

MATHEMATICS 1202

FUNCTIONS

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The material in this book is the product of the Lord's blessing and many individuals working together on the teams, both at Alpha Omega Publications and Christian Light Publications.

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2017 Printing

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Christian Light Publications, Inc.

Harrisonburg, Virginia 22802

Tel. 1-540-434-0768

Printed in USA

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FUNCTIONS

To continue with the concept of functions, we shall first look at the polynomial functions of degrees one and two. Next, higher degree polynomial functions will be introduced. Finally,

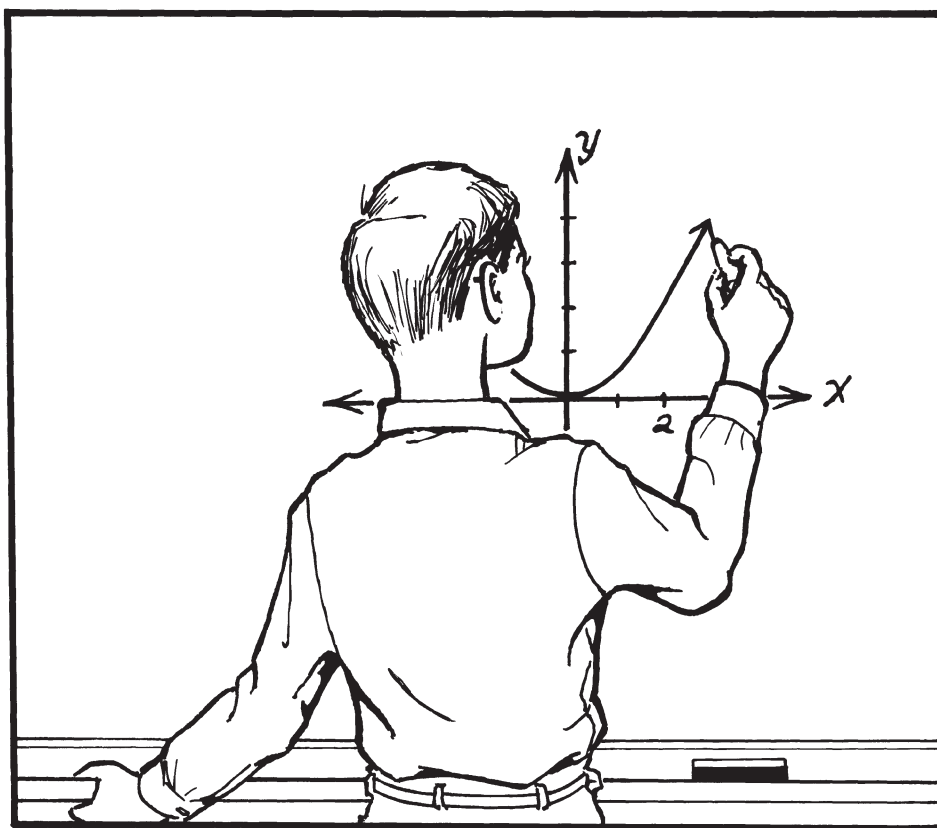
several special functions will be considered, including the greatest integer function, the exponential function, the logarithmic function, and combinations of these three functions.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this **LIGHTUNIT**.

When you have finished this **LIGHTUNIT**, you should be able:

1. To solve and graph linear equations and inequalities.
2. To solve and graph quadratic equations and inequalities.
3. To solve higher degree equations using factor theorems and synthetic division.
4. To solve and graph greatest integer, exponential, and logarithmic functions and function combinations.



I. LINEAR FUNCTIONS

The polynomial functions are a group of functions in the form $y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$ where $a_0, a_1, a_2, \dots$ are numerical coefficients and n , a positive integer, is the exponent of the variable x . The number n then becomes the degree of the polynomial.

SOLUTIONS AND GRAPHS

If $a_0 \neq 0$ and $n = 1$, the expression is a *linear equation* or *linear function*. Its degree is one. A linear function in x is one that can be written in the standard form $y = mx + b$, $m \neq 0$, where m and b are constants.

