

MATHEMATICS 1104

POLYNOMIALS

CONTENTS

I. PRODUCTS AND FACTORING	2
Multiplying With Monomials	2
Multiplying Polynomials by Polynomials	3
Using Special Products	5
Factoring Trinomials	7
Factoring Special Products	10
II. OPERATIONS	14
Addition and Subtraction of Polynominals	14
Division With Polynomials	17
Synthetic Division	23
III. VARIATIONS	31
Direct Variations	31
Inverse Variations	34
Joint and Combined Variations	38

2019 Printing

The material in this book is the product of the Lord's blessing and many individuals working together on the teams, both at Alpha Omega Publications and Christian Light Publications.

Original material copyright ©1978 by Alpha Omega Publications
All rights reserved. Revised by permission.

CLP Revision copyright ©1980

Christian Light Publications, Inc.

Harrisonburg, Virginia 22802

Tel. 1-540-434-0768

Printed in USA

Copyrighted material. May not be reproduced without permission from the publisher.

POLYNOMIALS

In this LIGHTUNIT you will be studying products and factoring as well as operations with polynomials and the patterns of variations.

You may have already studied multiplication of polynomials and factoring of polynomials. In this LIGHTUNIT you will review and learn some new factoring techniques.

Addition, subtraction, and division will be reviewed and synthetic division will be introduced.

Learning the operational aspects of polynomials will be useful in solving equations.

Direct variation and inverse variation are important relationships between variables that are found in many natural laws. These relationships are often used alone or in combination to solve practical problems in chemistry, physics, and other sciences.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT.

When you have finished this LIGHTUNIT, you will be able:

1. To multiply polynomials.
2. To identify special products and to factor completely.
3. To perform addition, subtraction, and division with polynomials.
4. To use direct, inverse, joint, and combined variations in writing equations and solving problems.

Survey the LIGHTUNIT. Ask yourself some questions about this study. Write your questions here.

OBJECTIVES

Read these objectives. When you have completed this section, you will be able:

I. PRODUCTS AND FACTORING

1. To multiply polynomials.
2. To identify special products and to factor completely.

This LIGHTUNIT section will deal with topics that you have studied before. Multiplying polynomials, recognizing special products, and factoring trinomials and special products were studied earlier.

New material included deals with a special product not previously covered and the factors of this product. The polynomial factors may surprise you.

You will need expertise in this section to complete later work with equations and word problems.

MULTIPLYING WITH MONOMIALS

Certain laws of exponents must be used in operations with polynomials.

LAWS OF EXPONENTS

A: $a^m a^n = a^{m+n}$

B: $(a^m)^n = a^{mn}$

C: $(ab)^m = a^m b^m$

(These must be memorized.)

Models: A: $x^3 \cdot x^5 = x^8$

B: $(x^3)^5 = x^{15}$

C: $(x^2 y^4)^3 = (x^2)^3 (y^4)^3$ or $x^6 y^{12}$

D: $4^x \cdot 10^y \neq 40^{x+y}$; the base numbers must be the same (as in Law A above) to permit adding the exponents.

The distributive property is the basis for multiplying monomials with polynomials.

DISTRIBUTIVE PROPERTY

$a(b + c) = ab + ac$

(This must be memorized.)

Model: $3x(a + y) = 3ax + 3xy$

The associative and commutative properties are used along with the distributive property and the laws of exponents to multiply.

ASSOCIATIVE PROPERTY

$$a(bc) = (ab)c$$

(These must be memorized.)

COMMUTATIVE PROPERTY

$$ab = ba$$

$$\begin{aligned} \text{Model: } 6x^2y(3xy^3 + 14x^2y^2) &= (6x^2y)(3xy^3) + (6x^2y)(14x^2y^2) \\ &= 6 \cdot 3 \cdot x^2 \cdot x \cdot y \cdot y^3 + 6 \cdot 14 \cdot x^2 \cdot x^2 \cdot y \cdot y^2 \\ &= 18x^3y^4 + 84x^4y^3 \end{aligned}$$

Find the indicated products.

