SECTION 8-1 USING RATIOS

A ratio is a mathematical way to represent the relationship between two or more numbers, or terms. A ratio is the mathematical method of making a comparison. The symbol used to indicate the relationship between terms in a ratio is the colon (:). A ratio can also be expressed as a fraction where the first term is the numerator and the second term is the denominator. **See Figure 8-1.**

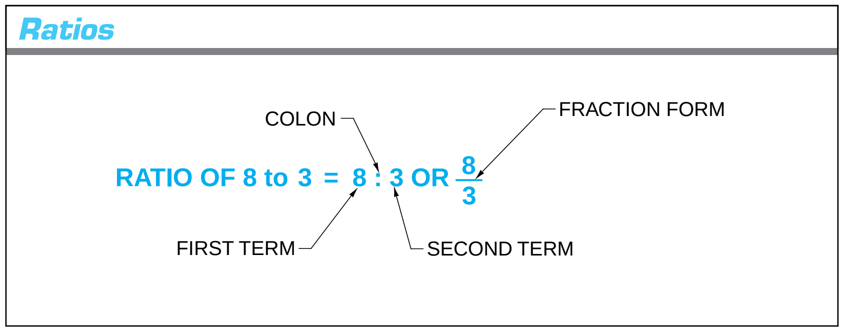

Figure 8-1. A colon can be used to separate the terms in a ratio or the terms can be expressed as a fraction.

Inverse Ratios

An *inverse ratio* is the ratio of the reciprocals of two quantities. To find an inverse ratio, simply invert the terms of the fraction. An inverse ratio can be used when two elements work opposite one another. **See Figure 8-2.**

In the following problem, fluid flow and time are inversely proportional because time increases when fluid flow decreases and time decreases when flow increases.

> *If it takes 14 hours to fill a barrel with a flow of 18 liters per minute, how long would it take to fill the same barrel if the flow were reduced to 7 liters per minute?*

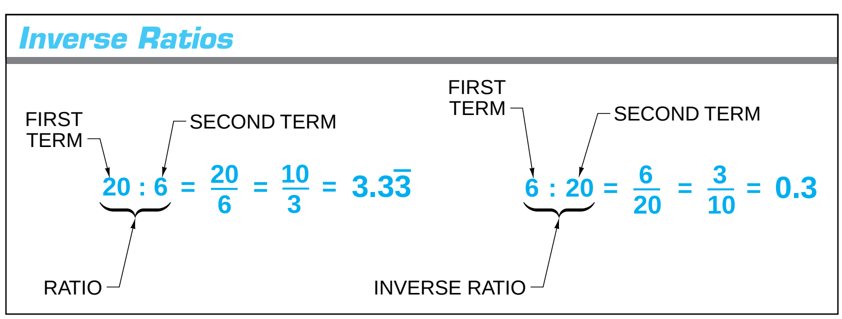

Figure 8-2. In an inverse ratio, the terms of the ratio are inverted and the ratio becomes the reciprocal of the two quantities.

Examples — Ratios

1. Reduce the ratio 21 : 3.

ANS: 7⁄1 or **7 : 1**

❶ Put ratio in fraction form.

❷ Reduce.

$$21:3 = \underbrace{\frac{21}{3}}_{❶} = \underbrace{\frac{7}{1}}_{❷} = 7:1$$

2. Invert the ratio 36 : 9 and reduce.

ANS: 1⁄4 or **1 : 4**

❶ Put ratio in fraction form.

❷ Invert.

❸ Reduce.

$$36:9 = \underbrace{\frac{36}{9}}_{❶} \qquad \underbrace{\frac{9}{36}}_{❷} = \underbrace{\frac{1}{4}}_{❸} = 1:4$$

QUICK REFERENCE

- *Put the ratio in fraction form.*
- *Reduce as required.*

Compound Ratios

A *compound ratio* is the product of two or more ratios. To find a compound ratio of two ratios, change the ratios to fractions, and multiply the numerators (first terms) and denominators (second terms). The product is a compound ratio in fraction form. **See Figure 8-3.**

Compound Ratios

$$8:3 \text{ and } 7:2 = \overbrace{\frac{8}{3} \times \frac{7}{2}}^{❶} = \frac{56}{6} = \frac{28}{3} = \underbrace{28:3}$$

REDUCED FRACTION

COMPOUND RATIO IN FRACTION FORM

COMPOUND RATIO

❶ CHANGE RATIOS TO FRACTION FORMS AND MULTIPLY FIRST TERMS AND SECOND TERMS.

Figure 8-3. A compound ratio is the product of two simple ratios.

When the terms include units of measure, a ratio can be formed only if the units of measure are common. It may be possible to convert one term to match the unit of measure of the other term. For example, 6 ft can be converted to yards to compare it to 3 yd (2 yd : 3 yd).

Example — Compound Ratios

1. Find the compound ratio of 6 : 1 and 3 : 2.

ANS: $\frac{9}{1}$ or **9 : 1**

❶ Put ratios in fraction form. Multiply the numerators (6 and 3), then multiply the denominators (1 and 2).

