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7.7 Dividing Rational Expressions

To divide rational expressions, follow the rules for dividing fractions. Multiply by the reciprocal. In other words, flip the second rational expression and multiply. Factors can be canceled after the problem is set up as multiplication by the reciprocal.

Example 1. Divide: $\frac{3y^5}{16x^7} \div \frac{9y^2}{4x}$

$$\frac{3y^5}{16x^7} \cdot \frac{4x}{9y^2}$$

Division changed to multiplication by the reciprocal.

$$\frac{\cancel{3}y^{\cancel{5}^3}}{\cancel{4}16x^{\cancel{7}^6}} \cdot \frac{\cancel{4}x}{\cancel{9}y^{\cancel{2}^1}}$$

Factors canceled.

$$\frac{y^3}{12x^6}$$

Multiplication completed.

Example 2. Divide: $\frac{6x^2 + 3x}{2y^2 + 6y} \div \frac{2x^3 + x^2}{y + 3}$

$$\frac{6x^2 + 3x}{2y^2 + 6y} \cdot \frac{y + 3}{2x^3 + x^2}$$

Division changed to multiplication by the reciprocal.

$$\frac{\cancel{3}x(\cancel{2}x+1)}{2y(\cancel{y}+3)} \cdot \frac{\cancel{y}+3}{x^2(\cancel{2}x+1)}$$

Expressions factored and factors canceled.

$$\frac{3}{2xy}$$

Multiplication completed.

Example 3. Divide: $\frac{5x + 20}{6} \div (x^2 + x - 12)$

$$\frac{5x + 20}{6} \cdot \frac{1}{(x^2 + x - 12)}$$

Division changed to multiplication by the reciprocal.

Remember the reciprocal for a whole number has a 1 for the numerator.

$$\frac{5(\cancel{x}+4)}{6} \cdot \frac{1}{(x-3)\cancel{(x+4)}}$$

Expressions factored and factors canceled.

$$\frac{5}{6(x-3)}$$

Multiplication completed.

Divide the rational expressions. Write your answers in lowest terms. 7.7

1. $\frac{5m^2n^3}{2} \div \frac{3mn^4}{4}$

2. $\frac{4x^3y}{3} \div \frac{2x^2y^2}{5}$

3. $\frac{3x-6}{4y} \div \frac{x-2}{y^2+y}$

4. $\frac{x^2-9}{3x} \div \frac{x+3}{12x^2}$

5. $(x-1) \div (x^2-1)$

6. $\frac{3xy+3y}{8} \div \frac{6y(x+1)}{4}$

7. $\frac{x^2-4x-12}{x^2-5x+6} \div \frac{x^2-8x+12}{x^2-x-6}$

8. $\frac{x^2+x-12}{5} \div \frac{x^2+2x-8}{15}$

9. $\frac{x^2-8x+7}{x+1} \div (49-x^2)$

Review

Compute each probability as a fraction and as a percent. Round to the nearest whole percent. 7.6

- ☆ 10. Each number from 1 through 15 is written on a card and placed face down on the table. One card is turned over. How likely is it that the number is . . .

a. . . 8?

b. . . a multiple of 3?

c. . . less than 20?

d. . . prime?

