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Sets

Words to Know

set: a collection of well-defined objects.

elements: the members (objects) of a set.

universal set: the set that contains all the elements or objects involved.

subset: a set that is part of a larger set.

empty set: a set with no elements.

Any well-defined collection of objects is a **set**. A set can refer to a collection of dishes, a set of tools, or the vowels of the English alphabet. This course will deal mostly with sets of numbers.

A set must be well-defined so it is easy to tell whether an object belongs to it. For example, a “set of letters” is not well-defined, because it does not specify which letters are to be included. It could be argued that Greek letters, as well as English ones, should be included in such a set. For the set to be well-defined, the description should have specified the letters *of the English alphabet*.

Objects in a set are called the **elements** or members of that set. If a set has a countable number of elements, such as the days in a week, it is a *finite* set. If a set has an uncountable number of elements (e.g. the set of all even numbers), it is an *infinite* set.

Sets can be described by either the rule method or the roster (list) method. The rule method uses words to describe the set. Notice that a capital letter designates both sets and that both sets are enclosed in braces, as per standard set notation.

$$\text{Set } V = \{\text{the vowels of the English alphabet}\}$$

The roster (list) method lists all the elements in a set. This method is used for smaller sets.

$$\text{Set } V = \{a, e, i, o, u\}$$

Sets that have the same number of elements, or the same *cardinal number*, are equivalent sets. Sets that contain the same elements are also equal sets. The order of the elements in a set is not important.

Example 1 Equivalent sets

$$\text{Set } A = \{1, 3, 5, 7, 9\}$$

$$\text{Set } B = \{2, 4, 6, 8, 10\}$$

$$\text{Set } X = \{\text{cows, horses, pigs, sheep}\}$$

$$\text{Set } Y = \{\text{horses, sheep, cows, pigs}\}$$

Sets A and B are equivalent, but not equal.

The cardinal number of Sets A and B is 5.

Sets X and Y are both equivalent and equal.

The cardinal number of Sets X and Y is 4.

Example 2 Equivalent sets

Set $R = \{\text{circle, rectangle, square, triangle, trapezoid, rhombus}\}$ Sets R and S are neither equivalent nor equal.

Set $S = \{\text{rectangle, triangle, rhombus}\}$ The cardinal number of Set R is 6, and the cardinal number of Set S is 3.

Subsets

If all the elements in Set N are also elements of Set M , then Set N is a subset of M . If Set N contains some but not all the elements of M , then Set N is considered a proper subset of M . If Set N contains *all* the elements of M , it is an *improper* subset of Set M .

Example 3 Proper Subsets

Set $M = \{1, 3, 6, 9, 12, 15, 18\}$ Set N is a proper subset of Set M .

Set $N = \{1, 6, 12, 18\}$

A **subset** may contain no elements (an **empty set**), one element, or any number of elements if they are all elements of the parent set, also called the **universal set**.

The empty set is a subset of every set, and every set is considered an improper subset of itself. Every set will have at least two subsets—the empty set and itself.

The number of subsets in a given set can be calculated using 2^n , where n is the number of elements in the set. For the example below, Set U has 4 elements, so $2^4 = 16$, which is the number of subsets listed.