Models of linear functions are $y = 3x + 2$, $x + y = 6$, and $7x = 3y$. For practice, identify a_0 , a_1 , and n in each of the examples. The graph of a linear function is a straight line. By the definition of a function, a vertical line is not a linear function. Review the definition of a function.

A function can be identified from a graph. If two or more points lie on the same vertical line, then x is multi valued and the relation is *not* a function.

REMEMBER?

STUDY THIS EXAMPLE:

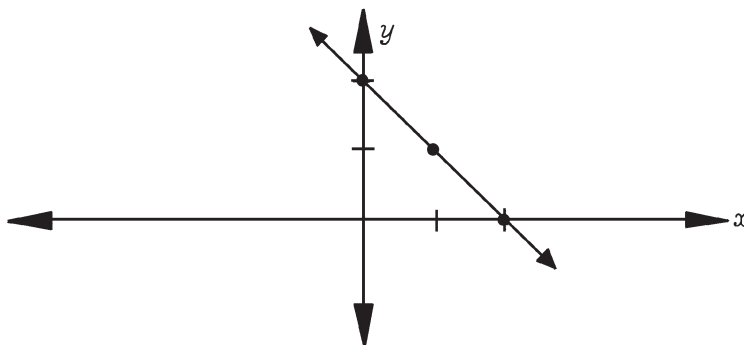
$$x + y = 2$$

$$y = -x + 2$$

Find at least three solutions, such as these

x	y
1	1
0	2
2	0

and locate the solutions as points on the coordinate plane.



A zero of a function is a value of x for which y , the value of the function, is zero. Thus, the zeros of a function are the roots or solutions of $f(x) = 0$.

The zero of the function in the preceding example, or the value of x when $y = 0$, is 2. That is, $f(x) = 0$ when $x = 2$. This value is also called the x -intercept. The y -intercept is the value of the second coordinate of the point where the line crosses the y -axis.

A special case of a linear function that we need to consider is of form $y = \frac{x^2 - 9}{x + 3}$. Implied in this rule is that $x \neq -3$; therefore, the domain of this function is the set of all real numbers except $x = -3$. If we exclude $x = -3$, then we may simplify $y = \frac{x^2 - 9}{x + 3}$ in the following manner.

Factoring the numerator, we get

$$y = \frac{(x - 3)(x + 3)}{(x + 3)}$$

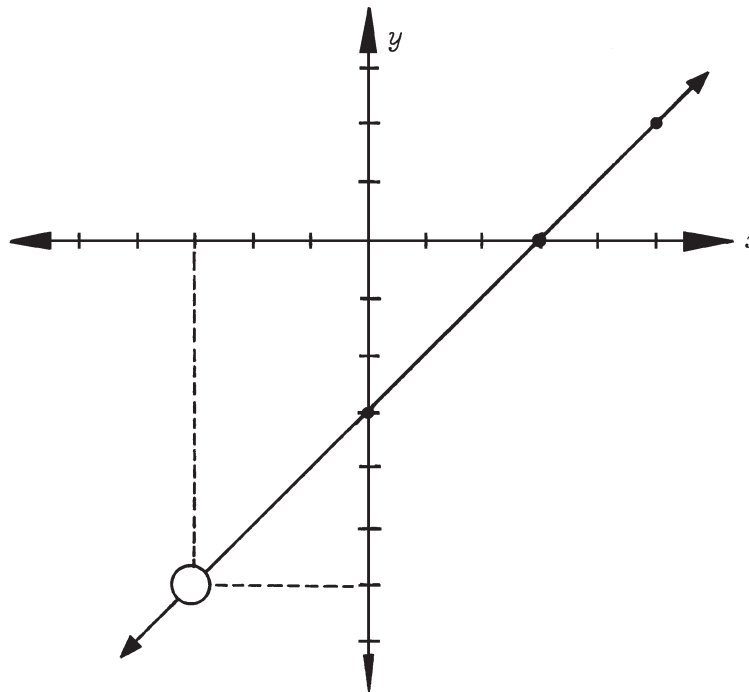
which can be reduced to

$$y = x - 3 ; x \neq -3$$

Since $y = x - 3$ is a straight line and $x \neq -3$, we have a line with a hole in it. Three solutions are

x	y
5	2
0	-3
3	0

Its graph is shown.

