

MATHEMATICS 1106

REAL NUMBERS

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2016 Printing

The material in this book is the product of the Lord's blessing and many individuals working together on the teams, both at Alpha Omega Publications and Christian Light Publications.

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Christian Light Publications, Inc.

Harrisonburg, Virginia 22802

Tel. 1-540-434-0768

Printed in USA

REAL NUMBERS

In this LIGHTUNIT you will study the real-number system and will learn about radicals and imaginary numbers. All of the previously learned properties apply to radicals, because they are real numbers. However, you will learn some special properties of radicals.

In this LIGHTUNIT you will learn to solve radical equations and quadratic equations. Since much of this LIGHTUNIT is necessary review, you should also acquire a better understanding of previous material and increased development of your skills.

OBJECTIVES

Read these objectives. These objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT.

When you have finished this LIGHTUNIT, you will be able:

1. To identify real numbers as rational or irrational .
2. To perform operations with radicals and simplify.
3. To solve radical equations.
4. To solve quadratic equations.
5. To write and use the quadratic formula to solve quadratic equations.
6. To define and use imaginary numbers.

Survey the LIGHTUNIT. Ask yourself some questions about this study. Write your questions here.

OBJECTIVES

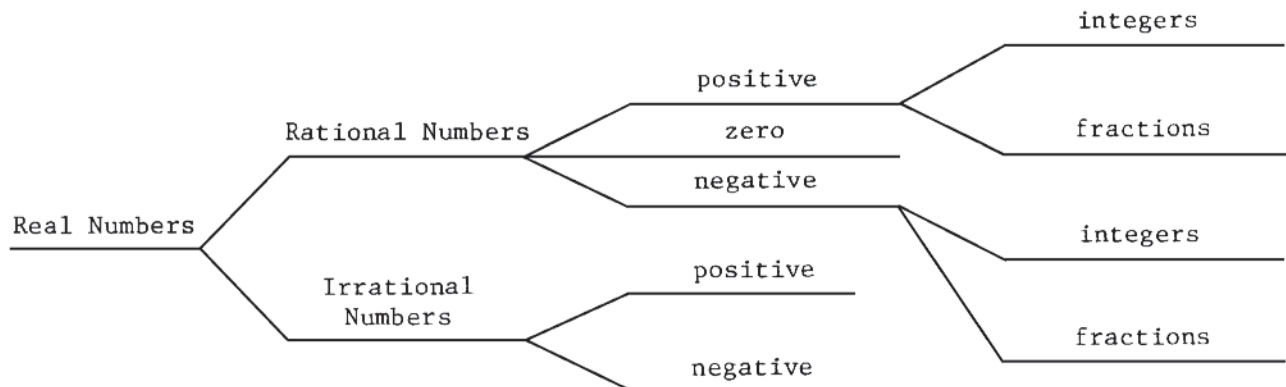
Read these objectives. When you have completed this section, you will be able:

I. REAL NUMBERS

1. To identify real numbers as rational or irrational.
2. To perform operations with radicals and simplify the resulting expressions.
3. To solve radical equations.

The simplest and earliest numbers were positive whole numbers. Soon man discovered a need for zero and fractions. Negative numbers also became useful; finally, the concept of *irrational numbers* was needed. To increase your understanding of our real-number system, you must learn more about *radicals* and how to solve equations that involve radicals.

RATIONAL AND IRRATIONAL NUMBERS



This chart shows the relationship of the real numbers.

Rational numbers are real numbers that can be represented in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. When considering rational numbers, remember that you are simply talking about all positive or negative whole numbers, zero, and the fractions you have always used in arithmetic.

Irrational numbers are the real numbers that are not rational numbers. They sometimes occur as special numbers, like π , which is the ratio of the circumference of a circle to its diameter. They may be indicated as nonrepeating, nonterminating decimals.

DEFINITIONS

Rational number: A number that can be expressed as a quotient of two other integers; a terminating or repeating decimal.

Irrational number: A number that cannot be fully expressed as a quotient of two other integers; a nonterminating, nonrepeating decimal.

The most commonly occurring type of irrational number is a number that contains a radical that cannot be simplified to a rational number.

$$\begin{array}{lcl} \text{Models: } \sqrt{16} = 4 & \left. \vphantom{\begin{array}{l} \sqrt{16} = 4 \\ \frac{4}{9} = \frac{2}{3} \\ \sqrt{3} \\ \frac{\sqrt{11}}{5} \end{array}} \right\} & \text{rational numbers} \\ \frac{4}{9} = \frac{2}{3} & & \\ \sqrt{3} & \left. \vphantom{\begin{array}{l} \sqrt{3} \\ \frac{\sqrt{11}}{5} \end{array}} \right\} & \text{irrational numbers} \\ \frac{\sqrt{11}}{5} & & \end{array}$$

A *radical* is an indicated root of a number. We know that 9 has two square roots, +3 and -3. Any positive real number has two square roots. When we use the radical symbol, we are referring only to the *principal*, or positive, *square root*.

