

MATHEMATICS 1006

CIRCLES

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CIRCLES

The circle is one of the basic geometric shapes that we study. The circle and its space partner, the sphere, can be found in all phases of life. The circle is even referred to in the Bible: Ezekiel, Chapter 1, mentions Ezekiel's vision of a wheel in a wheel; Isaiah 40:22 refers to God's power over "the circle of the earth." We use ideas related to circles in our cars, from the steering wheel to the gears that make them run.

Industry would be at a standstill if not for ideas based on the circle. Many games and sports activities depend on circles and spheres.

In our study of the circle, we shall learn about the different parts of a circle and how angles and segments are related to circles. We shall learn how to measure arcs, angles, and segment lengths that are formed in circles.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT.

When you have finished this LIGHTUNIT, you should be able:

1. To identify the characteristics of circles and spheres.
2. To define tangents.
3. To use theorems related to tangents.
4. To use arcs to measure central angles and central angles to measure arcs.
5. To find the measure of the angle formed by the hands of a clock at any given time.
6. To solve problems related to chords, secants, and inscribed angles.
7. To find the measure of angles formed by segments related to circles.
8. To find the measure of segments formed by chords, secants, and tangents.

Survey the LIGHTUNIT. Ask yourself some questions about this study. Write your questions here.

OBJECTIVES

Read this objective. When you have completed this section, you should be able:

I. CIRCLES AND SPHERES

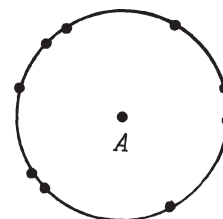
1. To identify the characteristics of circles and spheres.

To learn about circles and spheres we need to define the parts of the figures and the relationship those parts have to each other. Some of these terms you may already know; others will be completely new to you.

CHARACTERISTICS OF CIRCLES

Circles have specific parts, each with a geometric definition. After the parts of the circle have been learned, then the different types of circles become easy to define.

First we need to agree on a definition of a circle. A good definition of a circle follows.



Circle A

DEFINITION

Circle: the set of all points in a plane that are the same distance from a given point in that plane.

A circle, by this definition, consists of all the points in a plane that meet the requirement of being equally distant from some point in the plane. We call the other point the center of the circle. If we measure the distance from the center to the circle at any point, the distance will remain the same.

A circle is named by using its center point.

RADIUS

One important part of the circle is the radius.

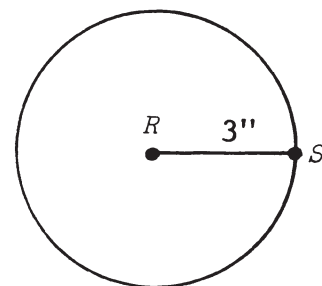
DEFINITION

Radius: a segment with endpoints on the circle and at the center of the circle.

The word *radius* will be used in two ways. First, as the definition states, a radius is a segment. Therefore, when we talk about the radius of a circle, we are referring to a segment. A second use of the word radius refers us to the length of that segment: The radius is a number. No confusion should arise as to which radius we mean because of the way it will be used in a sentence.

Model 1: The radius of circle R is 3".
(measure)

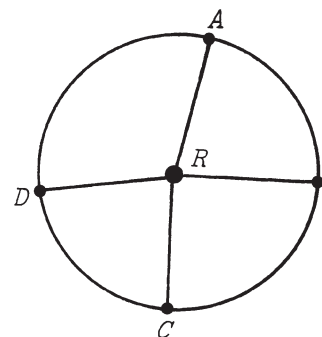
Model 2: The radius of circle R is \overline{RS} .
(segment)



When a doubt might exist as to which radius is intended, we will specify the *length* or the *measure* of the radius to mean the number.

Notice that all radii (plural of radius) in the same circle will have the same length. $RA = RB = RC = RD$ in circle R .

Radii of a circle are equal



DIAMETER

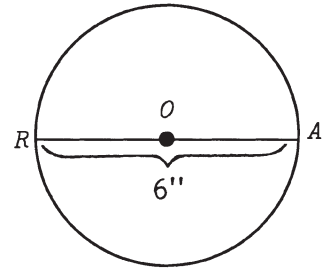
Another part of the circle is the diameter.

DEFINITION

Diameter: a segment passing through the center with endpoints on the circle.

The word *diameter* will also be used in two ways, just as radius is used. The segment \overline{RA} is the diameter of circle O . The diameter of circle O is 6 inches.

Since the diameter passes through the center of the circle, it is made up of two radii. \overline{OR} and \overline{OA} are radii and the three points R , O , and A are collinear with O being the midpoint of \overline{RA} . We then have $RO + OA = RA$ or radius + radius = diameter. Two radii equal a diameter. Therefore, the length of a diameter is twice the length of a radius.



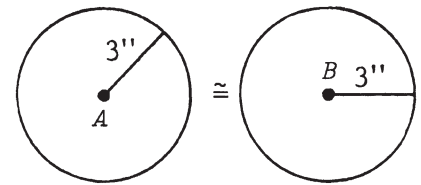
CONGRUENT CIRCLES

One way in which circles can be related is to be *congruent*.

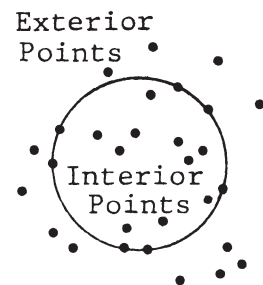
DEFINITION

Congruent circles: circles that have equal radii.