Graph each equation, using the given values for x . 4.9

11. $5x - 3y = -28$ $x = -8, -5, -2$

12. $6x - y = 15$ $x = 1, 2, 3$

Write equations for lines that have these slopes and points. 7.4

13. $m = -4$ $(2, -7)$

14. $m = 3$ $(3, 10)$

15. $m = \frac{4}{5}$ $(9, -2)$

Simplify the expressions. 6.11

16. $\frac{15xy - 5y}{10x - 30x^2}$

17. $\frac{x^2 - 36}{x^2 - 3x - 18}$

18. $\frac{x^2y - 11xy + 30y}{4xy - 20y}$

Determine the excluded values for the expression. 6.9

19. $\frac{10x+3}{8x^2-12x}$

20. $\frac{8}{(3x+1)^2}$

21. $\frac{1}{x^2-2x}$

Solve the systems of equations by the addition/elimination method. 6.12

22. $x + 4y = 24$

$3x - 4y = -8$

23. $7x - 3y = -51$

$x + 3y = 27$

Solve these inequalities. 5.7

24. $-\frac{x}{4} - 13 \geq 12$

25. $5x > 5 - 3x$

26. $9x + 16 \geq 22$

Draw solution graphs for these compound inequalities. 4.11

27. $x > 0$ and $x < 2.7$

28. $x \leq 0.6$ or $x > 1$

Divide the rational expressions. Write your answers in lowest terms. 7.7

29. $\frac{x^2 + 3x - 4}{9x^2 - 15x} \div \frac{3x^2 - 3x}{6x - 10}$

30. $\frac{x^2 + 6x + 9}{2x(x + 1)} \div \frac{x + 3}{x^2 - 1}$

31. $\frac{6x^2 - 30x}{5} \div 3x^2 - 21x$

7.8 Factoring Trinomials in the Form $ax^2 + bx + c$

In trinomials factored so far, the coefficient of the first term has been 1. This made it simple to find the first term of the binomial factors of the trinomial. However, the first term of trinomials is not always 1, as in $8x^2 + 2x - 3$.

When the first term of the trinomial has a coefficient greater than 1, the factors of its coefficient are involved in the factoring process. In the example, the first terms of the binomial factors could be

$$(8x \quad)(1x \quad) \quad (4x \quad)(2x \quad) \quad (-8x \quad)(-1x \quad) \quad (-4x \quad)(-2x \quad)$$

This creates four possible options for the binomial factors of $8x^2 + 2x - 3$. To find the correct option use the trial and error method. Substitute the factors of the trinomial's last term (-3, 1 and 3, -1) in the binomials and check using FOIL. If a pair of factors doesn't check, try reversing their order.

	Possible Combinations	Outer/Inner Term Products	Sum of Outer/Inner Products
Using factors 8, 1 with -3, 1 3, -1	$(8x - 3)(1x + 1)$	$8x, -3x$	$5x$
	$(8x + 1)(1x - 3)$	$-24x, 1x$	$-23x$
	$(8x - 1)(1x + 3)$	$24x, -1x$	$23x$
	$(8x + 3)(1x - 1)$	$-8x, 3x$	$-5x$
Using factors -8, -1 with -3, 1 3, -1	$(-8x - 3)(-1x + 1)$	$-8x, -3x$	$-5x$
	$(-8x + 1)(-1x - 3)$	$24x, -1x$	$23x$
	$(-8x - 1)(-1x + 3)$	$-24x, +1x$	$-23x$
	$(-8x + 3)(-1x - 1)$	$8x, -3x$	$5x$
Using factors 4, 2 with -3, 1 3, -1	$(4x - 3)(2x + 1)$	$4x, -6x$	$-2x$
	$(4x + 1)(2x - 3)$	$-12x, 2x$	$-10x$
	$(4x - 1)(2x + 3)$	$12x, -2x$	$10x$
	$(4x + 3)(2x - 1)$	$-4x, 6x$	$2x$
Using factors -4, -2 with -3, 1 3, -1	$(-4x - 3)(-2x + 1)$	$-4x, 6x$	$2x$
	$(-4x + 1)(-2x - 3)$	$12x, -2x$	$10x$
	$(-4x - 1)(-2x + 3)$	$-12x, 2x$	$-10x$
	$(-4x + 3)(-2x - 1)$	$4x, -6x$	$-2x$

Only the two factor combinations highlighted above produce the correct coefficient of the middle term +2.

The main purpose of illustrating this was to show how many possible factor combinations might have to be considered before finding the correct pairs and the right order to factor the polynomial. This can be very time-consuming.

