Defining Stability

so far stability has been defined by small perturbations.

- do you travel away from or the fixed point w/ a small perturbation?
- or do you return to the same fixed point.

this was for 1D systems (i.e. flow on the line).

\[ \begin{array}{ccc}
0 & \rightarrow & 1 \\
& \downarrow & \\
2 & \rightarrow & 3 \\
\end{array} \]

two possibilities: 1. stable fixed point
- unstable fixed point
- mixed stability fixed point.
- or blow up

But for 2D systems, we can flow/maneuver through a plane, so we have a wider range of possible behaviors.

conceptual analogy of unstable & stable fixed point

unstable pencil perfectly balanced on your finger

a ball at the bottom of a hill.
Attracting: a fixed point is attracting if all trajectories starting near the point approach the point as time $\to \infty$.

i.e. given enough time you will eventually end up at the fixed point if you start anywhere near it.

- says nothing about the path you take to get there.
- where you end up
- so you could travel away from the point before you approach it again.

ex.

Our fixed point is attracting even though you may travel away from your point.

Remember our flow on the circle - nonuniform oscillator

$$\dot{\theta} = \omega - \sin \theta$$
Liapunov Stable: a fixed point is Liapunov stable if all trajectories that start close to \((x^*)\) the fixed point you will stay close to \(x^*\) for all time.

- for all time: it does say something about the path; it must stay near; it cannot stray too far away.
- also again near must be defined.

attracting & Liapunov stable one can exist w/o the other.

our oscillator example is attracting but not Liapunov stable.
the term stable implies both attracting \& Liapunov stable.

we didn't have to differentiate for flows on a line.

unstable means: neither attracting nor Liapunov stable.