

# **MATHEMATICS 1003**

## **ANGLE RELATIONSHIPS**

## **AND PARALLELS**

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# ANGLE RELATIONSHIPS AND PARALLELS

In this LIGHTUNIT we shall study another basic geometric idea. Lines, segments, and rays can be placed in such a way as to form angles. Angles are present everywhere and are very important in the study of geometry. Many angle relationships will be presented, along with

methods for measuring angles. We shall also learn about parallels and how special angles are formed using parallels. Many theorems will be presented in connection with angles and parallels.

## OBJECTIVES

**Read these objectives.** The objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT .

When you have finished this LIGHTUNIT, you should be able

1. To identify angles as acute, right, or obtuse.
2. To find the measure of angles with a protractor.
3. To add and subtract measures of angles.
4. To find the measure of angles by their relationship with other angles.
5. To prove theorems about angles.
6. To state the definitions of terms related to parallels.
7. To prove theorems about parallel and related angles.
8. To classify triangles by their sides and by their angles.
9. To prove theorems about triangles and their related angles.

**Survey the LIGHTUNIT.** Ask yourself some questions about this study. Write your questions here.

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## SECTION OBJECTIVES

Read these objectives. When you have completed this section, you should be able:

### I. ANGLE DEFINITIONS AND MEASUREMENT

1. To identify angles as acute, right, or obtuse.
2. To find the measure of angles with a protractor.
3. To add and subtract measures of angles.

To continue our study of geometry, we must learn basic angle definitions and measurement methods. Definitions will help in identifying and classifying angles; measurement will allow us to add and subtract angles.

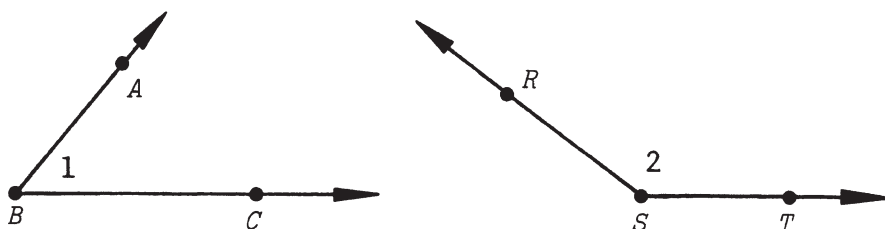
## ANGLE DEFINITIONS

These definitions relate to angles. Make sure you know them, because you will be using them many times in this LIGHTUNIT.

An *angle* is the union of two noncollinear rays that have a common end point.

The two rays that form the angle are called its *sides*, and the common end point is called the *vertex* of the angle. The symbol for angle is  $\angle$ .

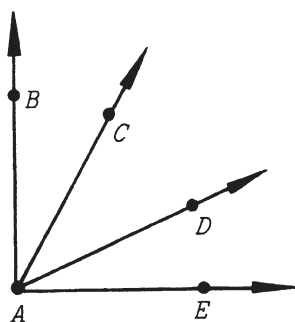
Models:



The angle to the left is formed by the union of  $\vec{BA}$  and  $\vec{BC}$ . Its sides are  $\vec{BA}$  and  $\vec{BC}$ . Its vertex is  $B$ . We can name the angle  $\angle ABC$ ,  $\angle CBA$ ,  $\angle B$ , or  $\angle 1$ . The angle to the right is the union of  $\vec{SR}$  and  $\vec{ST}$ . The vertex is point  $S$ . This angle can be called  $\angle RST$ ,  $\angle TSR$ ,  $\angle S$ , or  $\angle 2$ .

In geometry angles are often classified according to size.

DEGREE: The measure of an angle formed by rotating a ray around its endpoint  $\frac{1}{360}$  of a revolution. The symbol used for degrees is " $^\circ$ " above the right side of a number indicating the angle's measure.



Notice that when three letters are used to name an angle, the vertex letter is always placed between the other two.

If no confusion will result, the vertex letter can be used alone; or a numeral can be used. You should never use a single letter when several angles have the same vertex.