We studied congruent triangles in a previous LIGHTUNIT, and you should remember that congruent triangles are the same shape and size. Since all circles are the same shape, their size depends only on their radii. When we use this definition of congruent circles in a proof we can express it in this form: Radii of congruent circles are equal.



Our definition of a circle states that it lies in a plane. The circle divides the points of that plane into three sets: the points of the plane that are interior to the circle, the points that are exterior to the circle, and the points that are the circle itself.



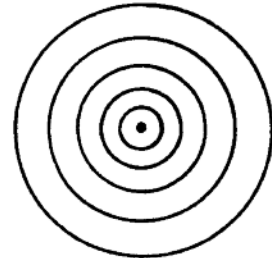
CONCENTRIC CIRCLES

Circles can also be related by being *concentric*.

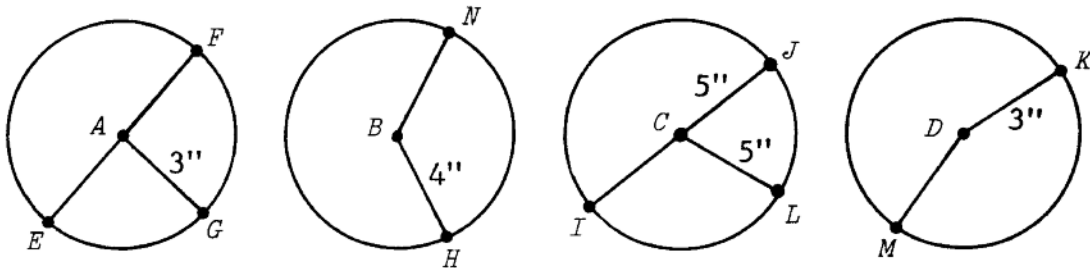
DEFINITION

Concentric circles: two or more circles that lie in the same plane and have the same center.

You see many examples of concentric circles every day. A bicycle wheel, a car wheel and tire, a bullseye target, and designs on clothing are a few.

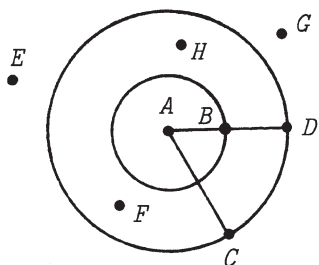


Refer to the circles to complete the following activities.



- 1.1 Name two radii of circle D . _____
- 1.2 Name a diameter of circle C . _____
- 1.3 Name two radii of circle B . _____
- 1.4 What is the name of the circle that has radius \overline{CL} . _____
- 1.5 Name three radii of circle A . _____
- 1.6 What is the length of the radius in circle C ? _____
- 1.7 What is the length of diameter \overline{EF} ? _____
- 1.8 What circles have the same length radii? _____
- 1.9 What circles are congruent? _____
- 1.10 What circles are concentric? _____

Refer to this circle to complete the following activities.



Given: $AB = 4$
 $AC = 6$

- 1.11 What is the name of the radius of the smaller circle? _____
- 1.12 What is the name of the radius of the larger circle? _____
- 1.13 Name six points in the exterior of the smaller circle. _____
- 1.14 Name four points in the interior of the larger circle. _____
- 1.15 What is the length of BD ? _____
- 1.16 What points are in the exterior of both circles? _____
- 1.17 What point is the center of both circles? _____
- 1.18 Does $AC = AB + BD$? _____
- 1.19 If $AC = 20$ and $BD = 8$, what is the radius of the smaller circle?

- 1.20 If $BD = 7$ and the radius of the smaller circle is 7, what is the diameter of the larger circle? _____

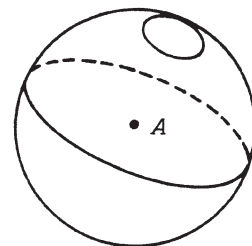
CHARACTERISTICS OF SPHERES

Many properties and parts of a circle suggest similar properties and parts of a sphere. Notice, for example, how close the definition of a circle and a sphere are to each other.

DEFINITION

Sphere: the set of all points that are the same distance from a given point.

Omitting the words *in a plane* from the definition of a circle gives the definition of a sphere. A sphere is named by its center just like a circle is.



Sphere A

Spheres and spherically shaped objects are all about us. Many sports and games are played with balls that are spheres: baseball, tennis, golf, marbles. We eat spherically shaped foods; apples, oranges, tomatoes, cherries. We even live on a sphere.

We can often define terms used for spheres by replacing the word *circle* with *sphere* in the definitions for the circle. The center of a sphere is the point from which all the rest are equidistant.

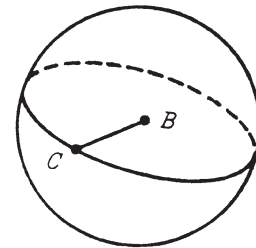
DEFINITIONS

Radius of sphere: a segment with endpoints at the center of the sphere and a point on the sphere.

Diameter of a sphere: a segment passing through the center with endpoints on the sphere.

Congruent spheres: spheres that have equal radii.

A sphere divides space into three sets. Points that are exterior to the sphere have a distance greater than the radius from the center. Points that are interior to the sphere have a distance that is less than the radius. Points of the sphere have a distance equal to the radius. Remember, the sphere itself is only the shell of points around its center. A sphere can be compared to a basketball or a balloon.



If a plane intersects a sphere at many points, the intersection will be a circle. If the plane does not contain the center of the sphere, the intersection is called a *small circle*. If the plane contains the center, the intersection is called a *great circle*. The equator of our earth is a great circle.

Spheres may be concentric to one another, similar to concentric circles.