1.1 $x^2 \cdot x^5$

1.2 $a^4 \cdot a^6$

1.3 $(a^4)^5$

1.4 $(x^6)^4$

1.5 $(x^2y)^3$

1.6 $(2a^2b)^4$

1.7 $-2x^2y^3 \cdot 14x^2y^3$

1.8 $(3x^2y^6)^7$

1.9 $a^2b(3a^2 + 4ab^2)$

1.10 $3a^n(a^n + a^{n-1})$

1.11 $6xy(\frac{1}{2}x^2 - \frac{1}{3}xy + \frac{1}{6}y^2)$

MULTIPLYING POLYNOMIALS BY POLYNOMIALS

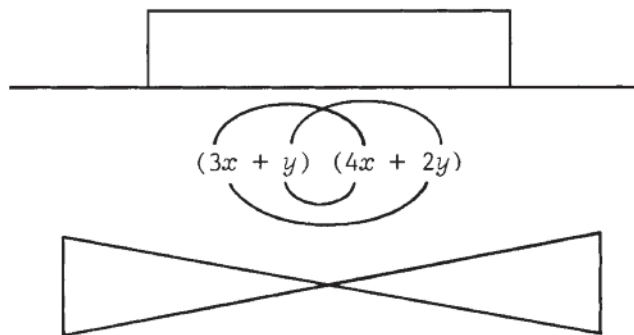
Multiplying two polynomials together involves the distributive, associative, and commutative properties reviewed in the previous section as well as some memory of products.

$$\begin{aligned}
 \text{Model: } (3x + y)(4x + 2y) &= (3x + y)4x + (3x + y)2y \\
 &= 12x^2 + \underbrace{4xy + 6xy} + 2y^2 \\
 &= 12x^2 + 10xy + 2y^2
 \end{aligned}$$

Using either the "foil" method or remembering the unforgettable face of George P. Trinomial can help you find the product. You should not have to write anything but the answer in finding this product.

The "foil" method involves using the letters in the word to remember that the product consists of the product of the first two first terms $(3x)(4x)$, plus the product of the outer terms $(3x)(2y)$ combined with the product of the innner terms $(y)(4x)$, and finally the product of the two last terms $(y)(2y)$.

George P. Trinomial is the name of the face you see.



George P. Trinomial

The crossed eyebrows, nose and chin help direct us to the products.

In certain polynomials you may group the terms and treat your selected binomials as monomials. With practice these products can be done mentally.

$$\begin{aligned}
 \text{Model: } (a + b - 3)(a + b + 5) &= [(a + b) - 3][(a + b) + 5] \\
 \text{let } x &= a + b \\
 (x - 3)(x + 5) &= x^2 + 2x - 15 \\
 \text{Now replace } x \text{ by } (a + b): &= (a + b)^2 + 2(a + b) - 15 \\
 &= a^2 + 2ab + b^2 + 2a + 2b - 15
 \end{aligned}$$

Find the indicated products mentally when possible.

1.12 $(a + 3)(a - 2)$ _____

1.13 $(3xy - 1)(4xy + 2)$ _____

1.14 $(2x - 3y)(4x - y)$ _____

1.15 $(ab - 9)(ab + 8)$ _____

1.16 $(m^3n + 8)(m^3n - 5)$ _____

1.17 $(3 - c^2d)(4 - 4c^2d)$ _____

1.18 $(1 - 7x)(1 + 9x)$ _____

1.19 $(3m^3 - \frac{1}{2}y)(3m^3 - \frac{1}{2}y)$ _____

1.20 $(a + b)(a^2 - ab + b^2)$ _____

1.21 $(a - b)(a^2 + ab + b^2)$ _____

Treat selected binomials as monomials to do these problems mentally.

1.22 $(x + y + 3)(x + y - 4)$ _____

1.23 $(a + b - c)(a + b + c)$ _____

1.24 $(x^2 + 2x - 1)(x^2 + 2x + 5)$ _____

1.25 $[4 - (3c - 1)][6 - (3c - 1)]$ _____

1.26 $(3x - 4y - 5z)(4x + 4y + 5z)$ _____

USING SPECIAL PRODUCTS

Although binomial products can always be found mentally as in the previous lesson, some special products require less mental energy if you can notice the patterns.

A *perfect square trinomial* is the name given to trinomials that result from squaring a binomial.