$\angle A$  could mean:

$\angle BAC$ , the angle formed by  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ ; or

$\angle CAD$ , the angle formed by  $\overrightarrow{AC}$  and  $\overrightarrow{AD}$ ; or

$\angle DAE$ , the angle formed by  $\overrightarrow{AD}$  and  $\overrightarrow{AE}$ ; or

$\angle BAD$ , the angle formed by  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$ ; or

$\angle CAE$ , the angle formed by  $\overrightarrow{AC}$  and  $\overrightarrow{AE}$ ; or

$\angle BAE$ , the angle formed by  $\overrightarrow{AB}$  and  $\overrightarrow{AE}$ .

An *acute angle* is an angle whose measure is less than  $90^\circ$ .

Models:

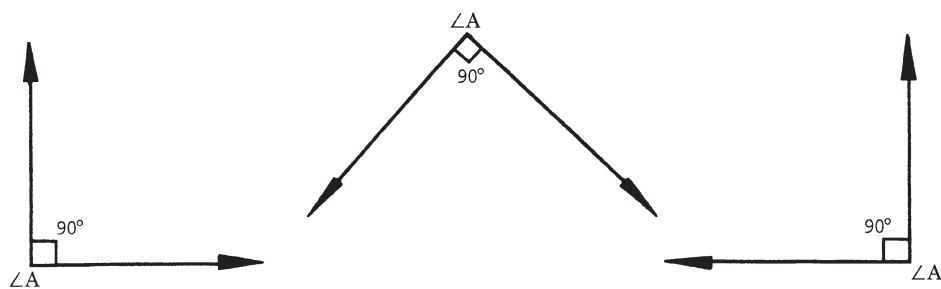


These angles are all acute angles.

A *right angle* (rt  $\angle$ ) is an angle whose measure equals  $90^\circ$ .

In symbols we say  $m\angle A = 90^\circ$ .

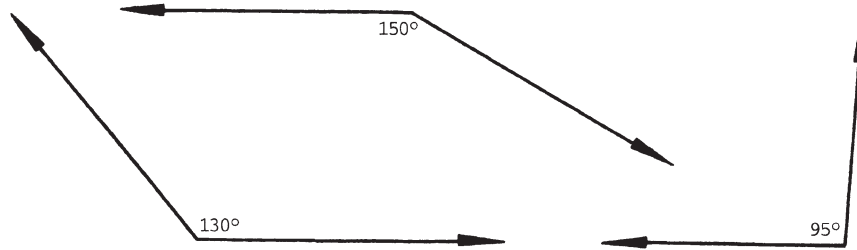
Models:



These angles are right angles. A little box at the vertex indicates a right angle.

An *obtuse angle* is an angle with a measure greater than  $90^\circ$  but less than  $180^\circ$ .

Models:

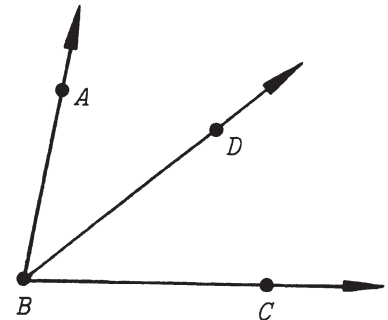


These angles are all obtuse angles.

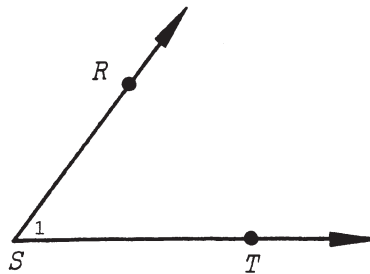
The *bisector of an angle* is a ray that is in the interior of the angle and divides the angle into two angles of equal measure.

Model:

If the measure of  $\angle ABD$  ( $m\angle ABD$ ) equals the measure of  $\angle DBC$  ( $m\angle DBC$ ), then  $\overrightarrow{BD}$  bisects  $\angle ABC$ . That is if  $m\angle ABD = m\angle DBC$ , then  $BD$  bisects  $\angle ABC$ .



Given the following angle, complete these items.