But there is an easier way. Using the same trinomial, here is how it works:

$$8x^2 + 2x - 3$$

1. Multiply the coefficients of the first and third terms of the trinomial.

$$(8)(-3) = -24$$

2. List all the possible pairs of factors for -24 . Remember to list the sets with opposite signs for each pair.

1, -24	-1, 24
2, -12	-2, 12
3, -8	-3, 8
4, -6	-4, 6

3. Choose the pair of factors whose sum is 2 (the trinomial's middle term coefficient).

4. Write the trinomial again, but express the middle term ($2x$) as a sum of two terms whose coefficients are the factors you chose in step three. (The order will not affect the final answer.)

$$8x^2 - 4x + 6x - 3$$

5. Group the first two terms and the last two terms together.

$$(8x^2 - 4x) + (6x - 3)$$

6. Factor the GCF out of each binomial.

$$4x(2x - 1) + 3(2x - 1)$$

7. Factor the common binomial ($2x - 1$) out of the expression.

$$(2x - 1)(4x + 3)$$

8. Check the factorization by multiplying the two binomials.

$$(2x - 1)(4x + 3) = 8x^2 + 2x - 3$$

As skill is developed in factoring, listing ALL factors in Step 2 may not always be necessary. With practice, many students soon find the right pair of factors mentally.

Example 1. Factor the trinomial $6x^2 - 7x - 3$.

$$6 \cdot (-3) = -18$$

First-term and third-term coefficients multiplied.

1, -18	-1, 18
2, -9	-2, 9
3, -6	-3, 6

Factor pairs for -18 listed.

Factors with a sum of -7 identified (trinomial's middle term coefficient).

$$6x^2 + 2x - 9x - 3$$

Trinomial written, re-expressing the middle term ($-7x$) as a sum of two terms, using the factor pair for the coefficients ($2x - 9x$).

$$(6x^2 + 2x) + (-9x - 3)$$

Polynomial grouped into two binomials.

$$2x(3x + 1) - 3(3x + 1)$$

GCFs factored out of the binomials.

$$(3x + 1)(2x - 3)$$

Common binomial factored out of the expression.

Example 2. Factor the trinomial $9x^2 - 15x + 4$.

$$9 \cdot 4 = 36$$

First-term and third-term coefficients multiplied.

1, 36	-1, -36
2, 18	-2, -18
3, 12	-3, -12
4, 9	-4, -9

Factor pairs for 36 listed.

Factors with a sum of -15 identified (trinomial's middle term coefficient).

$$9x^2 - 3x - 12x + 4$$

Trinomial written, re-expressing the middle term ($-15x$) as a sum of two terms, using the factor pair for the coefficients ($-3x - 12x$).

$$(9x^2 - 3x) + (-12x + 4)$$

Polynomial grouped into two binomials.

$$3x(3x - 1) + 4(-3x + 1)$$

GCFs factored out of the binomials. Notice the opposite signs in the two binomials.

$$3x(3x - 1) + (-4)(3x - 1)$$

(-1) factored out of second binomial to reverse its signs.

$$3x(3x - 1) - 4(3x - 1)$$

Simplified.

$$(3x - 1)(3x - 4)$$

The binomial $(3x - 1)$ factored out of the expression.