Example 4 Improper Subsets

Listed below are all the possible subsets of universal Set $U = \{1, 2, 3, 4\}$

Set $A = \{ \}$ or $\emptyset$

Set $B = \{1\}$

Set $C = \{2\}$

Set $D = \{3\}$

Set $E = \{4\}$

Set $F = \{1, 2\}$

Set $G = \{1, 3\}$

Set $H = \{1, 4\}$

Set $I = \{2, 3\}$

Set $J = \{2, 4\}$

Set $K = \{3, 4\}$

Set $L = \{1, 2, 3\}$

Set $M = \{1, 2, 4\}$

Set $N = \{1, 3, 4\}$

Set $O = \{2, 3, 4\}$

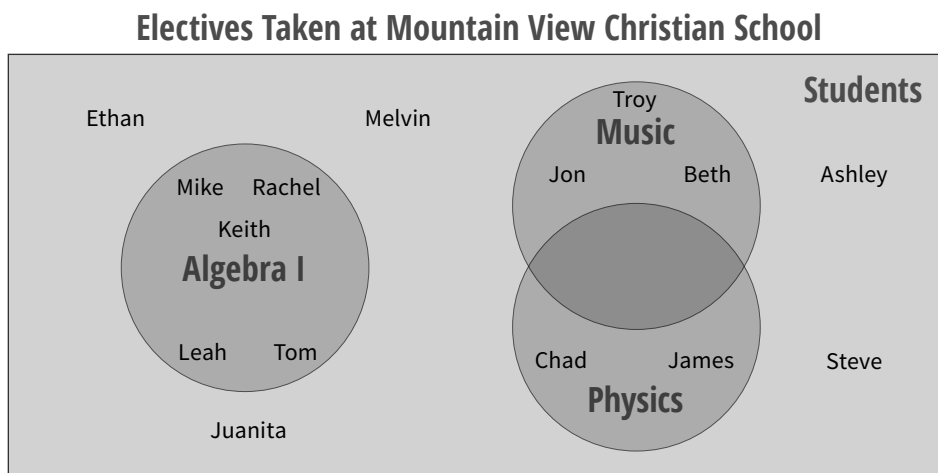
Set $P = \{1, 2, 3, 4\} = \text{Set } U$

All these subsets are proper subsets of Set U except Set P , which is an improper subset of U because it is equal to U .

Venn Diagrams

Sets can intersect (have elements in common), be disjoint (have no elements in common), or have union (be combined into one new set). Venn diagrams can illustrate the characteristics of sets.

The diagram below shows the sets of the electives taken by students at Mountain View Christian School.



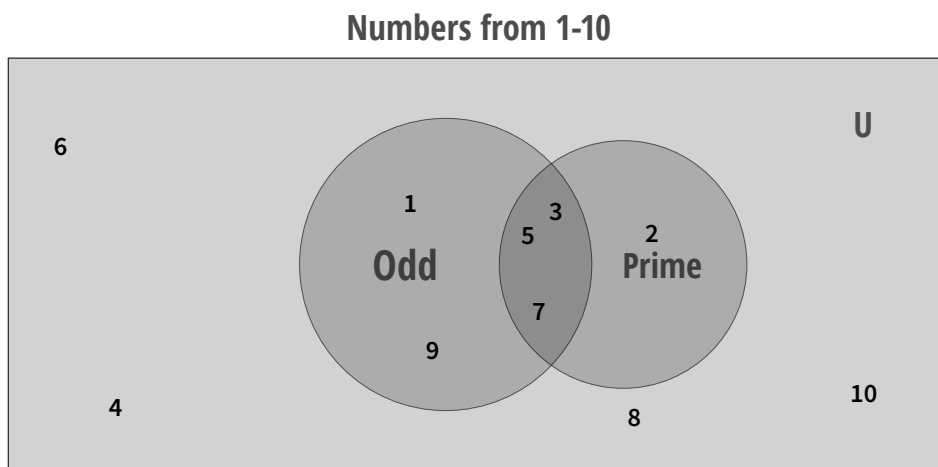
Set $S = \{\text{Students}\}$

Set $M = \{\text{Music}\}$

Set $A = \{\text{Algebra I}\}$

Set $P = \{\text{Physics}\}$

The diagram below shows the intersecting sets of the odd and prime numbers of the counting numbers 1-10.



Set $U = \{1, 2, 3, \dots, 8, 9, 10\}$

Set $O = \{\text{odd numbers 1-10}\}$

Set $P = \{\text{prime numbers 1-10}\}$

Mathematicians use symbols to identify sets and describe how different sets relate to one another. Below is a chart showing the most common set notation symbols.

Set Notation Symbols			
Symbol	Symbol Name	Example	Meaning
{ }	set	$A = \{1, 2, 3, 4, 5\}$ $B = \{4, 5, 8, 9\}$	a collection of elements
$\in$	element of	$5 \in A$	5 belongs to Set A
$\notin$	not an element of	$10 \notin A$	10 does not belong to Set A
$\subset$	subset	$A \subset U$	elements of Set A are part of Set U
$\not\subset$	not a subset	$B \not\subset A$	elements of Set B are not part of Set A
$\cap$	intersection	$A \cap B = \{4, 5\}$	elements that Set A and B have in common
$\cup$	union	$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$	elements of Sets A and B combined
'	complement	A'	elements that do not belong to Set A
-	difference	$A - B = \{1, 2, 3\}$	elements of Set A that are not in Set B

