MATHEMATICS 1205
IDENTITIES AND FUNCTIONS OF MULTIPLE ANGLES

CONTENTS

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IDENTITIES AND FUNCTIONS OF MULTIPLE ANGLES

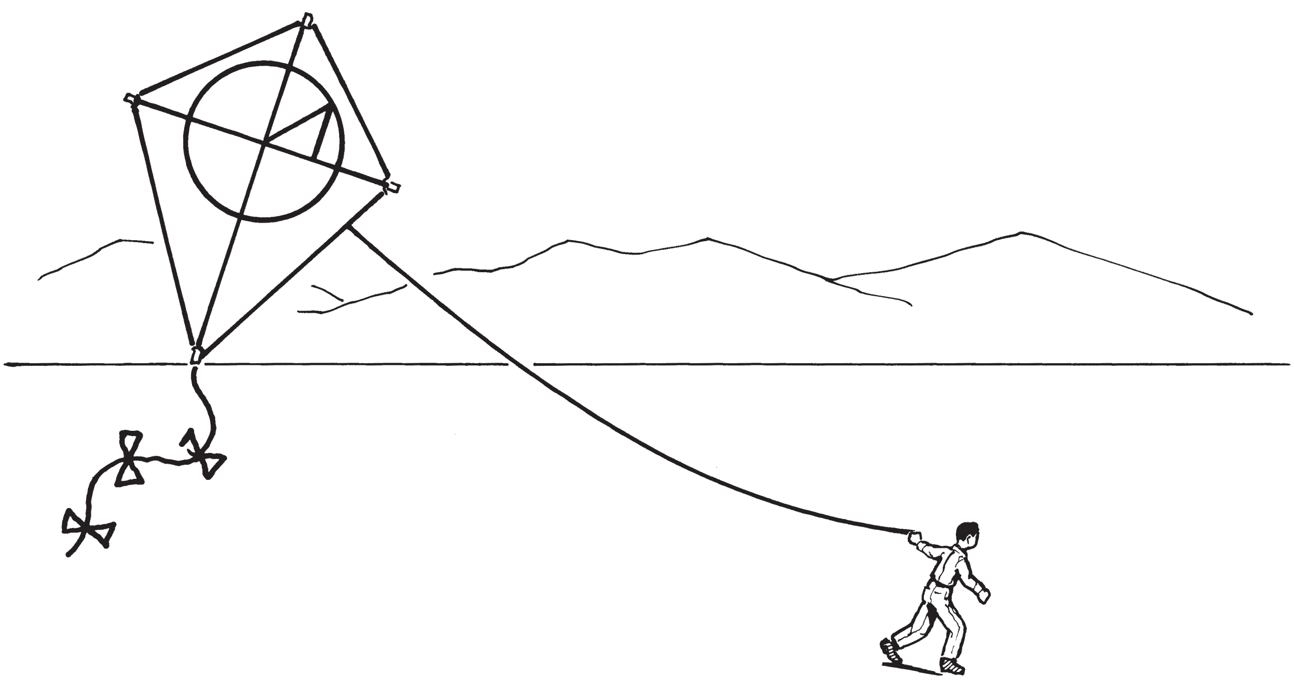
Identities are necessary to simplify expressions and equations involving the trigonometric functions. An identity is a sentence in which both members reduce to the same value for all values of the variable used. For example, in $3x + 1 = 3x + (1)$, for any value of x the sentence reduces to $1 = 1$. We shall define and verify the eight basic trigonometric identities. We shall also study functions of multiple angles, which include such expressions as $\sin(\alpha + \beta)$, $\cos(\alpha + \beta)$, $\sin 2x$, and $\tan \frac{\pi}{2}$. Their use and application occur throughout higher mathematics. These functions are usually called special trigonometric formulas. In this LIGHTUNIT, many of these identities will be derived and the activities will illustrate some of their applications.

OBJECTIVES

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIGHTUNIT.

When you have finished this LIGHTUNIT, you should be able:

1. To simplify trigonometric expressions.
2. To develop and apply the sum and difference formulas.
3. To develop and apply the double-angle formulas.
4. To develop and apply the half-angle formulas.
5. To develop and apply all multiple-angle formulas in identities and equations.



I. RECIPROCAL RELATIONS

The trigonometric functions have certain important relationships among themselves. These relationships are called *trigonometric identities* and are used for the simplification of more complex trigonometric expressions. The first of these identities is the *reciprocal* relations.

In Unit Three, the functions were defined as

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Notice that $\sin\theta$ and $\csc\theta$ are reciprocals of each other, as are $\cos\theta$ and $\sec\theta$, and $\tan\theta$ and $\cot\theta$.

Hence, if $\sin\theta = \frac{y}{r}$ and $\csc\theta = \frac{r}{y}$, then $\sin\theta\csc\theta = \frac{y}{r} \cdot \frac{r}{y} = 1$.

In the same way, $\cos\theta\sec\theta = \frac{x}{r} \cdot \frac{r}{x} = 1$ and $\tan\theta\cot\theta = \frac{y}{x} \cdot \frac{x}{y} = 1$.

You need to memorize these reciprocal relations. You will be using them very frequently in the next three units.

STUDY THIS EXAMPLE:

Simplify $\sin\theta\cos\theta\sec\theta$.

Since $\cos\theta$ and $\sec\theta$ are reciprocal functions, their product is one, $\cos\theta \cdot \sec\theta = 1$; substitute 1 for $\cos\theta\sec\theta$ and the result is

$$\sin\theta\cos\theta\sec\theta = \sin\theta \cdot 1$$

and, thus, $\sin\theta\cos\theta\sec\theta = \sin\theta$.

COMPLETE THESE ACTIVITIES.

1.1 Given the point T(3, 4), find $\sin\theta$ and $\csc\theta$ and show that their product is 1.

1.2 Given the point T(-3, -4), find $\tan\theta$ and $\cot\theta$ and show that their product is 1.

1.3 Given $\theta = \frac{\pi}{4}$, show that $\cos\theta \sec\theta = 1$.

1.4 Given $\theta = \frac{\pi}{3}$, show that $\sin\theta \csc\theta = 1$.

1.5 Simplify $\tan\theta \sin\theta \cot\theta$.

1.6 Simplify $\frac{1}{\cos\theta \sec\theta} \cdot \cot\theta$.

1.7 Simplify $1 + \tan\theta \cot\theta - \frac{\sin\theta \csc\theta}{2}$.

1.8 Simplify $\frac{\sin\theta}{\csc\theta}$ by using $\csc\theta = \frac{1}{\sin\theta}$; $\theta \neq 0$.

1.9 Simplify $\frac{\tan\theta}{\cot\theta}$.

1.10 Simplify $\frac{\sec\theta}{\cos\theta}$.
