

MATHEMATICS 1004

CONGRUENT TRIANGLES AND QUADRILATERALS

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CONGRUENT TRIANGLES AND QUADRILATERALS

You have learned in previous **LIGHTUNIT**s about geometry as a system. You have studied points and lines, induction and deduction, and angle relationships. The next step in your study of geometry is to learn about congruent triangles and some quadrilaterals related to congruent

triangles. Theorems and properties relating to these figures will be presented in this **LIGHTUNIT**. Completion of this **LIGHTUNIT** should prepare you for studying more complex and interesting geometric concepts.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this **LIGHTUNIT**.

When you have finished this **LIGHTUNIT**, you should be able:

1. To state the definition of congruent **triangles**.
2. To prove triangles congruent by using SSS, SAS, ASA, and AAS **statements**.
3. To prove right triangles congruent by using HL, LL, HA, and LA **statements**.
4. To prove corresponding parts equal when triangles are in normal **position**.
5. To prove corresponding parts equal when triangles are **overlapping**.
6. To prove properties of isosceles **triangles**.
7. To prove inequalities in one **triangle**.
8. To prove inequalities in two **triangles**.
9. To identify the properties of parallelograms, rectangles, squares, rhombuses, and trapezoids.

Survey the LIGHTUNIT. Ask yourself some questions about this study. Write your questions here.

SECTION OBJECTIVES

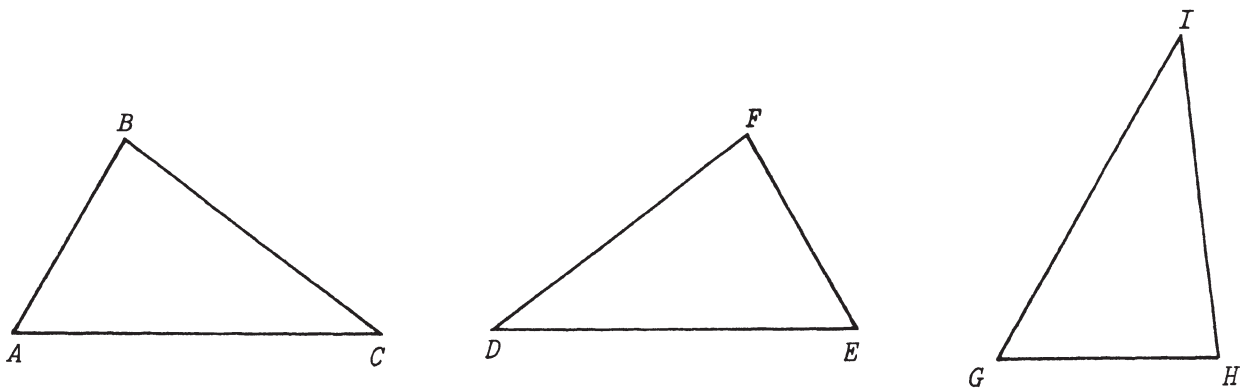
Read these objectives. When you have completed this section, you should be able:

I. TRIANGLES

1. To state the definition of congruent triangles.
2. To prove triangles congruent by using SSS, SAS, ASA, and AAS statements.
3. To prove right triangles congruent by using HL, LL, HA, and LA statements.

Most of the material goods we use today are mass produced. Every product is produced by the thousands, and all are exactly alike. They are the same size and the same shape. When your car needs a new part, the mechanic can replace the old part with a new one that is exactly the same as the old one. Figures, whether plane or solid, that have the same size and the same shape are called *congruent* figures.

DEFINING CONGRUENT TRIANGLES



All three triangles shown are congruent. One way of describing the situation is to say any one of these triangles can be moved onto any other one in such a way that it fits exactly. To show this fit we can match the vertices of the triangles. This matching can take place in several ways, but only one way will make one triangle fit exactly over the other.

Model 1: $A \leftrightarrow E$
 $B \leftrightarrow F$
 $C \leftrightarrow D$

When the vertices are matched as in Model 1,
then $\triangle ABC$ will fit over $\triangle EFD$.

Model 2: $A \leftrightarrow G$
 $B \leftrightarrow H$
 $C \leftrightarrow I$

When the vertices are matched as in Model 2,
then $\triangle ABC$ will fit over $\triangle GHI$.

