

Part 2: A Little Physical Interpretation

We use the Dimension-less version of the system

$$\begin{cases} (1) \Leftrightarrow \frac{dU}{ds} = \frac{U}{1+U} - \frac{BUV}{B_0+U} \\ (2) \Leftrightarrow \frac{dV}{ds} = \frac{CUV}{1+UV} - DV \end{cases}$$

1) Number of Steady Points

In general the addition of new terms in a Dynamic System results in the creation of new steady points. With previous system we had between 1 and 3 steady points. As we shall see we now have between 3 and 5 points.

2) Behaviour at Infinity

The Dynamic System now has one dissipative term $-DV$ (we assume D is strictly positive) that helps to control the flow along the direction V .

Better still, their control on the flow is strong enough to prevent the trajectories from escaping to infinity the same way as they were liable to with the previous system ($E=0$).

Consequently we expect the trajectories to remain bounded (as we shall see a simple Lyapunov function exists to ensure the result).

Proof

With the system $\begin{cases} \frac{dU}{ds} = \frac{U}{1+U} - \frac{BUV}{B_0+U} \\ \frac{dV}{ds} = \frac{CUV}{1+UV} - DV \end{cases}$ some combination of parameters would allow

some trajectories to escape to infinity as follows $\begin{cases} \frac{dU}{ds} = Cte > 0 \\ V \rightarrow C/D \end{cases}$

Now with $E>0$ the coordinate U is ruled by $\frac{dU}{ds} = Cte - EU$ so cannot escape to infinity.

3) Influence of D

The larger the D , the better the flow will be 'controlled' along the V axis. For large values of D all steady points will be stable – and we will not be able to get any oscillations.

As we shall see all but one of the steady points will disappear for large values of D and E . But before disappearing the steady point will go through a phase when they are stable.