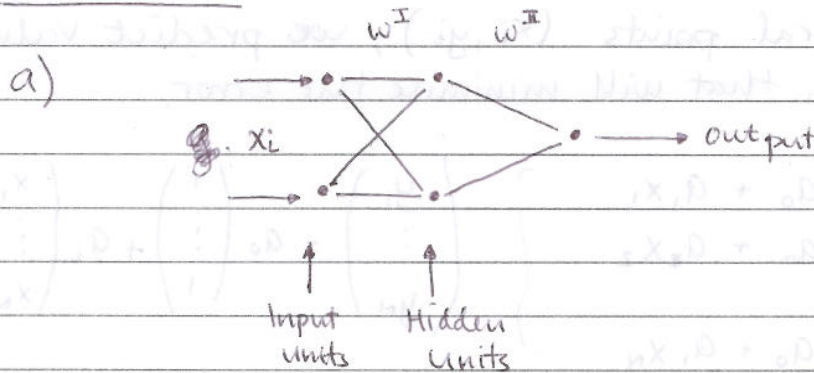


Question 1

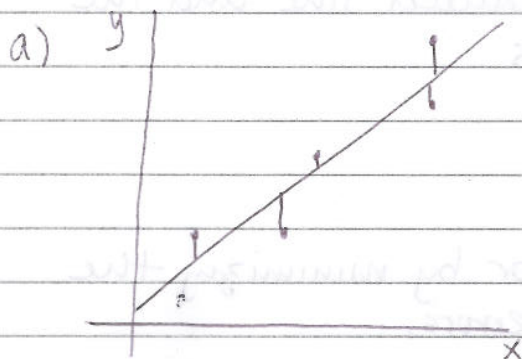


b) input to hidden units =  $g(x_i \cdot w_{ij}^{(I)})$

output =  $y = g(w_{ij}^{(II)} \cdot g(x_i \cdot w_{ij}^{(I)}))$

$g \equiv$  non-linear transformation

Question 2.



Given  $N \gg 2$  data points, we can establish the linear fit using the least squares method that minimizes the error (vertical, in the y-direction) of the fit. The solution

is optimal since we have minimized the error function. Total least squares fit can be done if we are unsure which variable is our independent and dependent variable. In this case, the normal distance from the points to the best fit line will be minimized.

## b) Matrix Formulation for least squares:

Given several points  $(x_i, y_i)$ , we predict values  $a_0$  and  $a_1$  that will minimise the error.

$$\left. \begin{array}{l} y_1 = a_0 + a_1 x_1 \\ y_2 = a_0 + a_1 x_2 \\ \vdots \\ y_N = a_0 + a_1 x_N \end{array} \right\} \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = a_0 \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + a_1 \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

Our predicted line can be written as

$$\hat{y} = \begin{pmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_N \end{pmatrix} = a_0 \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + a_1 \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_N \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$\hat{y} = Xa$$

The error between our predicted line and the points can be written as

$$e = \hat{y} - y$$

We can minimise the error by minimizing the sum of all the squared errors.

$$E = e^T e = (\hat{y} - y)^T (\hat{y} - y)$$

Taking the gradient of  $E$ , we can obtain

$$(\nabla E)_a = 2X^T X a - X^T y - X^T y.$$

The least squares solution is given by the coefficients  $a$  that will minimize the error.

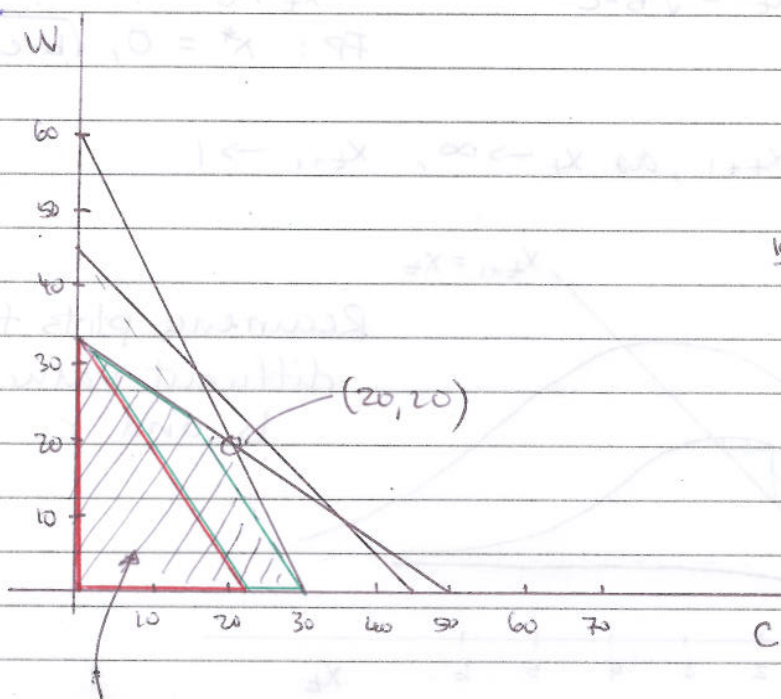
$$a = (X^T X)^{-1} X^T y.$$

### Question 3

a) We want to optimize the profit based upon the given constraints. ( $C$  = Ha of corn,  $W$  = Ha of wheat)

