

SCIENCE 1202

DYNAMICS

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2013 Printing

The material in this book is the product of the Lord's blessing and many individuals working together on the teams, both at Alpha Omega Publications and Christian Light Publications.

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Christian Light Publications, Inc.

Harrisonburg, Virginia 22802

Tel. 1-540-434-0768

Printed in USA

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III. UNIFORM CIRCULAR MOTION

We briefly mentioned that planets orbit the sun because of the sun's gravitational field. This type of motion and all motion in a circle at a constant speed is called uniform circular motion. (We shall see later that the orbits only approximate circles.) Motion in a circle at a

constant speed is accelerated motion (see Section Four of Science **LIGHTUNIT 1201**). This circular acceleration is **centripetal acceleration** and the force that produces this acceleration is **centripetal force**.

SECTION OBJECTIVES

Read these objectives. When you have completed this section, you should be able:

9. To explain the cause of centripetal acceleration.
10. To calculate problems involving centripetal acceleration and centripetal force.

VOCABULARY

Study these words to enhance your learning success in this section.



centripetal acceleration

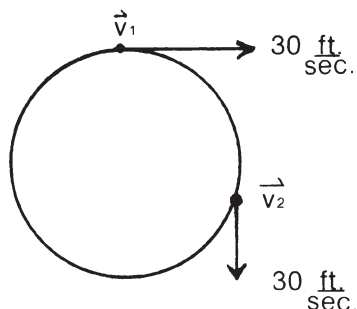
centripetal force

CENTRIPETAL ACCELERATION

Remember that acceleration is a change in velocity with respect to a change in time. An object in straight-line motion that speeds up has a positive acceleration, and when it slows down it has a negative acceleration. The third case, in

which $\frac{\Delta \vec{v}}{\Delta t} = \vec{a}$, is constant velocity *magnitude* but changing velocity *direction*. Here again vector diagrams become important, and use of a straightedge is a must.

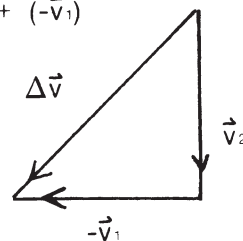
Consider an object moving in a circle at $30 \frac{\text{ft.}}{\text{sec.}}$



To find the direction of the acceleration that results from the change in velocity direction, find the direction of the change in velocity ($\Delta \vec{v}$).

$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\text{or } \Delta \vec{v} = \vec{v}_2 + (-\vec{v}_1)$$



Scale: 1 inch = $30 \frac{\text{ft.}}{\text{sec.}}$

Notice that $\Delta \vec{v}$ points into the circle, since the triangle is a right triangle with the legs 1 inch long, and the hypotenuse approximately 1.4 inches long. If the two points taken are closer together on the circle, the $\Delta \vec{v}$ and $\vec{a}$ vectors point at the center of the circle.

If 1 inch = 30 ft./sec., then 1.4 inches = ?

$$\frac{30 \frac{\text{ft.}}{\text{sec.}}}{1 \text{ in.}} = \frac{\Delta \vec{v}}{1.4 \text{ in.}}$$

$$\Delta \vec{v} = 42 \frac{\text{ft.}}{\text{sec.}}$$

If the object took ten seconds to travel from one point to the other, then

$$\vec{a} = \frac{\Delta v}{\Delta t}$$

$$\vec{a} = \frac{42 \frac{\text{ft.}}{\text{sec.}}}{10 \text{ sec.}}$$

$$\vec{a} = 4.2 \frac{\text{ft.}}{\text{sec}^2}$$

This acceleration that points towards the center is called **centripetal acceleration**. The centripetal acceleration is the *center-pointing* acceleration that produces uniform circular motion.

Since the object goes around with constant speed,

$$v = \frac{2\pi R}{T}$$

where R is the radius of the circle and T is the time required for one revolution, called the *period* of the motion. The velocity changes

direction always tangent to the circle, and it takes a full cycle to come back to its original direction so that

$$a = \frac{\Delta v}{\Delta t} = \frac{2\pi v}{T}$$

Combining these two equations,

$$a = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

In addition to the graphical method, a mathematical method is available to calculate the magnitude of the acceleration; it is given by the three equations. The direction is always toward the center. (Note: hereafter, the symbols v, d, and a are assumed to be vectors, although the → may be missing.)

