

MATHEMATICS 1001

A MATHEMATICAL SYSTEM

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A MATHEMATICAL SYSTEM

A mathematical system generally is a logical study of shape, arrangement, and quantity. Algebra, geometry, trigonometry, and calculus are examples of mathematical systems. Geometry is a logical mathematical study of points, lines, planes, and solids, their properties, measurement,

and relationship to each other in space. Historically, the word “geometry” comes from two Greek words—*geo*, which means: “land” or “the earth” and *metria*, meaning “measurement.”

There are four items involved in any mathematical system, including geometry:

1. Basic undefined terms.
2. All other terms carefully and clearly defined.
3. Some basic statements about these terms are assumed true without using logical proof. These are called postulates.
4. All other statements are established by logical reasoning and formal proof. These are called theorems.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LightUnit.

When you have finished this LightUnit, you should be able:

1. To identify, draw, and label models of undefined terms.
2. To define some basic geometric terms.
3. To define *postulate* and *theorem*, and to recognize a model of each.
4. To describe the four items that are involved in a mathematical system

Survey the LightUnit. Ask yourself some questions about this study. Write your questions here.

SECTION OBJECTIVE

Read this objective. When you have completed this section, you should be able:

I. UNDEFINED TERMS

1. To identify, draw, and label models of undefined terms.

Euclid, a famous Greek mathematician, lived in Alexandria, Egypt, about 300 B.C. His most famous book, *The Elements*, sounds like a chemistry book, but is actually a work on algebra, geometry, and theory of whole numbers. Except for the Bible, Euclid's book has gone through more translations and editions than any other book in the world. Although it contains very little original material, its uniqueness comes from Euclid's organization of geometric statements. He began with simple terms and statements and derived or proved everything else logically.

In general, geometry develops one's skill in critical thinking and mathematical reasoning. Even in daily living, these skills may improve interpersonal communication and reduce misunderstandings.

The word “bear” can mean at least three different things:

1. To carry or hold up as a burden.
2. To give birth to a child.
3. A large four-footed beast with shaggy fur and short tail.

Usually we can easily identify the meaning of a word by its context in a sentence or a paragraph. On the other hand there are many other words in the dictionary with only one meaning. But the sense of that meaning depends on our

previous experience and knowledge.

To understand the need for some undefined terms, suppose we try to define “point.” We can say, “A point is a dot.” But exactly what is a dot? A dot may be defined as a tiny round speck. But what is a tiny round speck? You see it is impossible to define everything without going around in circles. Eventually we would end up back where we started. To avoid such circular reasoning, mathematicians have decided to accept “point” as an undefined term in geometry. This may seem like surrendering to failure, but to define any particular word requires a group of other familiar words. But what about these words? Do we need to clearly define them also? If we do, then we need still other words. This process of thinking must stop somewhere, or else we will come back to where we began. (This is what the dictionary really does.)

To avoid this endless process of circular reasoning, we choose a few simple basic terms on which we build other definitions and statements. In geometry there are three terms that we do not try to define. These terms are “point,” “line,” and “plane.” We have to rely on our experience to understand these terms. Starting with these terms as a foundation, we build our mathematical system of geometry.

POINTS

A point cannot be seen. It has no size, shape or color and it has no dimensions or physical properties. In a sense, it is imaginary. A point does, however, have position or location. When we talk about a point, we often represent it by a dot on a paper. This dot we name with a capital letter, such as A , B , C , etc.

The five locations to the right of this paragraph are particular points named A , B , C , D , and E . There are more than five points on this page. In fact, the number of points on this page is infinite!

$$\begin{array}{ccc} \cdot A & \cdot B & \cdot C \\ & \cdot D & \cdot E \end{array}$$

➔ **Complete these activities.**

- 1.1 We think of a point as having _____ .
- 1.2 A point is represented by a _____ .
- △1.3 Place names on these points.
- . . .
- . .
- 1.4 How many points are located on this page? _____ .
- 1.5 What physical characteristics does a point have? _____ .
- △1.6 Draw and label four points.