Radicals occur not only as second-degree radicals (square roots) but as third-degree or higher-degree radicals.

DEFINITIONS

Radical: An expression consisting of a number with a radical sign indicating some root of the number beneath the radical sign.

Index: (plural is *indices*) A small number written over the radical sign indicating which root of the number is being sought.

Radicand: The number inside the radical sign.

Radical sign: A symbol indicating that an expression is a radical.

Model: 5 is the index.
 $\sqrt[5]{79}$ 79 is the radicand.
 $\sqrt[5]{79}$ is the radical.
 $\sqrt{\quad}$ is the radical symbol.

Notice that if the radical has no index number, the root indicated is the square root. Therefore, $\sqrt{\quad} = \sqrt[2]{\quad}$.

Any decimal that does not terminate or repeat is an irrational number. If a decimal does terminate or repeat, it can be written as a rational number (in the form $\frac{a}{b}$).

Models: $0.51 = \frac{51}{100}$

$$0.25 = \frac{1}{4}$$

$$0.1666\dots = \frac{1}{6}$$

A repeating decimal can be changed to a common fraction, as shown in the following model, by multiplying by the same power of 10 as the number of digits repeating.

Model 1: 0.2525...

Let 0.2525... be x . Multiplying by 10^2 or 100, because two digits are repeating, gives $100x = 25.25\dots$

$$\begin{array}{r} \text{Subtract.} \quad 100x = 25.25\dots \\ \quad \quad \quad x = 0.2525\dots \\ \hline 99x = 25 \\ \quad \quad x = \frac{25}{99} \end{array}$$

Model 2: 1.533...

$$\begin{array}{r} 10x = 15.333 \\ 100x = 153.333 \\ 100x = 153.333 \\ -10x = 15.333 \\ \hline 90x = 138.0 \\ x = \frac{138}{90} = \frac{23}{15} \end{array}$$

Multiplying by a power of 10 that matches the number of digits repeating will allow use of this procedure for a decimal with any number of repeating digits.

Identify each number as rational or irrational.

- | | | | | | |
|-----|-------------|-------|------|------------------------------|-------|
| 1.1 | 1384 | _____ | 1.6 | -19.78 | _____ |
| 1.2 | π | _____ | 1.7 | $-\frac{\sqrt{3}}{5} + 10.8$ | _____ |
| 1.3 | -0.28764... | _____ | 1.8 | 0.86258625... | _____ |
| 1.4 | $\sqrt{3}$ | _____ | 1.9 | -7.832198653... | _____ |
| 1.5 | 0 | _____ | 1.10 | -0.8662866239... | _____ |

Change to the form $\frac{a}{b}$ in lowest terms.

- | | | |
|------|--------------------|-------|
| 1.11 | 0.225 | _____ |
| 1.12 | 0.625 | _____ |
| 1.13 | $-0.66\frac{2}{3}$ | _____ |
| 1.14 | 0.888... | _____ |
| 1.15 | -3.143143... | _____ |

Complete with the proper word.

- 1.16 In $\sqrt[3]{18}$ the 3 is called the _____.
- 1.17 In $\sqrt[3]{18}$ the 18 is called the _____.
- 1.18 In $\sqrt[3]{18}$, $\sqrt{\quad}$ is called the _____.
- 1.19 The name for an expression like $\sqrt[3]{13}$ is a _____.
- 1.20 The square root of 16 is 4; 4 is called the _____ root of 16.

LAWS OF RADICALS

The laws of radicals are rules about how to combine radicals. Study them carefully. The results of some of the combinations may surprise you!

If m is any positive integer, and if x , y , $\sqrt[m]{x}$, and $\sqrt[m]{y}$ are real numbers, then these laws of radicals will be true for any second-degree or higher-degree radicals.