Factor. Show your work. 7.8

1. $2x^2 + 3x - 9$

2. $3x^2 + 14x + 8$

3. $10x^2 - x - 3$

4. $7x^2 + 10x + 3$

5. $6x^2 + x - 1$

6. $12x^2 - 7x - 10$

Review

Divide the rational expressions. Write your answers in lowest terms. 7.7

7. $\frac{4r^3}{3s} \div \frac{5r}{12s^2}$

8. $\frac{2mn - 8m}{5n^4} \div \frac{4m^3}{n^5}$

9. $\frac{x^2 + x - 6}{3x^4} \div \frac{x + 3}{6x}$

Simplify the radicals. Show the steps. 6.8

10. $\sqrt{180x^7y^5}$

11. $\sqrt{54w^{21}}$

12. $\sqrt[3]{128r^{11}s^6}$

Simplify the binomials. 4.14

13. $(2r - 3s)^2$

14. $(5x + 5)^2$

15. $(\frac{1}{2}x - 1)^2$

Graph each of the equations by the x- and y-intercepts. 6.2

16. $6x + 5y = -15$

17. $3x - 14y = -21$

Graph the equations, using slope-intercept form. 5.12

18. $y = \frac{1}{6}x - 6$

19. $2x + 5y = 35$

Factor by grouping. 7.3

20. $2mn + 4m - 3n - 6$

21. $15xy - 3y + 10x - 2$

22. $4x^2 + x^2y^2 - 3y^2 - 12$

Multiply the sum and difference binomials mentally. 5.8

23. $(x - \sqrt{y})(x + \sqrt{y})$

24. $(10x - 1)(10x + 1)$

25. $(7 - \sqrt{x})(7 + \sqrt{x})$

Factor the trinomials. If any is not a perfect square, write *not a perfect square*. 6.3

26. $x^2 + 30x + 225$

27. $x^2 - 6x + 9$

28. $x^2 + x + \frac{1}{4}$

Solve the systems of equations by using substitution. 6.4

29. $x - 5y = -41$

$x + 2y = 15$

30. $3x + y = 15$

$3x - 2y = 24$

Solve the systems of equations by using the multiplication/addition method. 7.2

31. $2x - 7y = 32$

$3x + 4y = 19$

32. $6x - 2y = -19$

$8x + 6y = -21$

Multiply the rational expressions. Write your answers in lowest terms. 7.1

33. $\frac{4xy + 8x}{3x^2y} \cdot \frac{2y}{7x}$

34. $\frac{x + 3}{x - 1} \cdot \frac{x^2 - 5x + 4}{x^2 + x - 6}$

35. $\frac{3x^2 - 9x}{2y^2} \cdot \frac{6y^3}{x^2 - 3x}$

Compute each probability as a fraction and as a percent. Round to the nearest whole percent. 7.6

36. Each letter in the word *honorificabilitudinitatibus* is written on a piece of paper and placed in a basket. One paper is drawn from the basket. How likely is the letter to be . . .

a. . . a vowel?

b. . . a consonant?

c. . . an i?

d. . . an e?

Solve for the requested information. 6.14

☆ 37. At 9:00 a.m. two jets leave from Portland, Oregon and New York, New York, each flying to the opposite city. In calm air the jets average 550 mph, but there is an average westerly (from the west) wind at 40 mph. The cities are about 2,750 miles apart. When the jets cross paths how far will each have traveled? (Hint: For the jet flying east, add the wind speed; flying west, subtract the wind speed.)

Factor. Show your work. 7.8

38. $5x^2 + 21x - 20$

39. $63x^2 - 11x - 2$

7.9 Adding and Subtracting Radicals

Radicals with the same radicand and the same index are *like* radicals and can be combined the same as *like* variables, by adding or subtracting the coefficients.

$$2\sqrt{3} + \sqrt{3} = (2 + 1)\sqrt{3} = 3\sqrt{3}$$

$$3x\sqrt{2} - x\sqrt{2} = (3x - x)\sqrt{2} = 2x\sqrt{2}$$

$$2\sqrt[3]{5} + 3\sqrt[3]{5} = (2 + 3)\sqrt[3]{5} = 5\sqrt[3]{5}$$

$$7\sqrt{2x} + 5\sqrt{2x^2} = ?$$

These two radicals cannot be combined because their radicands are different.

$$4\sqrt[3]{5} + 3\sqrt{5} = ?$$

These two radicals cannot be combined because their indices are different.

Sometimes unlike radicals can be simplified and then combined. Check to make sure unlike radicals are in their simplified form.