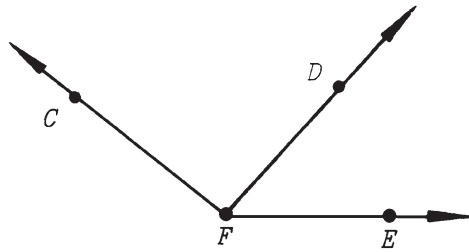


- 1.1 State four ways of naming this angle. \_\_\_\_\_
- 1.2 What point is the vertex of the angle? \_\_\_\_\_
- 1.3 Name the sides of this angle. \_\_\_\_\_

Complete these items.

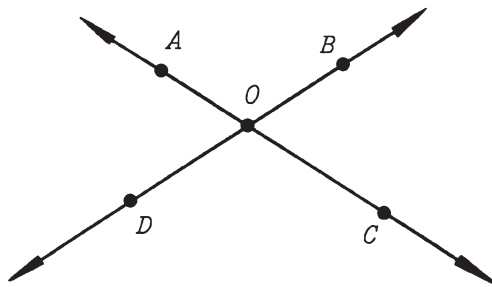
- 1.4 Name the sides of  $\angle ABC$ . \_\_\_\_\_
- 1.5 Name the vertex of  $\angle ABC$ . \_\_\_\_\_

Given the following angles, complete these items.



- 1.6 Name three angles with vertex  $F$ . \_\_\_\_\_
- 1.7  $\overrightarrow{FC}$  and  $\overrightarrow{FE}$  are the sides of what angle? \_\_\_\_\_
- 1.8 What ray is the common side of  $\angle CFD$  and  $\angle DFE$ ? \_\_\_\_\_
- 1.9 If  $m \angle DFE = m \angle CFD$ , name the bisector of  $\angle CFE$ . \_\_\_\_\_

Given the following angles, complete these items.



- 1.10 Name the acute angles. \_\_\_\_\_
- 1.11 Name the obtuse angles. \_\_\_\_\_

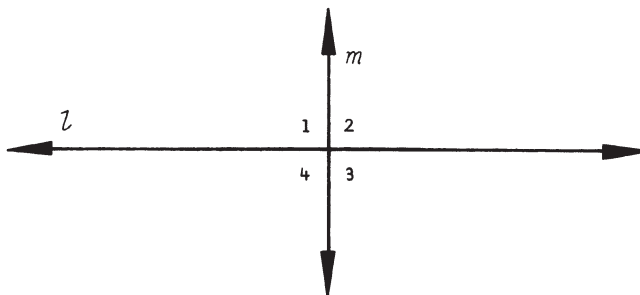
Complete this activity.

- 1.12 In the space provided,
  - a. Draw a right angle.
  - b. Draw a bisector of the right angle. Label all parts.
  - c. What kind of angles are formed by the bisector?

\_\_\_\_\_

Perpendicular lines are two lines that intersect such that the four angles formed are equal to each other.

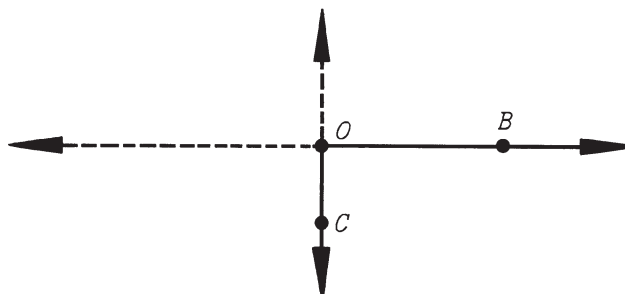
Model:



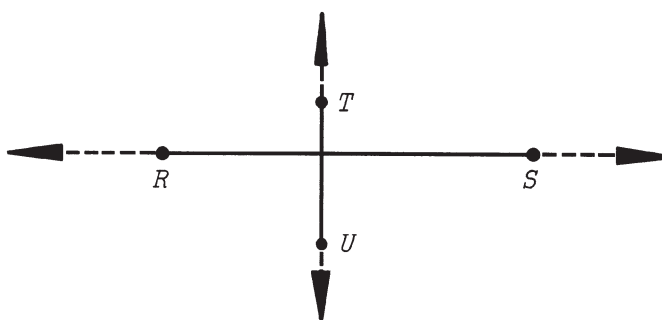
When  $m \angle 1 = m \angle 2 = m \angle 3 = m \angle 4$ , we say that line  $l$  is perpendicular ( $\perp$ ) to line  $m$ .