Example

A rock tied to a string is swung in a circle at a constant speed of $25 \frac{\text{ft.}}{\text{sec.}}$. Calculate the magnitude of the centripetal acceleration if the rock takes five seconds to make ten swings, and calculate the radius of this circle.

$$T = \frac{5 \text{ sec.}}{10 \text{ swings}} = 0.5 \text{ sec.}$$

$$a = \frac{2\pi v}{T}$$

$$a = \frac{2(3.14)(25 \frac{\text{ft.}}{\text{sec.}})}{0.5 \text{ sec.}}$$

$$a = 314 \frac{\text{ft.}}{\text{sec.}^2}$$

$$\text{Since } a = \frac{v^2}{R}, R = \frac{v^2}{a}$$

$$R = \frac{(25 \frac{\text{ft.}}{\text{sec.}})^2}{314 \frac{\text{ft.}}{\text{sec.}^2}}$$

$$R = 2 \text{ ft.}$$



Complete these activities.

- 3.1 If an object moves in uniform circular motion in a circle of radius $R = 1$ meter, and the object takes four seconds to complete ten revolutions, calculate the magnitude of the velocity around the circle. _____
- 3.2 Calculate the centripetal acceleration for the preceding example. _____
- 3.3 An object moves in uniform circular motion at $25 \frac{\text{m}}{\text{sec}}$, and takes one second to go a quarter circle. What is the radius of the circle? _____
- 3.4 Calculate the centripetal acceleration for the preceding problem. _____
- 3.5 In 3.1 through 3.4, which way does the acceleration vector point? _____

-

CENTRIPETAL FORCE

According to Newton's Second Law, $F = ma$, if the acceleration in uniform circular motion points toward the center, the force that produces the acceleration also points toward the center. This center-pointing force is called **centripetal force**. If an object on a string is swung in a circle, you must continually pull the string to keep the object in uniform circular motion.

When a car rounds a curve, a force is exerted

by the road on the tire to pull it into the curve. If the tires hit a slick portion in the road, the car skids out of the curved path because the force on the tires was not great enough.

$$\vec{F} = m\vec{a}$$

$$F = m \left(\frac{v^2}{R} \right) = m \left(\frac{2\pi v}{T} \right)^2 = m \left(\frac{4\pi^2 R}{T^2} \right)$$

Example

From the problem stated in the previous Example, calculate the centripetal force if the object has a weight of 4 lbs.

First, find the mass of the object:

$$m = \frac{W}{g} = \frac{4 \text{ lbs}}{32 \frac{\text{ft}}{\text{sec}^2}} = \frac{1}{8} \text{ slug}$$

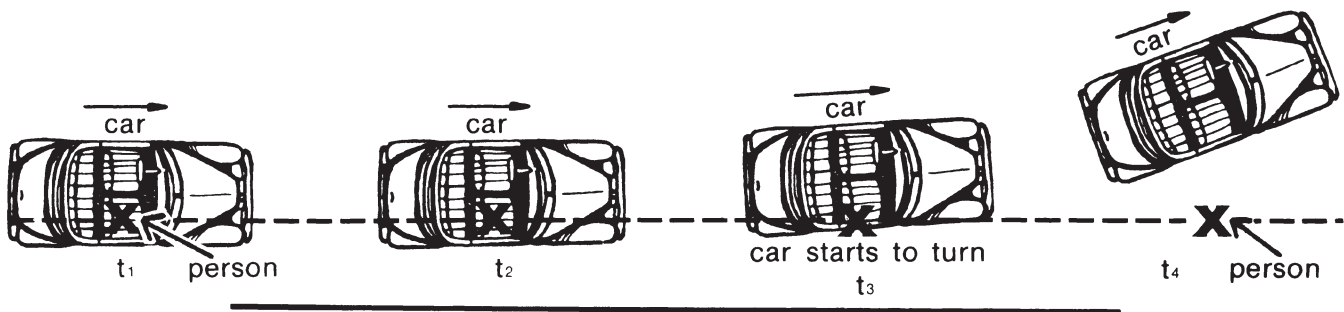
From the previous Example, $a = 314 \frac{\text{ft}}{\text{sec}^2}$

$$F = ma$$

$$F = \left(\frac{1}{8} \text{ slug}\right) \left(314 \frac{\text{ft}}{\text{sec}^2}\right)$$

$F = 39.3 \text{ lbs.}$, pointing toward the center.