DEFINITION

Perfect square trinomial: the square of a binomial.

$$\text{Model 1:} \quad (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{Model 2:} \quad (2x - 4y^2)^2 = 4x^2 - 16xy^2 + 16y^4$$

Note that the first term in the first trinomial is a^2 , which is the square of a . The middle term is twice the product of the two terms in the binomial, $2ab$. The third term in the trinomial is the square of the second term in the binomial. Is $4x^2$ the same as $(2x)^2$? Is $-16xy^2$ the same as $2(2x)(-4y^2)$? Is $16y^4$ the same as $(-4y^2)^2$?

Another special product is the difference of two perfect squares. This product is the result of multiplying the sum of two numbers and the difference of those same two numbers.

$$\text{Model 3:} \quad (a + b)(a - b) = a^2 - b^2$$

It is important to remember patterns such as in these models in multiplying and factoring polynomials so you can recognize them during the exercises and tests.

$$\text{Model 4:} \quad (3x^2 + 4y)(3x^2 - 4y) = 9x^4 - 16y^2$$

Most surprising is the product of $(a + b)$ and $(a^2 - ab + b^2)$.

$$\begin{array}{r} \text{Model 5:} \quad a^2 - ab + b^2 \\ \quad \quad \quad \underline{a + b} \\ a^3 - a^2b + ab^2 \\ \quad \quad \quad + a^2b - ab^2 + b^3 \\ \hline a^3 \qquad \qquad \qquad + b^3 \end{array}$$

The product of $(a - b)$ and $(a^2 + ab + b^2)$ is $a^3 - b^3$.

To find this product mentally, you must notice that the binomial factor (here: $a - b$) is a difference when the product (here: $a^3 - b^3$) is the difference of two perfect cubes; it is a sum when the product is the sum of two perfect cubes. Another observation is that the trinomial $(a^2 + ab + b^2)$ is not a perfect square trinomial, simply because it lacks the factor of 2 in its middle term.

$$\text{Model 6:} \quad (a + b)(a^2 - ab + b^2) = a^3 + b^3$$

$$\text{Model 7:} \quad (2a - 3b)(4a^2 + 6ab - 9b^2) = 8a^3 - 27b^3$$

Find the indicated products.

- | | | |
|------|---|-------|
| 1.27 | $(w + x)(w + x)$ | _____ |
| 1.28 | $(y - x)(y - x)$ | _____ |
| 1.29 | $(wx - y)^2$ | _____ |
| 1.30 | $(x + 2y)^2$ | _____ |
| 1.31 | $(4x^3 + 7y^3z^4)^2$ | _____ |
| 1.32 | $(10^x + 4^y)^2$ | _____ |
| 1.33 | $(2x - \frac{1}{2})^2$ | _____ |
| 1.34 | $(0.1x + 0.4y)^2$ | _____ |
| 1.35 | $(2a^x + \frac{1}{2}b^x)^2$ | _____ |
| 1.36 | $(3x^{2a} - 4y^a z^{3a})^2$ | _____ |
| 1.37 | $(x + 2)(x - 2)$ | _____ |
| 1.38 | $(4a + x)(a - x)$ | _____ |
| 1.39 | $(y + \frac{1}{3})(y - \frac{1}{3})$ | _____ |
| 1.40 | $(b^2 + 8)(b^2 - 8)$ | _____ |
| 1.41 | $(5c + \frac{2}{3})(5c - \frac{2}{3})$ | _____ |
| 1.42 | $(x + 0.2)(x - 0.2)$ | _____ |
| 1.43 | $(y^n - 5)(y^n + 5)$ | _____ |
| 1.44 | $(a^x - b^y)(a^x + b^y)$ | _____ |
| 1.45 | $(4y^a - 6x^b)(4y^a + 6x^b)$ | _____ |
| 1.46 | $(a + b - c)(a + b + c)$ (See Model 3 on p.6
for mental shortcut after grouping) | _____ |
| 1.47 | $(x + y)(x^2 - xy + y^2)$ | _____ |
| 1.48 | $(x - y)(x^2 + xy + y^2)$ | _____ |
| 1.49 | $(2x - y)(4x^2 + 2xy + y^2)$ | _____ |
| 1.50 | $(3a + 2c)(9a^2 - 6ac + 4c^2)$ | _____ |
| 1.51 | $(4a^2 - 3b^2)(16a^4 + 12a^2b^2 + 9b^4)$ | _____ |

FACTORIZING TRINOMIALS

Since factoring is essentially "unmultiplying," we must use our multiplication skills backwards to