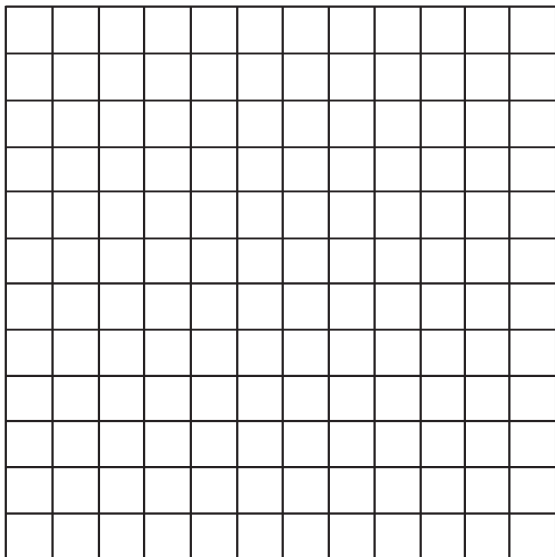


The zero of this function is 3; that is, $f(x) = 0$ when $x = 3$.

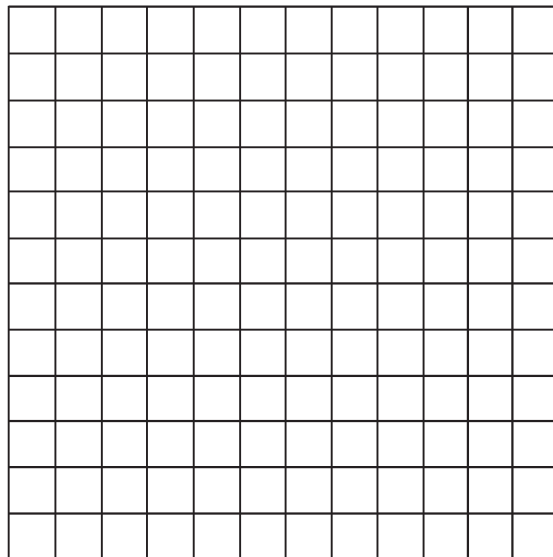
You will prove in the calculus that the closer the value of x gets to -3, the closer the function value, y , will get to -6.

GRAPH THE FOLLOWING LINEAR FUNCTIONS BY FINDING THREE SOLUTIONS FOR EACH.