A matching of vertices in this way is called
a *one-to-one correspondence* between the vertices of
the two triangles. The angles at the vertices that
are matched are called *corresponding angles*. Three
corresponding sides also match.

Model 3: $\overline{AB} \leftrightarrow \overline{GH} \leftrightarrow \overline{FE}$
 $\overline{BC} \leftrightarrow \overline{HI} \leftrightarrow \overline{FD}$
 $\overline{CA} \leftrightarrow \overline{IG} \leftrightarrow \overline{ED}$

DEFINITIONS

One-to-one correspondence: the situation when each member
of a set, such as angles of a triangle, can be paired with
one and only one member of another set.

Corresponding angles: angles paired with one another in a
one-to-one correspondence.

Corresponding sides: sides paired with one another in a *one-*
to-one correspondence.

For each part (angle or side) of one triangle
a corresponding part of the other triangle exists.
Therefore, we have a one-to-one correspondence
between all six parts of one triangle with all six
parts of another triangle.

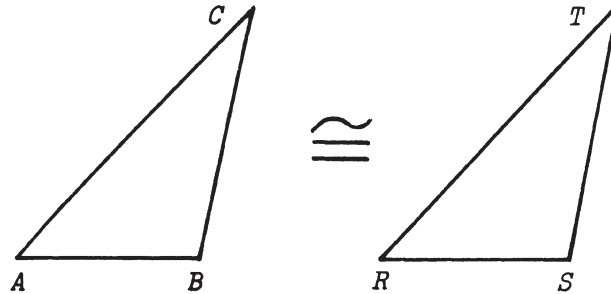
If the one-to-one correspondence of all six
parts leads to one triangle fitting over the other
exactly, then the triangles are *congruent*. The
symbol for congruent is $\cong$.

DEFINITION

Congruent triangles: If a one-to-one correspondence between the parts of two triangles is such that the corresponding parts are equal, then the triangles are congruent.

To show which parts correspond to each other, we name the triangles in a special way.

Model 4:



First write the name of one triangle, then write the vertices of the other triangle so that the corresponding vertices are in matching position in the name.

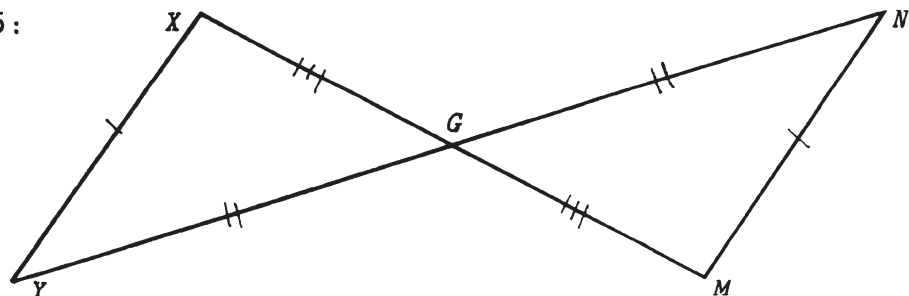
$$\begin{array}{c} \text{A} \quad \text{B} \quad \text{C} \\ \text{A} \quad \text{B} \quad \text{C} \\ \Delta ABC \cong \Delta RST \\ \text{or} \end{array}$$

$$\begin{array}{c} \text{B} \quad \text{C} \quad \text{A} \\ \text{B} \quad \text{C} \quad \text{A} \\ \Delta BCA \cong \Delta STR \\ \text{or} \end{array}$$

$$\begin{array}{c} \text{C} \quad \text{A} \quad \text{B} \\ \text{C} \quad \text{A} \quad \text{B} \\ \Delta CAB \cong \Delta TRS \end{array}$$

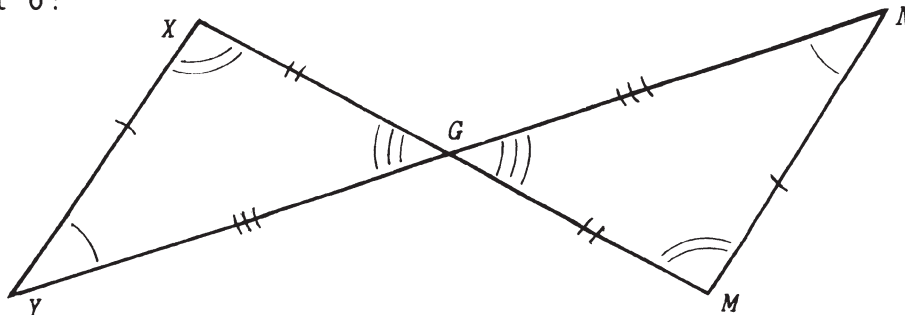
When we draw models of congruent triangles, we often mark pairs of corresponding parts in the same way, to show which parts are equal.

