_2 \frac{q}{R}$$

$$Rq(u_2 - c_2) = yV_2 q$$

$$R(u_2 - c_2) = yV_2$$

$$y^* = \frac{R}{V_2} (u_2 - c_2^*)$$

$$(4) \frac{dy}{dt} = yR \min \left\{ \frac{c_1}{K_1 + c_1}, \frac{c_2}{K_2 + c_2} \right\} - ay = 0$$

$$ay = yR \min \left\{ \frac{c_1}{K_1 + c_1}, \frac{c_2}{K_2 + c_2} \right\}$$

$$\boxed{\frac{a}{R} = \min \left\{ \frac{c_1}{K_1 + c_1}, \frac{c_2}{K_2 + c_2} \right\}}$$

$$\frac{dc_1}{dt} = au_1 - ac_1 - yV_1 \min \left\{ \frac{c_1}{K_1 + c_1}, \frac{c_2}{K_2 + c_2} \right\} = 0$$

$$\underbrace{\hspace{10em}}_{a/R}$$

$$a(u_1 - c_1) = yV_1 \frac{a}{R}$$

$$R(u_1 - c_1) = yV_1$$

$$y^* = \frac{R}{V_1} (u_1 - c_1^*)$$

$$\frac{dc_2}{dt} = au_2 - ac_2 - yV_2 \min \left\{ \frac{c_1}{K_1 + c_1}, \frac{c_2}{K_2 + c_2} \right\} = 0$$

$$\underbrace{\hspace{10em}}_{a/R}$$

$$a(u_2 - c_2) = yV_2 \frac{a}{R}$$

$$\boxed{y^* = \frac{R}{V_2} (u_2 - c_2^*)}$$