- Total land available = 45 Ha :  $C + W \leq 45$
- Total labor available = 100 <sup>workers</sup> :  $2C + 3W \leq 100$
- Total fertilizer available = 1200 kg :  $40C + 20W \leq 1200$

Our profit function :  $P = 300C + 200W$



$$W = 45 - C$$

$$W = 33\frac{1}{3} - \frac{2}{3}C$$

$$W = 60 - 2C$$

$$\frac{100}{3} - \frac{2}{3}C = 60 - 2C$$

$$\frac{4}{3}C = \frac{80}{3}$$

$$C = 20, W = 20$$

Feasible region

The maximum profit will be when  $W=20$  and  $C=20$

$$P = 300(20) + 200(20) = \underline{10,000}$$

b) i) The maximum profit for wheat alone =  $33\frac{1}{3}(200) = 6667$ .

The amount of corn it takes to make the same profit is 22.2 Ha, so if the corn and wheat values lie within the red triangle above, then it is advantageous to only produce wheat.

ii) Similarly, if cultivation lies within the green box, then it is worthwhile to only produce corn for a profit of 9000.

Question 4

$$x_{t+1} = \frac{x_t^2 + bx_t}{x_t^2 + x_t + c}$$

a) To find the fixed points,  $x_{t+1} = x_t$ .

$$x_t = \frac{x_t^2 + bx_t}{x_t^2 + x_t + c} \Rightarrow x_t^* = 0$$

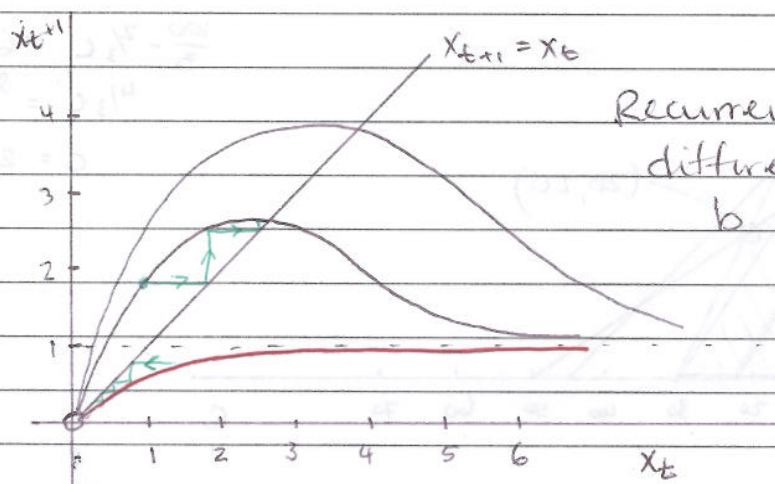
$$x_t^2 + x_t + c = x_t + b$$

$$x_t^2 = \sqrt{b-c}$$

Z.F.P. in the region where  $x_t > 0$

$$\text{FP: } x^* = 0, \sqrt{b-c}$$

Graph of  $x_{t+1}$ , as  $x_t \rightarrow \infty$ ,  $x_{t+1} \rightarrow 1$



Recurrence plots for different values of  $b$  and  $c$ .

For  $b > c$

From cobweb analysis, the  $\sqrt{b-c}$  fixed point is stable, while the 0 fixed point is unstable.

For  $b < c$  (red line)

From the cobweb analysis, the 0 fixed point is the only existing and stable fixed point.

b)

$$x_{t+1} = \frac{x_t^2 + bx_t}{x_t^2 + x_t + c} - Kx_t$$

proportion of the population.

Find fixed points  $x_t = x_{t+1}$ .

$$x_t = \frac{x_t^2 + bx_t}{x_t^2 + x_t + c} - K$$

$$x_t(1+K) = \frac{x_t^2 + bx_t}{x_t^2 + x_t + c}$$

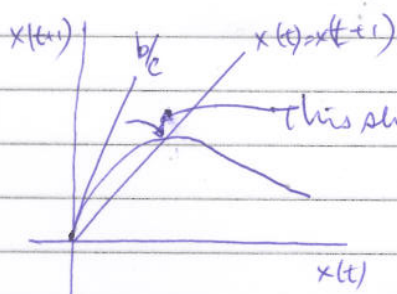
For the fish population to go to extinction, we can see that the  $(0,0)$  fixed point must be the only FP in the system. This will occur if  $K = \text{tangent}$  to the curve at  $(0,0)$ . If  $K < \text{tangent}$ , then there will be two fixed points.

So we want tangent of  $y = \frac{x^2 + bx}{x^2 + x + c}$  at  $(0,0)$ .

$$\frac{dy}{dx} = \frac{(x^2 + bx)(2x + 1)}{(x^2 + x + c)^2} + \frac{2x + b}{x^2 + x + c}$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{b}{c}$$

Hence, the limiting value of  $K = b/c - 1$ . Any value of  $K$  above  $b/c$  will drive the population to extinction.



This shift =  $b/c - 1$ , needs to be subtracted to get a line with only one fixed point.