

## Biol 398/Math 388 Week 4 Assignment:

### Two nutrients and population growth in a Chemostat

We consider in this assignment the 2-nutrient “and” model, which has a population growth rate that depends on the product of the two nutrient concentrations. The model, as mentioned in class, is given by three differential equations for the biomass concentration ( $x$ ), nutrient 1 concentration ( $y$ ), and nutrient 2 concentration ( $z$ ):

$$\begin{aligned}\frac{dx}{dt} &= -qx + V \frac{y}{K+y} \frac{z}{M+z} x \\ \frac{dy}{dt} &= qu - qy - \frac{1}{\varepsilon} V \frac{y}{K+y} \frac{z}{M+z} x \\ \frac{dz}{dt} &= qv - qz - \frac{1}{\delta} V \frac{y}{K+y} \frac{z}{M+z} x\end{aligned}$$

Your assignment is to find the equilibria of this system. One equilibrium is ( $x=0, y=u, z=v$ ), which is not very interesting as all the biomass is gone. There is a second one (if the parameters obey some constraints you’ll have to work out). Here’s what we’ll do.

- (1) The  $x$ -equation will give you a relationship that  $y$  and  $z$  must satisfy. You should be able to solve for  $z$  as a function of  $y$  (and of the parameters, too). Show your work in finding this function, and sketch the graph of this function.
- (2) The  $y$ - and  $z$ -equations will give you exactly the relationship we discovered in class. Work this out for yourself and provide  $z$  as a function of  $y$ .
- (3) Use the results of (1) and (2) to identify the values of  $y$  and  $z$  at equilibrium. These will be somewhat complicated algebraic expressions in terms of the parameters.
- (4) Using (2) and (3), find the resulting equilibrium value for the biomass  $x$ .
- (5) Sketch a graph (comparable to Figure 1 of the terSchure paper) of the nutrients’ and biomass’ concentrations as a function of the feed rate  $u$  of the first nutrient.

You may do all this with pencil-and-paper and scan your work to upload to your individual journal, or you may type it up in Word or latex or some other typesetting package.