LAWS OF RADICALS

$$A. \quad \sqrt[m]{x} \sqrt[m]{y} = \sqrt[m]{xy}; \quad \sqrt[m]{xy} = \sqrt[m]{x} \sqrt[m]{y}$$

$$B. \quad \left(\sqrt[m]{x^a}\right)^b = \sqrt[m]{x^{ab}}; \quad \sqrt[m]{x^{ab}} = \left(\sqrt[m]{x^a}\right)^b$$

$$C. \quad \frac{\sqrt[m]{x}}{\sqrt[m]{y}} = \sqrt[m]{\frac{x}{y}}; \quad \sqrt[m]{\frac{x}{y}} = \frac{\sqrt[m]{x}}{\sqrt[m]{y}}, \quad \sqrt[m]{y} \neq 0$$

$$D. \quad a\sqrt[m]{x} + b\sqrt[m]{x} = (a + b)\sqrt[m]{x}$$

THESE SHOULD BE
MEMORIZED.

The laws of radicals shown can help you simplify and compute with radicals. Notice that in each law the indices are the same.

$$A. \quad \text{Model:} \quad \sqrt[3]{9} \sqrt[3]{5} = \sqrt[3]{45}$$

$$B. \quad \text{Model:} \quad (\sqrt[4]{a^3})^2 = \sqrt[4]{a^6}$$

$$C. \quad \text{Model:} \quad \frac{\sqrt[5]{64}}{\sqrt[5]{2}} = \sqrt[5]{32} = 2$$

Radicals can be combined by addition and subtraction only when radicals (both index and radicand) are the same.

$$D. \quad \text{Model 1:} \quad 6\sqrt[4]{5} + 2\sqrt[4]{5} = (6 + 2)\sqrt[4]{5} = 8\sqrt[4]{5}$$

$$\text{Model 2:} \quad 3\sqrt[2]{8} + 2\sqrt[3]{4} \quad \begin{array}{l} \text{These two radicals} \\ \text{cannot be combined} \\ \text{into one term.} \end{array}$$

$$\begin{aligned} \text{Model 3:} \quad & (\sqrt{2} + 3\sqrt{5})(2\sqrt{2} - \sqrt{5}) \\ & 2(\sqrt{2})^2 + 6\sqrt{2}\sqrt{5} - \sqrt{2}\sqrt{5} - 3(\sqrt{5})^2 \\ & 2 \cdot 2 + 5\sqrt{10} - 3 \cdot 5 \\ & 4 + 5\sqrt{10} - 15 \\ & 5\sqrt{10} - 11 \end{aligned}$$

Radical symbols can be eliminated by fractional exponents. In some work with algebraic expressions or equations, fractional exponents are useful.

In a *fractional exponent*, the numerator of the fraction represents the power of the number and the denominator represents the degree or index of the radical.

DEFINITION

Fractional exponent: An exponent in the form of a fraction, with the numerator representing the power to which the base is to be raised and the denominator representing the index of the radical.

Models: $x^{\frac{a}{b}} = b\sqrt[b]{x^a}$

MEMORIZE THIS LAW.

$$7^{\frac{2}{3}} = \sqrt[3]{7^2}$$

As fractions must always be simplified to lowest terms, radicals must also be simplified. Three conditions must not exist in the final answer of a radical problem.

- A. No perfect n^{th} power factors of a radicand can be in an n^{th} degree radical.
- B. No fractions can be in the radicand.
- C. No radical can be in the denominator of a fraction.

MEMORIZE
THESE
WITH THEIR
MODELS

Radicals must usually be simplified until all three of these conditions are fulfilled.

A. Model: $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} \text{ or } \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$

B. Model: $\sqrt[5]{\frac{3}{4}} = \sqrt[5]{\frac{3 \cdot 8}{4 \cdot 8}} \text{ or } \sqrt[5]{\frac{24}{32}}$
 $= \frac{\sqrt[5]{24}}{\sqrt[5]{32}} = \frac{\sqrt[5]{24}}{2}$

C. Model: $\frac{6}{\sqrt[4]{27}} = \frac{6\sqrt[4]{3}}{\sqrt[4]{27}\sqrt[4]{3}}$
 $= \frac{6\sqrt[4]{3}}{\sqrt[4]{81}}$
 $= \frac{6\sqrt[4]{3}}{3} = 2\sqrt[4]{3}$

Also notice that $\sqrt{x^5} = \sqrt{x^2 \cdot x^2 \cdot x} = x \cdot x\sqrt{x} = x^2\sqrt{x}$

and $\sqrt[3]{x^5} = \sqrt[3]{x^3x^2} = x\sqrt[3]{x^2}$

Change these radicals to algebraic expressions with fractional exponents.

1.21 $\sqrt[5]{x^3}$

1.22 $\sqrt{y^2}$

1.23 $\sqrt[3]{a}$

1.24 $\sqrt[4]{x^5}$

1.25 \sqrt{b}