Model 5:



The marks ', ", and "' show that
 $YX = MN$, $YG = GN$, and $XG = GM$.

Model 6:



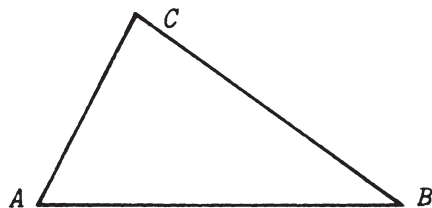
The marks $\frown$, $\smile$, and $\bigcirc$ show that
 $\angle Y = \angle N$, $\angle X = \angle M$, and $\angle XGY = \angle MGN$.

Two more definitions that we will be using
 are the definitions for *included angle* and *included side*.

DEFINITION

Included angle: the angle formed by two sides of a triangle.

Model:

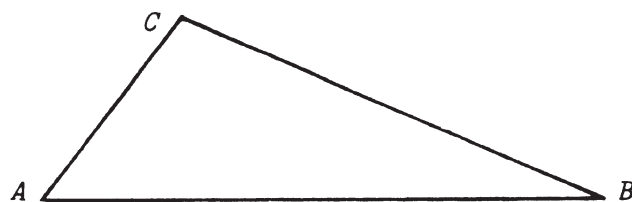


$\angle C$ is the included angle between sides $\overline{AC}$ and $\overline{BC}$. $\angle B$ is the included angle between sides $\overline{AB}$ and $\overline{BC}$. $\angle A$ is the included angle between sides $\overline{AB}$ and $\overline{AC}$.

DEFINITION

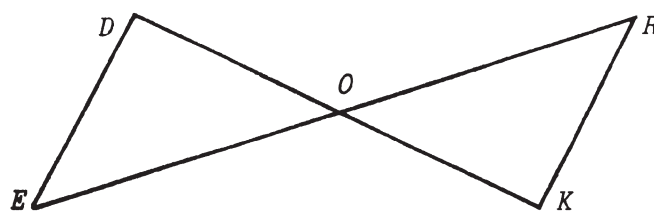
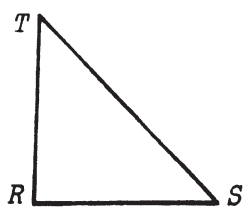
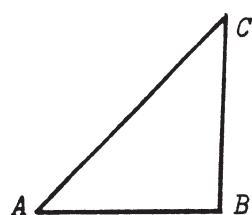
Included side: the side of a triangle that is formed by the common side of two angles.

Model :



$\overline{AB}$ is the included side between $\angle A$ and $\angle B$. $\overline{BC}$ is the included side between $\angle B$ and $\angle C$. $\overline{AC}$ is included between $\angle A$ and $\angle C$.

Complete the correspondence so a congruence can be established.



1.1 $A \leftrightarrow$ _____

1.4 $E \leftrightarrow$ _____

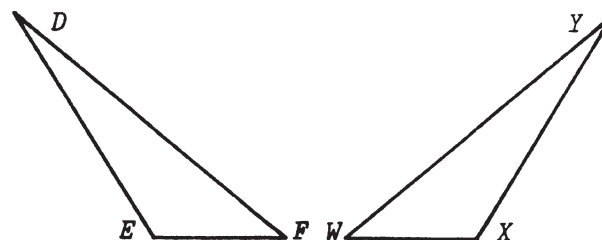
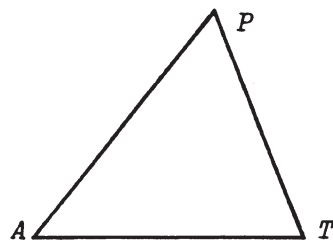
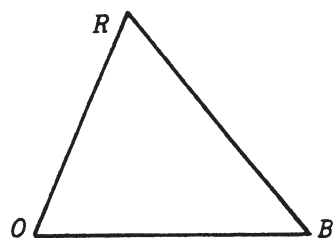
1.2 $B \leftrightarrow$ _____

1.5 $D \leftrightarrow$ _____

1.3 $C \leftrightarrow$ _____

1.6 $O \leftrightarrow$ _____

Basing your answer on the appearance of the figures, write true or false.