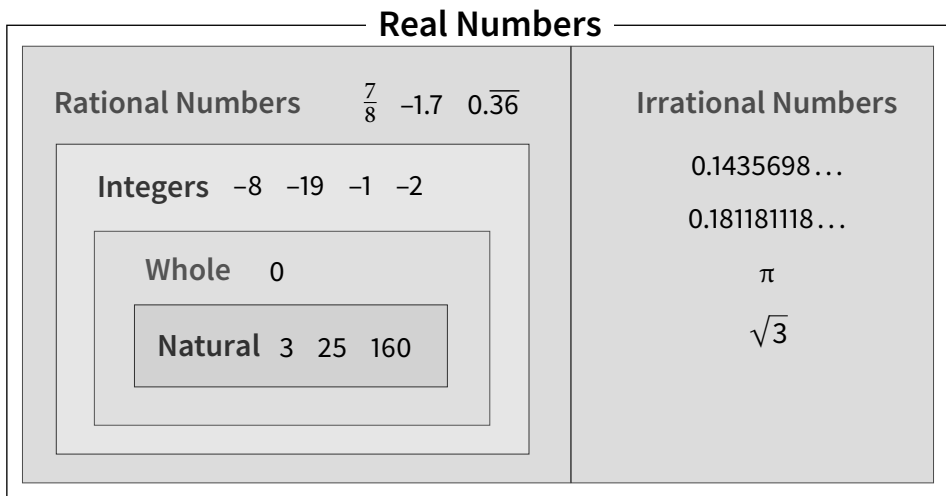
Real Number Categories

Real numbers are the numbers most often used in mathematics. They are known as real because there is a different group of numbers called imaginary numbers. Imaginary numbers exist, but they are called *imaginary* because their unique properties were imagined before being proven.

Categories of Real Numbers

- Rational Numbers** ► all the numbers that can be expressed as a quotient of two integers. In other words, these numbers can be expressed as fractions (denominator cannot be 0). All positive and negative whole numbers, mixed numbers, fractions, terminating and repeating decimals can be expressed as fractions, so they are all rational numbers.
- $5, \frac{1}{2}, -\frac{7}{1}, 4\frac{1}{2}, 1.5, 0.\overline{36}$
- Natural Numbers** ► the counting numbers with which the study of math begins.
- $1, 2, 3, 4, \dots$
- Whole Numbers** ► the counting numbers *plus* 0. (counting never begins with 0).
- $0, 1, 2, 3, 4, \dots$
- Integers** ► the whole numbers *and* their opposites (the negative numbers). Zero does not have an opposite.
- $\dots -3, -2, -1, 0, 1, 2, \dots$
- Irrational Numbers** ► the numbers that can be *expressed* as a quotient of two integers. In decimal form these are non-terminating and non-repeating.
- $2.645751312\dots, \pi$

The following chart illustrates the categories or subsets of real numbers in standard set notation. Each larger rectangle contains all the elements of each smaller rectangle within it. For example, the set of whole numbers includes all the elements of the natural numbers as well. The set of rational numbers includes all the integers, the whole numbers, and the natural numbers.



Today's Lesson

Given these sets, answer the questions below with *yes* or *no*.

Set $D = \{\text{odd numbers from one to nine}\}$

Set $J = \{2, 4, 8, 10\}$

Set $F = \{\text{even numbers from two to ten}\}$

Set $L = \{ \}$

Set $H = \{9, 7, 5, 3, 1\}$

1. Are Sets H and J listed in roster method?
2. Are Sets D and F equal?
3. Are Sets D and H equal?
4. Are Sets F and J equal?
5. Are Sets D and H equivalent?
6. Is 5 the cardinal number for Set H ?
7. Are Sets F and H infinite?

State the answers.

8. What is the cardinal number of Set J ?
9. Set J is a proper subset of Set ____?
10. Set H is an improper subset of Set ____?

11. Set L is a proper subset of Sets ____?____, ____?____, ____?____, and ____?____.
12. What is the number of possible subsets for Set H ? (Use the formula).

List the real-number categories each constant belongs to.

13. π 14. -52 15. $0.7653457\dots$
 16. 0 17. 3 18. $2.\overline{33}$

Write the answer. If there is none, use the symbol for the empty set.

19. Name the number that is a whole number but not a natural number.
20. List the first three counting numbers.
21. Name a month that has 34 days.
22. Write an irrational number.
23. Write a negative number that is a natural number.

Write the term for each definition.

24. A collection of well-defined objects
25. the members of a set

Write the number of elements that satisfy these set operations.

26. $O \cap H = \underline{\hspace{2cm}}$
27. $H \cup Cl = \underline{\hspace{2cm}}$
28. $C - O = \underline{\hspace{2cm}}$
29. $C \cap O \cap H = \underline{\hspace{2cm}}$
30. $C \cap O \cap H \cap Cl = \underline{\hspace{2cm}}$
31. $C \cup Cl = \underline{\hspace{2cm}}$
32. $H - (O \cap C) = \underline{\hspace{2cm}}$
33. $(C \cup Cl)' = \underline{\hspace{2cm}}$
34. $O \cap H \cap Cl = \underline{\hspace{2cm}}$