LINES

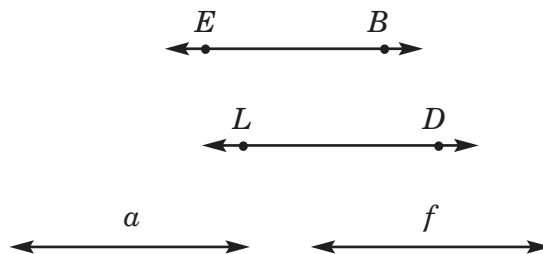
A line is an infinite set of points that are arranged one after another in a row. A line has no “holes” or gaps between adjacent points. The points contained in a straight line extend infinitely far in both directions. A line in geometry is assumed to be straight with no curves or bends. It has only one dimension — length. We represent a line on a paper like this:



The arrowheads on the ends tell us that the line extends indefinitely in the direction indicated by the arrow. Since a line is made up of infinitely many points, it has no thickness or other physical characteristics. A line may be identified by naming any two of its points and placing a double-headed arrow above the two capital letters.

To the right are two lines named $\overleftrightarrow{EB}$ and $\overleftrightarrow{LD}$.

The line $\overleftrightarrow{EB}$ is the same as $\overleftrightarrow{BE}$. Sometimes we wish to name a line by a lower-case letter; that is, only one letter. In the diagram are shown lines a and f . Thus we see that we can draw a line between any two points.



➔ **Supply the information required.**

- 1.7 We think of a line as a collection of _____ .
- 1.8 Unless otherwise indicated, a line in geometry is understood to be (straight, curved, either) _____ .
- 1.9 Name the following lines.



a. _____



b. _____



c. _____

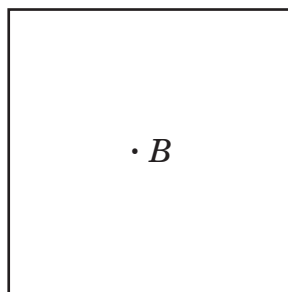
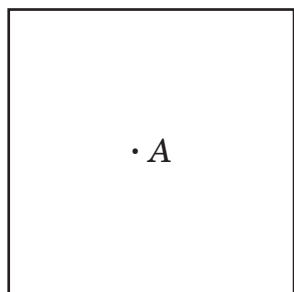
- △1.10 Draw two lines. Use arrowheads. Label the first with two capital letters and the second with one lowercase letter.

- 1.11 What is indicated by arrowheads on a line? _____

PLANES

A *plane* is an infinite collection of points arranged on a flat surface. This surface has no thickness, holes, pockets, or ridges and extends infinitely far in all directions. A plane has only two dimensions, length and width. The area of a plane is infinite just as the length of a line is infinite.

We represent a plane on a paper like this:



(Remember that the edges are not really there.) A plane may be named by using one or more of its points. Thus we say “plane A,” or “plane B,” etc. As with points and lines, planes do not have any physical properties. Since a point has no thickness and planes are made up of points, you could

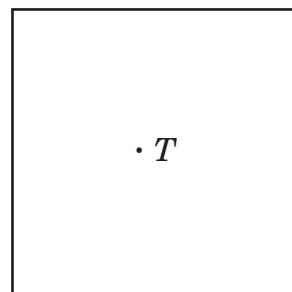
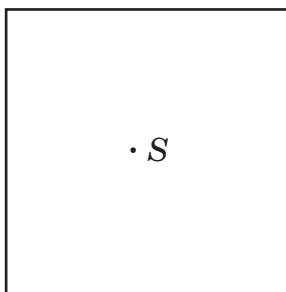
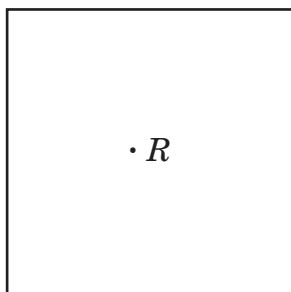
stack a million planes together and they would still have no thickness. (A million times zero gives zero.)

Any two points in a plane can name a line. So as the number of points is infinite, the number of lines in a plane is also infinite.

➔ **Write the following information.**

1.12 A plane can be thought of as a (flat, curved) a. _____ surface, made up of b. _____ .

1.13 Name each of the following planes.



a. _____ b. _____ c. _____

1.14 How long is a plane? _____

1.15 How thick is a plane? _____

1.16 List five examples of points, lines, and planes in your room.

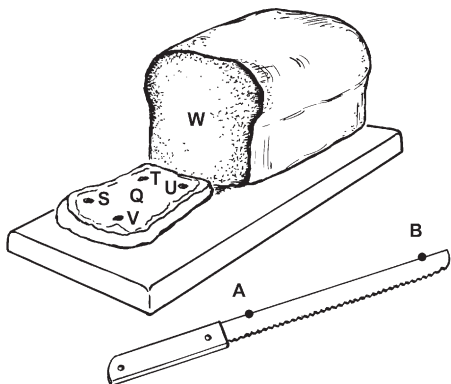
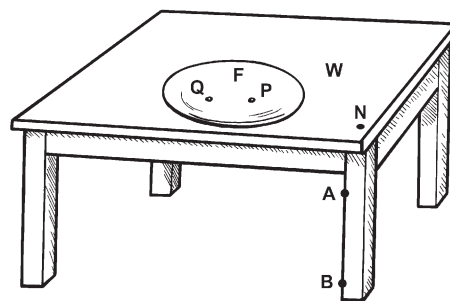
points _____