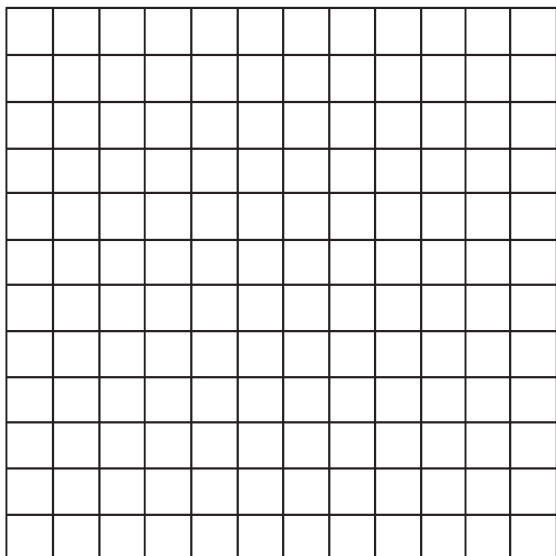
1.1 $F(x) = x + 3$



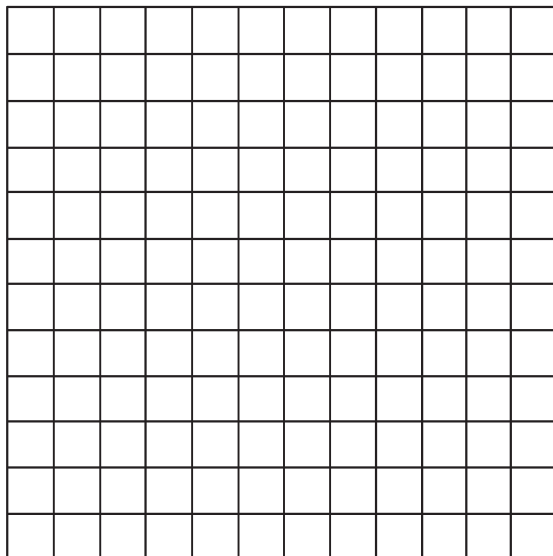
1.3 $F^{-1}(x)$ if $F(x) = x + 3$



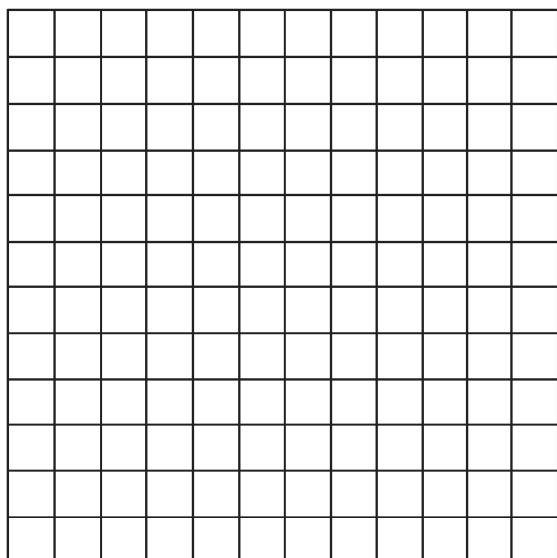
1.2 $G(x) = 3x - 2$



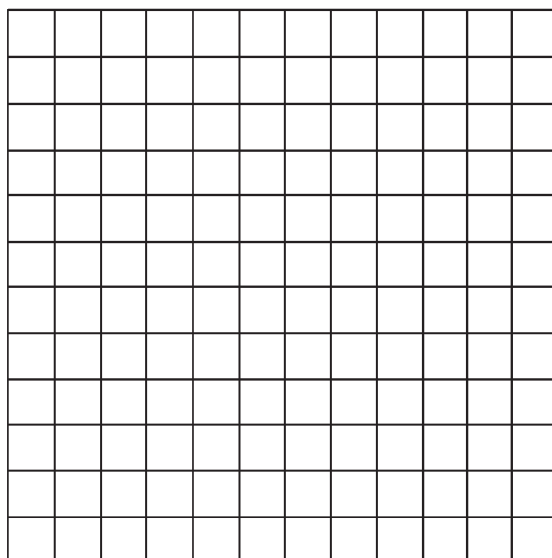
1.4 $g(x) = \frac{3x - 5}{4}$



1.5 $h(x) = \frac{x^2 - 16}{x - 4}$



1.6 $A(x) = \frac{9x^2 - 25}{3x + 5}$

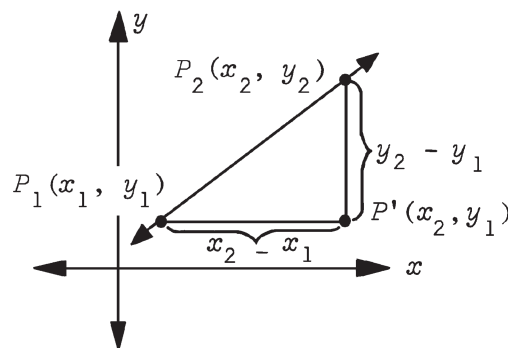


COMPLETE THE FOLLOWING SENTENCES.

- 1.7 The zero of $F^{-1}(x)$ in 1.1 is _____.
- 1.8 The y -intercept of $g(x)$ in 1.4 is _____.
- 1.9 The rule for $g^{-1}(x)$ in 1.4 is $g^{-1}(x) =$ _____.
- 1.10 The zero of the function in 1.6 is _____.
- 1.11 The y -intercept of $h(x)$ in 1.5 is _____.

