

MATHEMATICS 1005

SIMILAR POLYGONS

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SIMILAR POLYGONS

In our study of congruent triangles, we learned that congruent triangles have the same size and the same shape. We are now going to study objects that have the same shape, but not necessarily the same size.

We have all looked at photographs. The photograph shows a smaller or larger version of the object that was photographed. The picture is the same shape but a different size.

We have all taken trips and used a road map to help get us from here to there. A map is a smaller version of the real thing.

Some of you may have built model airplanes or may have a model train. These models are scaled-down versions of a real item. The model is the same shape as the real thing but the size is different.

These examples are all practical examples of similarly shaped objects. In mathematics we also study similar shapes. In this LIGHTUNIT we shall learn and use some principles of algebra in our study of similarity and its application to the right triangle.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT.

When you have finished this LIGHTUNIT, you should be able:

1. To write ratios in simplest form.
2. To name the properties of proportions.
3. To solve proportion problems.
4. To identify similar polygons by using definitions, postulates, and theorems.
5. To use theorems about special segments in similar triangles.
6. To solve problems about similar right triangles.
7. To use the Pythagorean Theorem.
8. To solve triangle problems using trigonometry.
9. To find measurements indirectly.

Survey the LIGHTUNIT. Ask yourself some questions about this study. Write your questions here.

SECTION OBJECTIVES

Read these objectives. When you have completed this section, you should be able:

I. PRINCIPLES OF ALGEBRA

1. To express ratios in simplest form.
2. To name the properties of proportions.
3. To solve proportion problems.

Before we learn and use some of the properties of similar polygons, we should review some basic principles from algebra.

RATIOS AND PROPORTIONS

Two ideas from algebra that we need to review at this time are ratios and proportions.

DEFINITION

Ratio: the comparison of two numbers by division. The quotient is the ratio of the two numbers.

The ratio of 3 to 15 is $\frac{1}{5}$. The ratio of 8 to 2 is $\frac{4}{1}$. The ratio of a to b is $\frac{a}{b}$. Notice the quotients $\frac{1}{5}$, $\frac{4}{1}$, $\frac{a}{b}$ are written as fractions. We can arrive at the ratio by dividing the "to" number into the "of" number. Keep in mind that you are not finding the ratio of one object to another, but rather the ratio of two numbers that are measures of the object in the same unit.

Rather than go through a division process to find a ratio, we can set up the two numbers as a fraction and reduce the fraction.

The "of" number will be the numerator (top) and the "to" number will be the denominator (bottom).

Model 1: Find the ratio of 6 to 8.

$$\frac{6}{8} = \frac{3}{4}$$

Model 2: If $AB = 6$ inches and $CD = 18$ inches, find the ratio of AB to CD .

$$\frac{AB}{CD} = \frac{6}{18} = \frac{1}{3}$$

Model 3: If $\angle A = 35^\circ$, $\angle B = 50^\circ$, find the ratio of $\angle A$ to $\angle B$.

$$\frac{\angle A}{\angle B} = \frac{35}{50} = \frac{7}{10}$$

Model 4: If the side of one triangle is 2 feet and the side of another triangle is 18 inches, find the ratio of the small side to the large side.

$$\frac{18}{24} = \frac{3}{4} \text{ (2 feet was changed to 24 inches)}$$

$$\frac{1\frac{1}{2}}{2} = \frac{\frac{3}{2}}{2} = \frac{3}{4} \text{ (18 inches was changed to } 1\frac{1}{2} \text{ feet)}$$

$$\text{Note: } \frac{1\frac{1}{2}}{2} \text{ means } 1\frac{1}{2} \div 2 = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

Ratio is simply a number. No units are connected to a ratio.

The ratio of two numbers can also be written in the form $a:b$. This form is useful when we are comparing three or more numbers.

Model 5: The ratio of 3 to 4 to 5 can be written as 3:4:5.

This ratio means that the ratio of the first to the second is $\frac{3}{4}$, the ratio of the second to the third is $\frac{4}{5}$ and the ratio of the first to the third is $\frac{3}{5}$.

The following sets of numbers all have the ratio 3:4:5, since they can all be reduced to that ratio.

$$\{12, 16, 20\} \quad \frac{12}{16} = \frac{3}{4}, \quad \frac{16}{20} = \frac{4}{5}, \quad \frac{12}{20} = \frac{3}{5}$$

$$\{3x, 4x, 5x\} \quad \frac{3x}{4x} = \frac{3}{4}, \quad \frac{4x}{5x} = \frac{4}{5}, \quad \frac{3x}{5x} = \frac{3}{5}$$

$$\{15x^2, 20x^2, 25x^2\} \quad \frac{15x^2}{20x^2} = \frac{3}{4}, \quad \frac{20x^2}{25x^2} = \frac{4}{5}$$

$$\frac{15x^2}{25x^2} = \frac{3}{5}$$

Express each ratio in simplest form.

1.1 $\frac{6}{12} =$ _____

1.5 $21:28 =$ _____

1.2 $\frac{6}{9} =$ _____

1.6 $25:45 =$ _____

1.3 $\frac{18}{29} =$ _____

1.7 $10:20 =$ _____

1.4 $\frac{36}{72} =$ _____

1.8 $15:25:35 =$ _____

If $a = 3$ and $b = 5$, find each ratio.

1.9 $\frac{a}{b} = \underline{\hspace{2cm}}$

1.11 $\frac{a}{(a + b)} = \underline{\hspace{2cm}}$

1.10 $\frac{b}{a} = \underline{\hspace{2cm}}$

1.12 $\frac{b}{(a + b)} = \underline{\hspace{2cm}}$

If $c = 4$ and $d = 5$, find each ratio.