lines _____

planes _____

Some concrete illustrations may help to visualize relationships between abstract points, lines and planes.

In the adjacent diagram table leg (line AB) intersects table top (plane W) at a nail (point N .) Peas (points Q and P) are on the plate (plane F).

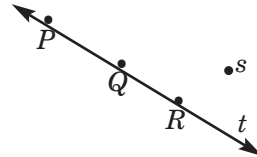
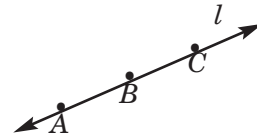


Cutting knife (line AB) cuts the loaf of bread to make a slice (plane W) and (plane Q). Plane Q is spread with butter. The points (S, T, U, V) are raisins on plane Q .

We may use many models to represent points, lines, and planes. The sharp end of a pin, tack, or pencil can be a model for a point. For a line we could use a pencil or a meterstick and a table top, the floor, or a windowpane as a plane.

Here are some ways points, lines, and planes are related. Note the terms used in talking about points, lines, and planes.

Points A and B are in line l . Line l contains points A , B , and C . Point C is a point of $\overleftrightarrow{AB}$. D is not a point of $\overleftrightarrow{AB}$.

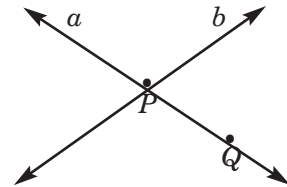


1.17 Using the same language talk about this diagram:

Points ____ and ____ are on line ____.

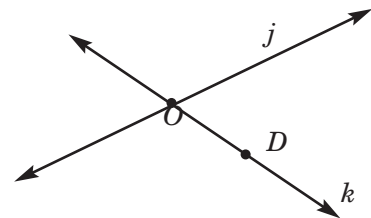
Line ____ contains points ____ , ____ , and ____ . ____ is a point on $\overleftrightarrow{PQ}$.

Lines a and b intersect each other at point P . Point Q is not a point of line b . Point P is a point of both lines a and b .

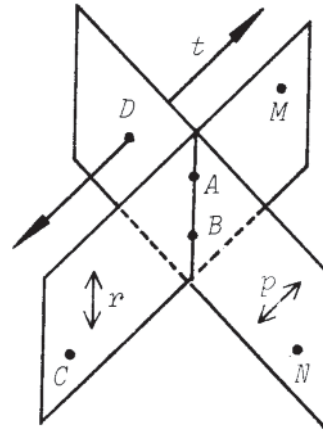


1.18 Using language similar to the above, talk about this diagram:

Lines ____ and ____ intersect each other at point ____ . Point ____ is not a point of line j . Point ____ belongs to both line ____ and line ____ .



Plane M and plane N intersect at $\overleftrightarrow{AB}$. (A dotted line indicates a part of a plane is “hidden” behind the front section of another plane.) The intersection of planes M and N consists of an infinite line. Point C is in plane M . Point C is not in plane N . Line t intersects plane N only at point D . Plane M contains all of line r . Line r lies completely in plane M . Every part of line r is contained in plane M . Points A and B and all of line $\overleftrightarrow{AB}$ are in both plane M and plane N .



1.19 Complete these exercises.

Write the set of points common to plane M and plane N . _____ .

Point ____ is in plane N but not in plane M .

Line ____ and plane N have point ____ in common.

Every point of line ____ is in plane N .

How many points do lines t and $\overleftrightarrow{AB}$ have in common? _____

How many points do $\overleftrightarrow{AB}$ and plane N have in common? _____

How many points do line t and plane N have in common? _____

How many points do lines p and t have in common? _____

The student is referred to the following items which follow the Glossary of this LightUnit:

1) Appendix I – Formulas – Mathematics 808 LightUnit.

2) Appendix II – Glossary – Mathematics 808 LightUnit.

For further review or study, consult this LightUnit.



Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.