Segments and rays that are parts of lines can also be called perpendicular if the lines of which they are part are perpendicular.

Models:

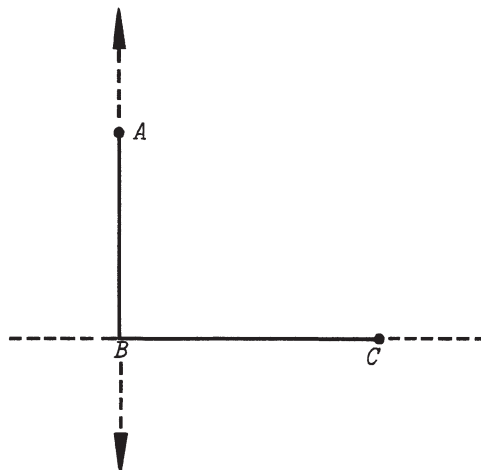


$$\vec{OB} \perp \vec{OC}$$



$$\overline{RS} \perp \overline{TU}$$

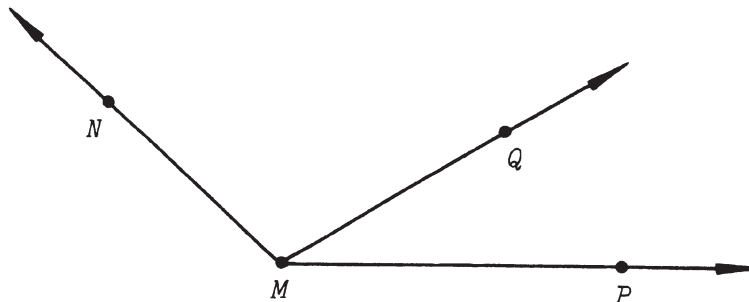
$$\overline{AB} \perp \overline{BC}$$



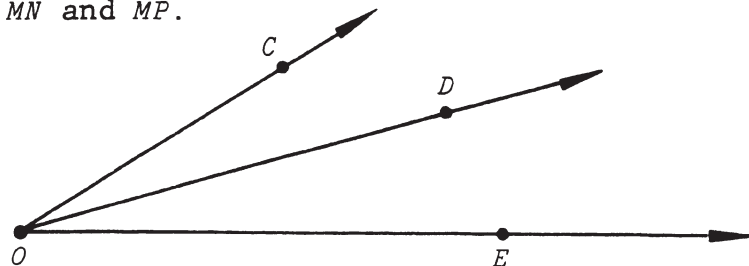
Betweenness of rays means that for any  $\angle AOC$ ,  
 $\overrightarrow{OB}$  is between  $\overrightarrow{OA}$  and  $\overrightarrow{OC}$  when  
 all three rays have the same end point  
 and when

$\overrightarrow{OB}$  lies in the interior of  $\angle AOC$ . Both  
 of these conditions must be true for  
 $\overrightarrow{OB}$  to be between  $\overrightarrow{OA}$  and  $\overrightarrow{OC}$ .

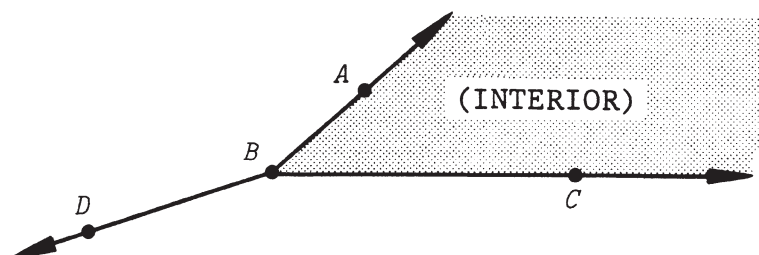
Models:



$\overrightarrow{MQ}$  is between  $\overrightarrow{MN}$  and  $\overrightarrow{MP}$ .



$\overrightarrow{OD}$  is between  $\overrightarrow{OC}$  and  $\overrightarrow{OE}$ .



$\overrightarrow{BD}$  is not between  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ .  
 It is not in the interior of  
 $\angle ABC$ .