EQUATIONS

Consider the linear function $L = \{(x, y): ax + by = c; b \neq 0\}$. Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two distinct solutions or roots of this function, L , a nonvertical line.



The figure shows such a linear function. In the figure, P' is the point (x_2, y_1) , and the points P_1 , P_2 , and P' are vertices of a right triangle. $P_1P' = x_2 - x_1$ gives the measure of the change in the value of x from P_1 to P_2 or the horizontal change from P_1 to P_2 and is denoted Δx . $P_2P' = y_2 - y_1$ gives the change in the value of y from P_1 to P_2 or the vertical change from P_1 to P_2 and is denoted Δy . You should see that the value of Δy can be positive, negative, or zero. The value of Δx can be positive or negative but not zero because $x_2 \neq x_1$ since the line L is not vertical. For all solutions P_1 and P_2 of this linear function L , the quotient

$$\frac{y_2 - y_1}{x_2 - x_1}$$

is constant. This constant is called the *slope* of the line.

DEFINITION

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are any two distinct solutions, points on a nonvertical line L , then the *slope* m of L is denoted by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } m = \frac{\Delta y}{\Delta x}.$$

STUDY THIS EXAMPLE:

Let L be the line containing the points $P_1(-3, 9)$ and $P_2(5, -7)$. Find the slope of L .

The slope of this line by definition is given by

$$\begin{aligned} m &= \frac{-7 - 9}{5 - (-3)} \text{ where } \Delta y = -16 \text{ and } \Delta x = 8. \\ &= -\frac{16}{8} \\ &= -2 \end{aligned}$$

Since the two points $P_1(-3, 9)$ and $P_2(5, -7)$ determine a unique line--that is, one and only one line exists containing the points P_1 and P_2 --we should be able to obtain the equation of this line through P_1 and P_2 .

If we let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two given points on a line with slope m , then

$$m = \frac{y_2 - y_1}{x_2 - x_1};$$

and if $P(x, y)$ is any point on the line except (x_1, y_1) , the slope is also

$$m = \frac{y - y_1}{x - x_1}.$$

This point $P(x, y)$ will also be on the line containing P_1 and P_2 if and only if the slope remains constant, that is, if and only if

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Multiplying both sides of this equation by $(x - x_1)$, we obtain

$$y - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1} \right] (x - x_1).$$

This form of a linear function is called the *two-point* form. It will give you the equation of the line if two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, are known.

STUDY THIS EXAMPLE:

Find the equation of the line containing the points $(-1, 2)$ and $(4, 3)$.

Letting $P_1(-1, 2)$ and $P_2(4, 3)$ be the given points and let $P(x, y)$ be a general point on the line and using

$$y - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1} \right] (x - x_1),$$

we obtain

$$y - 2 = \left[\frac{3 - 2}{4 - (-1)} \right] (x - (-1))$$

$$y - 2 = \frac{1}{5}(x + 1)$$

$$5y - 10 = x + 1$$

$$y = \frac{1}{5}x + \frac{11}{5} \text{ or } -x + 5y = 11.$$

Since in the equation $y - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1} \right] (x - x_1)$, $\frac{y_2 - y_1}{x_2 - x_1}$ is the value of m , or the slope of the line, we can write

$$y - y_1 = m(x - x_1).$$

This equation is called the *point-slope* form of a linear function. This form will give you the equation of the line if you know a point $P_1(x_1, y_1)$ on the line and the slope m of the line.

STUDY THIS EXAMPLE:

Find the equation of the line containing the point $(2, 7)$ and having a slope of 3.

Letting $P_1(2, 7)$ and $m = 3$, and using

$$y - y_1 = m(x - x_1),$$

we obtain

$$y - 7 = 3(x - 2)$$

$$y - 7 = 3x - 6$$

$$y = 3x + 1 \text{ or } -3x + y = 1.$$