1.7 _____ $\triangle ROB \cong \triangle PTA$

1.12 _____ $\triangle DFE \cong \triangle YWX$

1.8 _____ $\triangle ROB \cong \triangle PAT$

1.13 _____ $\triangle FED \cong \triangle WXY$

1.9 _____ $\triangle RBO \cong \triangle PTA$

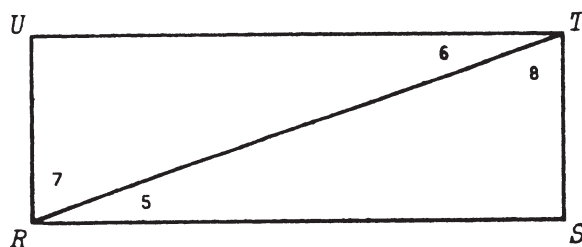
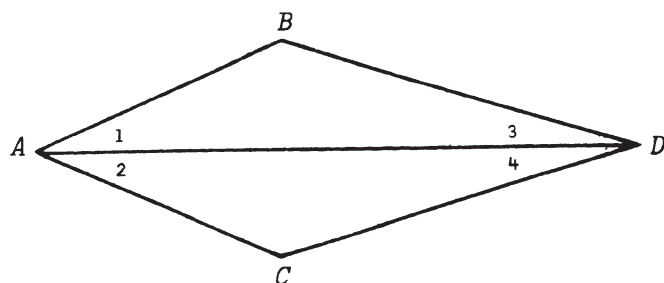
1.14 _____ $\triangle PAT \cong \triangle WXY$

1.10 _____ $\triangle OBR \cong \triangle APT$

1.15 _____ $\triangle ROB \cong \triangle DEF$

1.11 _____ $\triangle DEF \cong \triangle WXY$

In the following pairs of congruent triangles, complete the pairs of corresponding parts.



1.16 $\overline{AB} \leftrightarrow$ _____

1.17 $\overline{CD} \leftrightarrow$ _____

1.18 $\overline{AD} \leftrightarrow$ _____

1.19 $\angle 1 \leftrightarrow$ _____

1.20 $\angle 3 \leftrightarrow$ _____

1.21 $\angle B \leftrightarrow$ _____

1.22 $\overline{RS} \leftrightarrow$ _____

1.23 $\overline{TS} \leftrightarrow$ _____

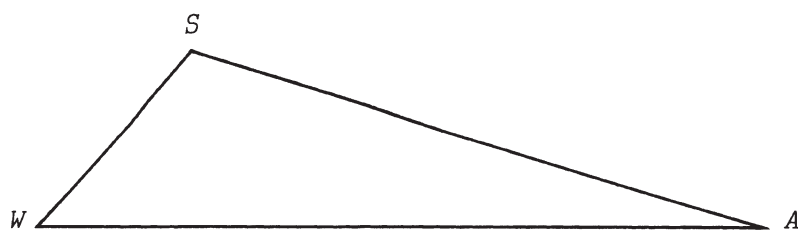
1.24 $\overline{RT} \leftrightarrow$ _____

1.25 $\angle 5 \leftrightarrow$ _____

1.26 $\angle 7 \leftrightarrow$ _____

1.27 $\angle U \leftrightarrow$ _____

Name the included angle or the included side asked for.



1.28 Included angle between $\overline{WA}$ and $\overline{AS}$: _____

1.29 Included angle between $\overline{SW}$ and $\overline{WA}$: _____

1.30 Included side between $\angle S$ and $\angle A$: _____

1.31 Included side between $\angle S$ and $\angle W$: _____

1.32 Included angle between $\overline{SW}$ and $\overline{AS}$: _____

1.33 Included side between $\angle A$ and $\angle W$: _____