

MATHEMATICS 1203

THE TRIGONOMETRIC FUNCTIONS

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THE TRIGONOMETRIC FUNCTIONS

The trigonometric functions have a wide variety of applications. Many of these applications have to do with the study of motion and sound: for example, a moving car; an object moving in a circular motion; a vibrating string.

The skills involving these functions must be developed and understood to be able to apply them. During the next few LIGHTUNITS, you will encounter many applications of these functions. Another important aspect of the trigonometric functions will show the relationships between angles, arc lengths, variables, coordinate

systems, and certain aspects of geometry.

The six trigonometric functions are defined in terms of the ordered pairs (u, v) . The domain of the functions is the angles found when a ray is drawn from the origin to a point (u, v) in the coordinate system.

A measure used in circular functions is the radian measure, which has implications and applications in the calculus. This measure will be developed by means of the unit circle in the plane.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT.

When you have finished this LIGHTUNIT, you should be able:

1. To define the six trigonometric functions.
2. To evaluate the six trigonometric functions.
3. To locate a given angle in standard position.
4. To use correctly the trigonometric tables.
5. To reduce any angle to a first quadrant angle.
6. To evaluate the quadrantal function values.
7. To evaluate special function values of 30-, 45-, and 60-degree angles.
8. To define the radian unit of measure.

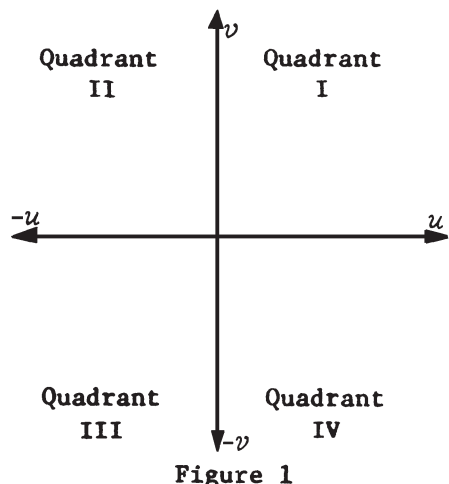
TANGENT = HYPOTENUSE + ADJACENT?

$$\text{TANGENT} = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}?$$



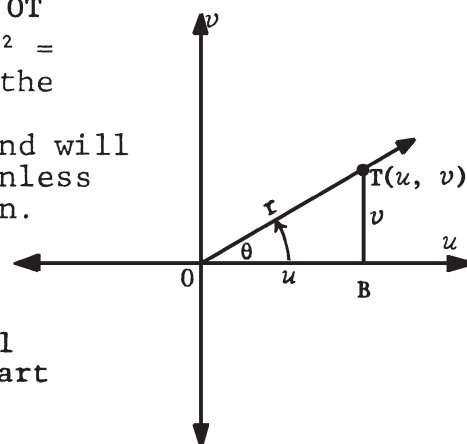
$$\text{TANGENT} = \text{OPPOSITE} \div \text{ADJACENT}?$$

I. DEFINITION OF THE TRIGONOMETRIC FUNCTIONS

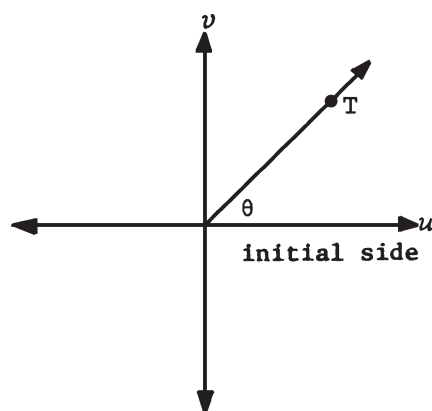


Choose a rectangular coordinate system such that the u - and v -axes intersect at the origin 0. The number u is called the *abscissa* and v , the *ordinate* of the ordered pair (u, v) . The intersection of the u - v axes forms the four quadrants as shown in Figure 1.

Now choose a point $T(u, v)$ in the u - v coordinate system and construct a perpendicular line from T to the u -axis forming a right triangle TOB . Angle TOB will be designated as angle *theta*, $\angle \theta$, and the distance \overline{OT} as r . In figure 2, $r^2 = u^2 + v^2$ by applying the pythagorean theorem. Thus, $r = \sqrt{u^2 + v^2}$ and will always be positive unless point T is the origin.



Angle θ is said to be in *standard position* when the vertex is at the origin and the initial side of the angle coincides with the positive part of the u -axis (Figure 3).



NAMES OF FUNCTIONS

Using the preceding concepts, the trigonometric functions are defined in this manner:

Full Name	Abbreviation	Definition
Sine	\sin^*	$(\theta, \frac{v}{r}): \sin \theta = \frac{v}{r}$
Cosine	\cos^*	$(\theta, \frac{u}{r}): \cos \theta = \frac{u}{r}$
Tangent	\tan^*	$(\theta, \frac{v}{u}): \tan \theta = \frac{v}{u}$
Cotangent	\cot^*	$(\theta, \frac{u}{v}): \cot \theta = \frac{u}{v}$
Secant	\sec^*	$(\theta, \frac{r}{u}): \sec \theta = \frac{r}{u}$
Cosecant	\csc^*	$(\theta, \frac{r}{v}): \csc \theta = \frac{r}{v}$

Does any relationship exist between \sin and \csc ? \cos and \sec ? \tan and \cot ? Explain. _____

The domain θ is the set of *all* angles. Usually our discussion will be limited to a domain of $-360^\circ \leq \theta \leq 360^\circ$.