A similar situation is a car turning a corner at high speed. You *feel* as if you are being pushed out and away from the center of the curve. You would not actually be thrown away from the center, nor would the rock actually fly away from the center of its curve along the radius. Newton's First Law states that an object in motion stays in *straight-line* motion unless an external force acts on it. Therefore, as the car moves forward you move forward in a straight-line motion ("x" is your position). If the driver turns the wheel, the road pulls on the tires, exerting a centripetal force and your reaction is to slide across the seat toward the outside of the curve. As the car turned it did not throw you *radially* away from the center. You continued in straight line motion *tangential* to the curve.



An explanation is needed here. The centripetal force acted on the car alone because you were not anchored to it with a seat belt. Since no force acted on you, you continued in a straight line. However, rather than seeing yourself moving in a straight line as the car moved away from you, which is what it did from one *frame of reference* (the bird's-eye view), you looked at it from your reference frame and said that you were thrown radially outward.

While we are discussing centripetal forces, let us consider the earth revolving around the sun. The centripetal force in this case is the gravitational force of attraction between the sun and earth.

$$F_{\text{centripetal}} = F_{\text{gravitational}}$$

$$\frac{m_e v^2}{R} = \frac{G m_e m_s}{R^2}$$

$$v^2 = \frac{G m_s}{R}$$

We could have used any planet to discover that the speed (squared) with which any planet orbits the sun is proportional to the sun (not the planet) and is inversely proportional to the distance of separation. Planets four times the earth's distance from the sun orbit at half the

speed (remember that v is squared, or $v = \sqrt{\frac{G m_s}{R}}$).

The equation holds true for all satellites. Our moon revolves with a velocity proportional to the earth's mass, as do all earth's artificial satellites.

$$v^2 = \frac{G m_e}{R}$$

Example

If a satellite's orbit at one earth radius above the surface of the earth ($2R$ from the center of

the earth) has a speed of $5.6 \cdot 10^3 \frac{\text{m}}{\text{sec.}}$, what speed has a satellite at $4R$ from the earth's center.

$$v_1 = \sqrt{\frac{Gm_e}{2R}} = 5.6 \cdot 10^3 \frac{\text{m}}{\text{sec.}}$$

$$\begin{aligned} v_2 &= \sqrt{\frac{Gm_e}{4R}} = \frac{\sqrt{1}}{\sqrt{2}} \frac{\sqrt{Gm_e}}{\sqrt{2R}} = \frac{\sqrt{1}}{\sqrt{2}} \sqrt{\frac{Gm_e}{2R}} \\ &= \frac{1}{\sqrt{2}} (5.6 \cdot 10^3 \frac{\text{m}}{\text{sec.}}) \\ &= (.707) (5.6 \cdot 10^3 \frac{\text{m}}{\text{sec.}}) \\ &= 4.0 \cdot 10^3 \frac{\text{m}}{\text{sec.}} \end{aligned}$$

This result is as expected: that is, the farther away a satellite is, the slower it orbits. This problem could have been solved tediously by letting $G = 6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$, $m_e = 5.98 \cdot 10^{24} \text{ kg}$, and $R_e = \text{radius of earth} = 6.37 \cdot 10^6 \text{ m}$



Complete this activity.

- 3.6 Calculate the speed with which the moon orbits the earth given the center to center distance from earth to moon as $R = 3.8 \cdot 10^8 \text{ m}$. _____
- 3.7 In Activity 3.1, calculate the magnitude of the centripetal force if the object has a mass of 2 kg. _____
- 3.8 An object of mass 0.5 kg is swung in uniform circular motion. The radius is 2 meters, and the force exerted is 4 N. Calculate the magnitude of the velocity.