\[ \frac{dx}{dt} = \sin(x) \]

- Viewed as a flow on the line
- Viewed as a Flow on the circle

0 and \(2\pi\) are not the same point

\[ \frac{d\theta}{dt} = f(\theta) \]

- so a 1D system oscillates only when if can be viewed as flow on a circle
- only works if \(f(\theta)\) is a \(2\pi\) periodic function
  \[ f(\theta) = f(\theta + 2\pi) \]
- This ensures that each point on the circle has a unique velocity \(\frac{d\theta}{dt}\)
- so it doesn’t work for \(\frac{d\theta}{dt} = \theta\)
Uniform Oscillators

- Uniform Oscillator
  - velocity is doesn’t vary with \( \theta \)
  
  \[ \dot{\theta} = \omega \]

- \( \omega \) is the angular velocity
- \( T = \frac{2\pi}{\omega} \)
- \( T \) is the period

Nonuniform Oscillators

- Nonuniform Oscillator
  - velocity varies with \( \theta \)
  - example:
    
    \[ \dot{\theta} = \omega - a \sin(\theta) \]

  behavior depends on the value of \( a \) relative to \( \omega \)

what happens at \( a = \omega \)?
Bifurcation at $a = \omega$

<table>
<thead>
<tr>
<th>$a &lt; \omega$</th>
<th>$a = \omega$</th>
<th>$a &gt; \omega$</th>
</tr>
</thead>
</table>

- $a < \omega$
- $a = \omega$
- $a > \omega$

Saddle node ... and its ghost for $a < \omega$ but close to $a = \omega$

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### Basic properties of an excitable system

1. The system contains a unique and global resting state that it returns to when not being stimulated.

2. The system contains a threshold value such that when an input stimulates the system above that threshold value the phase point travels a long excursion through the phase space before returning to the rest state.

**Neurons are an example of Excitable Cells**
Oscillations / Rhythms Occur in Nature

- Circadian Rhythms (24 hours)
  - sleep wake cycles
- Biochemical Oscillations (1 – 20 min)
  - metabolites oscillate
- Neuronal Oscillations (ms – s)
- Cardiac Rhythms (1 s)
- Hormonal Oscillations (10 min - 24 hour)
- Communication in Animals
  - firefly