*Always pronounce these abbreviations as the full name is pronounced.

The functions and the cofunctions are related in that the cofunction is the same function being considered in the complementary angle. That is considered a 30° , 60° , 90° triangle: The $\cos 30^\circ$ is the \sin of the complementary angle, 60° . So $\sin 30^\circ = \cos 60^\circ$ and $\cos 30^\circ = \sin 60^\circ$. Cosine literally means the sine of the complementary angle. This goes for the other cofunctions also.

Law of cofunctions: Any function of an acute angle is equal to the cofunction of its complement.

SIGNS OF FUNCTIONS

The signs of the functions depend on the quadrant in which (u, v) is located.

DEFINITION

If (u, v) is located in the
 first quadrant, all functions are positive.
 second quadrant, \sin and \csc are positive; the rest are negative, since u is negative.
 third quadrant, \tan and \cot are positive; the rest are negative, since u and v are both negative.
 fourth quadrant, \cos and \sec are positive; the rest are negative, since v is negative.

The diagram in Figure 4 illustrates the definition.

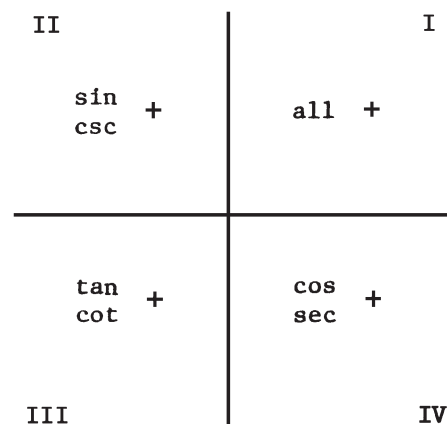


Figure 4

GIVE THE QUADRANT IN WHICH EACH OF THE FOLLOWING POINTS ARE LOCATED, AND DETERMINE WHICH OF THE FUNCTIONS ARE POSITIVE AND WHICH ARE NEGATIVE.

		QUADRANT	SIN	COS	TAN	CSC	SEC	COT
	(4, 3)	I	+	+	+	+	+	+
1.1	(-3, 4)	_____	_____	_____	_____	_____	_____	_____
1.2	(20, -21)	_____	_____	_____	_____	_____	_____	_____
1.3	(-12, 5)	_____	_____	_____	_____	_____	_____	_____
1.4	(-1, -1)	_____	_____	_____	_____	_____	_____	_____
1.5	(8, -15)	_____	_____	_____	_____	_____	_____	_____
1.6	(-5, -30)	_____	_____	_____	_____	_____	_____	_____
1.7	$(\sqrt{3}, \sqrt{10})$	_____	_____	_____	_____	_____	_____	_____
1.8	$(-6\frac{1}{2}, 4\sqrt{7})$	_____	_____	_____	_____	_____	_____	_____
	$(-10 + \sqrt{5}, \sqrt{3})$	III	-	-	+	-	-	+
1.9	(5, 12)	_____	_____	_____	_____	_____	_____	_____
1.10	(-6, 10)	_____	_____	_____	_____	_____	_____	_____

	QUADRANT	SIN	COS	TAN	CSC	SEC	COT
1.11	(5, -7)	_____	_____	_____	_____	_____	_____
1.12	(-14, 2)	_____	_____	_____	_____	_____	_____
1.13	(6, -8)	_____	_____	_____	_____	_____	_____

NAME THE QUADRANT(S) IN WHICH

- 1.14 $\tan \theta$ and $\sec \theta$ are positive. _____
- 1.15 $\sec \theta$ and $\cos \theta$ are positive. _____
- 1.16 $\tan \theta$ and $\cot \theta$ are positive. _____
- 1.17 $\sin \theta$ and $\cos \theta$ and $\tan \theta$ are positive. _____
- 1.18 $\csc \theta$ and $\sin \theta$ are positive. _____
- 1.19 $\sin \theta < 0$ and $\cos \theta > 0$. _____
- 1.20 $\sin \theta > 0$ and $\cos \theta > 0$. _____
- 1.21 $\sin \theta < 0$ and $\cos \theta < 0$. _____
- 1.22 $\tan \theta < 0$ and $\cos \theta > 0$. _____
- 1.23 $\sec \theta < 0$ and $\csc \theta < 0$. _____

IN WHICH QUADRANT IS θ LOCATED

- 1.24 if $\sin \theta$ is positive and $\tan \theta$ is positive? _____
- 1.25 if $\cos \theta$ is negative and $\tan \theta$ is negative? _____
- 1.26 if $\sec \theta$ is negative and $\tan \theta$ is positive? _____
- 1.27 if $\cos \theta$ is positive and $\cot \theta$ is negative? _____
- 1.28 if $\cos \theta = \frac{4}{7}$ and $\sin \theta$ is negative? _____
- 1.29 if $\cos \theta = -\frac{3}{5}$ and $\cot \theta$ is positive? _____
- 1.30 if $\sec \theta = -\frac{13}{5}$ and $\tan \theta$ is positive? _____
- 1.31 if $\sin \theta = -\frac{3}{5}$ and $\cos \theta$ is positive? _____
- 1.32 if $\sin \theta = \frac{3}{5}$ and $\cos \theta = -\frac{4}{5}$? _____

(Hint: is $\frac{4}{7}$ positive or negative?)



Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.