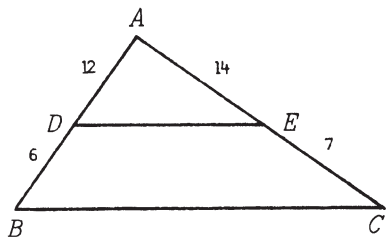
1.13 c to $d = \underline{\hspace{2cm}}$

1.15 $\frac{d}{(c + d)} = \underline{\hspace{2cm}}$

1.14 d to $c = \underline{\hspace{2cm}}$

1.16 $\frac{(d - c)}{(d + c)} = \underline{\hspace{2cm}}$

Use the figures to find each ratio in simplest form.



1.17 $\frac{AD}{DB} = \underline{\hspace{2cm}}$

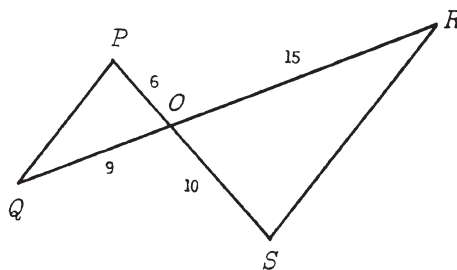
1.18 $\frac{AD}{AB} = \underline{\hspace{2cm}}$

1.19 $\frac{DB}{AB} = \underline{\hspace{2cm}}$

1.20 $\frac{AE}{AC} = \underline{\hspace{2cm}}$

1.21 $\frac{AC}{AE} = \underline{\hspace{2cm}}$

1.22 $\frac{EC}{AC} = \underline{\hspace{2cm}}$



1.23 $\frac{PO}{OS} = \underline{\hspace{2cm}}$

1.24 $\frac{PO}{PS} = \underline{\hspace{2cm}}$

1.25 $\frac{OS}{PS} = \underline{\hspace{2cm}}$

1.26 $\frac{OR}{QO} = \underline{\hspace{2cm}}$

1.27 $\frac{OR}{QR} = \underline{\hspace{2cm}}$

1.28 $\frac{QR}{PS} = \underline{\hspace{2cm}}$

1.29 The measures of the angles of a triangle are in the ratio of 1:2:3. Find the measure of each angle. (Hint: The sum of the angles of a triangle = 180° .)

- 1.30 The distance from A to B is 60 feet. The distance from B to C is 10 yards. The distance from C to D is 20 inches. Find the ratio of $AB:BC:CD$. (Hint: Convert all measures to inches.)

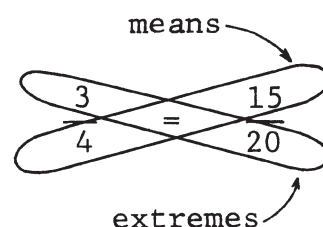
DEFINITION

Proportion: an equation that states that two ratios are equal.

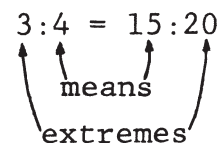
The proportion $\frac{a}{b} = \frac{c}{d}$ tells us that the ratio a to b and the ratio c to d are equal ratios. The proportion can be read, a is to b as c is to d ; or, the quotient of a and b equals the quotient of c and d . The proportion can also be written $a:b = c:d$.

Each of the four numbers a , b , c , and d is called a *term* of the proportion: a is the first term, b is the second term, c is the third term, and d is the fourth term.

The first and the fourth terms are known as the *extremes* of the proportion. The second and third terms are called the *means*.



The fact that more than two ratios are equal is often expressed in the form of an *extended proportion*. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$ is an extended proportion stating that all four ratios are the same number. Another extended proportion is $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$. We can pick any two of the ratios and form a regular proportion: $\frac{3}{6} = \frac{6}{12}$, $\frac{7}{14} = \frac{2}{4}$.



Since a proportion is an equation, we can use the properties of equality to transform a proportion to another form. For example, $\frac{a}{b} = \frac{c}{d}$ can be written as $ad = bc$ by using the multiplication property of equality (multiply each side of the equation by bd).

$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ (bd) \frac{a}{b} &= (bd) \frac{c}{d} \\ ad &= bc \end{aligned}$$

Name the means and the extremes in these proportions.

1.31 $\frac{3}{4} = \frac{15}{20}$ means : _____ extremes : _____

1.32 $\frac{5}{7} = \frac{20}{28}$ means : _____ extremes : _____

1.33 $\frac{6}{11} = \frac{x}{y}$ means : _____ extremes : _____

1.34 $3:9 = 2:6$ means : _____ extremes : _____

1.35 $1:2 = 4:8$ means : _____ extremes : _____

1.36 $x:y = 3:7$ means : _____ extremes : _____

Find the value of x in each of these proportions.

1.37 $\frac{x}{25} = \frac{2}{5}$ $x =$ _____ 1.42 $\frac{1}{2}:x = \frac{2}{3}:\frac{3}{4}$ $x =$ _____

1.38 $\frac{x}{6} = \frac{3}{2}$ $x =$ _____ 1.43 $\frac{(x+3)}{6} = \frac{5}{4}$ $x =$ _____

1.39 $\frac{9}{x} = \frac{3}{12}$ $x =$ _____ 1.44 $\frac{(x+1)}{(x+2)} = \frac{2}{3}$ $x =$ _____

1.40 $\frac{10}{7} = \frac{x}{5}$ $x =$ _____ 1.45 $\frac{3}{2} = \frac{x}{4}$ $x =$ _____

1.41 $9:x = x:4$ $x =$ _____

Find the ratio of x to y .

1.46 $2x = 3y$ 1.49 $2x - 3y = 0$

1.47 $5x = 7y$ 1.50 $x - 5y = 0$

1.48 $\frac{x}{3} = \frac{y}{2}$