Example 1. add $3\sqrt{2} + 7\sqrt{50}$.

$$3\sqrt{2} + 7\sqrt{50}$$

$\sqrt{2}$ and $\sqrt{50}$ cannot be combined in their present form.

$$3\sqrt{2} + 7(5\sqrt{2})$$

$\sqrt{50}$ simplified to $5\sqrt{2}$.

$$3\sqrt{2} + 35\sqrt{2}$$

Multiplication completed ($7 \cdot 5$).

$$38\sqrt{2}$$

Like radicals combined.

Note: If you cannot combine the coefficients of like radicals, group the coefficients in parentheses.

$$3\sqrt{2} + x\sqrt{2} = (3 + x)\sqrt{2}$$

Sometimes when there are three or more terms, two terms will combine and the rest will not.

$$2\sqrt{3} + 2\sqrt[3]{3} - \sqrt{3} = 2\sqrt[3]{3} + \sqrt{3}$$

Combine like radicals. If they cannot be combined, write *cannot be combined*. 7.9

1. $3\sqrt{7} + 4\sqrt{7}$

2. $3\sqrt{2} - \sqrt{2}$

3. $2\sqrt{3} + 3\sqrt{7}$

4. $2x\sqrt{3} + x\sqrt{3}$

5. $6\sqrt{x} + 8\sqrt{x^2}$

6. $5\sqrt[3]{4} - 2\sqrt[3]{4}$

7. $10\sqrt{3} + \sqrt[3]{5} - \sqrt{3}$

8. $4\sqrt{2} - \sqrt{2} + 3\sqrt{2}$

9. $3\sqrt{x} + 3\sqrt{x^2} - \sqrt{x}$

Review

Factor. Show your work. 7.8

10. $8x^2 + 10x + 3$

11. $56x^2 - x - 1$

Determine the excluded values for the expression. 6.9

12. $\frac{5}{2x-1}$

13. $\frac{7x}{3x^2-3x}$

14. $\frac{x+6}{4x^2+12x+9}$

15. $\frac{4}{1-x^2}$

Simplify the expressions. 6.11

16. $\frac{16r^2s^2 + 48rs^2}{rs + 3s}$

17. $\frac{28jk - 84k}{-14j + 42}$

18. $\frac{x^2 - 4}{x + 2}$

Factor by grouping. 7.3

19. $4x - y - yz + 4xz$

20. $6rs + 9r - 14s - 21$

21. $4mn + 1 + 2n + 2m$

Solve the systems of equations by the addition/elimination method. 6.12

22. $3x - 5y = 40$

$3x + 10y = -35$

23. $12x - 2y = -66$

$8x + 2y = -34$

Solve the systems of equations by using the multiplication/addition method. 7.2

24. $5x + 6y = 11$

$2x - 3y = -1$

25. $2x - 5y = 28$

$3x + 2y = -15$

Write equations for lines that have these slopes and points. 7.4

26. $m = 10$ $(-3, -3)$

27. $m = \frac{1}{5}$ $(-5, 4)$

28. $m = -\frac{3}{7}$ $(-2, 6)$

Multiply the rational expressions. Write your answers in lowest terms. 7.1

29. $\frac{6-3x}{4x^2} \cdot \frac{2}{(x-2)}$

30. $\frac{x^2-4}{2-x} \cdot \frac{3}{x+2}$

31. $\frac{x^2}{9} \cdot \frac{4}{y} \cdot \frac{3}{y^2} \cdot \frac{y^4}{x^4}$

Divide the rational expressions. Write your answers in lowest terms. 7.7

32. $\frac{x^2-25}{x-2} \div \frac{x^2+10x+25}{x^2-4}$

33. $\frac{3y^4}{6x^5} \div \frac{12y}{3x^7}$

34. $\frac{8m^2n-4m^2}{3n^3} \div 2mn$

Combine like radicals. If they cannot be combined, write *cannot be combined*. 7.9

35. $3\sqrt{2x} - 2\sqrt{2x} + \sqrt{x}$

36. $2\sqrt[3]{x^2} - 3\sqrt[3]{x} + \sqrt[3]{x}$

37. $x\sqrt{2x} - 5x\sqrt{2x}$