❷ Reduce $\frac{18}{2}$ to $\frac{9}{1}$.

$$6:1 \text{ and } 3:2 = \overbrace{\frac{6}{1} \times \frac{3}{2} = \frac{18}{2}}^{❶} = \overbrace{\frac{9}{1} = 9:1}^{❷}$$

QUICK REFERENCE

- *Change the ratios to fraction form and multiply both numerators by numerators and denominators by denominators.*
- *Reduce as required.*

Ratios with Fractions

Sometimes a ratio expression can have a fraction as the first term, second term, or both terms. To solve ratios with fractions, any whole number needs to be changed to a fraction. When both ratios are fractions, they can be divided. To do this, invert the second ratio, multiply the numerators, and multiply the denominators. **See Figure 8-4.**

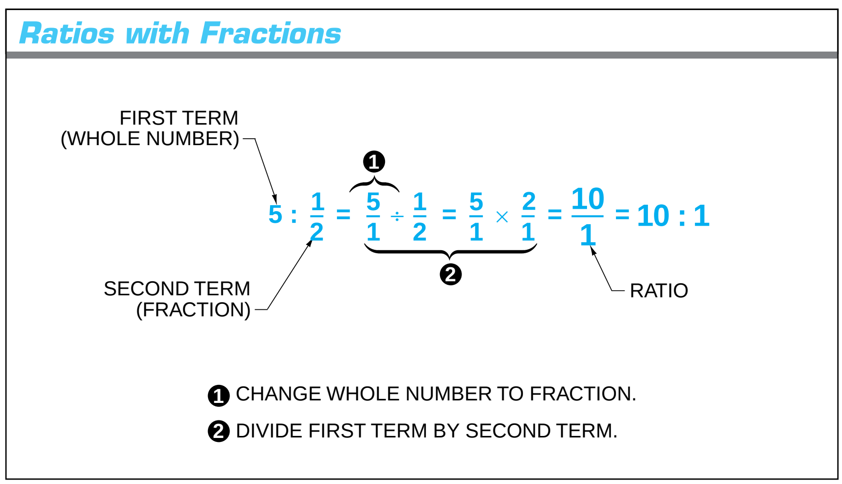

Figure 8-4. To find ratios that include a fraction, change the whole number to a fraction and divide the first term by the second term.

When using a calculator to solve ratios with fractions, convert each fraction to a decimal number and then divide the first term by the second term.

Examples — Ratios with Fractions

1. Find the ratio of ¾ : 8.

ANS: **3 : 32**

❶ Change whole number 8 to a fraction.

❷ Divide fractions by inverting second fraction (8⁄1 becomes 1⁄8) and multiplying.

SECOND TERM (WHOLE NUMBER)

$$\frac{3}{4} : 8 = \frac{3}{4} \div \overbrace{\frac{8}{1}}^{❶} = \underbrace{\frac{3}{4} \times \frac{1}{8}}_{❷} = \frac{3}{32} = \underbrace{3 : 32}_{\text{RATIO}}$$

FIRST TERM (FRACTION)

QUICK REFERENCE

- *Change a whole number to a fraction.*
- *Divide the fractions by inverting the second fraction and multiplying.*
- *Reduce as required.*

2. Find the ratio of ⅖ : ⅔.

ANS: **3 : 5**

❶ Divide fractions by inverting second fraction (⅔ becomes 3⁄2) and multiplying.

❷ Reduce to ⅗.

SECOND TERM (FRACTION)

$$\frac{2}{5} : \frac{2}{3} = \frac{2}{5} \div \frac{2}{3} = \overbrace{\frac{2}{5} \times \frac{3}{2}}^{❶} = \overbrace{\frac{6}{10} = \frac{3}{5}}^{❷} \text{ or } \underbrace{3 : 5}_{\text{RATIO}}$$

FIRST TERM (FRACTION)

MATH EXERCISES — Ratios

____________________ **1.** Reduce the ratio 36 : 4.

____________________ **2.** Find the compound ratio of 9 : 2 and 10 : 3 and reduce.

____________________ **3.** Find the inverse ratio of 3 to 9 and reduce.

____________________ **4.** Find the ratio of ⅔ to 5.

____________________ **5.** Find the compound ratio of 22 : 1 and 3 : 4 and reduce.

____________________ **6.** Find the inverse ratio of 24 to 48 and reduce.

____________________ **7.** Find the ratio of 3⅓ to 16⅔.

______________ **8.** Find the compound ratio of 10 : 3 and 6 : 4 and reduce.

PRACTICAL APPLICATIONS—Ratios

______________ **9.** **Manufacturing:** A prototype will be produced at actual size while the print for the prototype is drawn at ¼ scale. What is the ratio of the prototype to the print?

______________ **10.** **Welding:** The ratio of oxygen to acetylene for welding is approximately 1 : 1. For cutting operations, the oxygen is increased by one and one-half times. What is the ratio of oxygen to acetylene for cutting?

Proportions

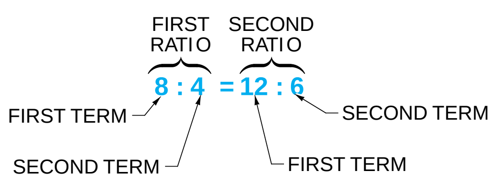

Figure 8-5. All proportions are composed of two equal ratio expressions.

SECTION 8-2 USING PROPORTIONS

A *proportion* is an expression of equality between two ratios. **See Figure 8-5.** For example, the ratio expressions 8 : 4 and 12 : 6 can both be simplified as 2 : 1, and are therefore equal. The proportion is read, "8 is to 4 as 12 is to 6."

The cross-product rule states that the product of the two inner numbers of the proportions (the means) equals the product of the two outer numbers of the proportions (the extremes). This means that in