~~Implications of the Atomic Constitution of Matter:~~ (Real Processes)

Macroscopic Objects and Classical Mechanics

§ 1

In the light of our knowledge about the atomic constitution of matter, any "object" of macroscopic size is composed of an enormous number of constituent particles. The wave function for such an object is then in a space of fantastically high dimension (3N, if N is the number of particles). Our present problem ~~now~~ is to understand the existence of macroscopic objects, and to relate their ordinary (classical) behavior ~~of such objects~~ in the three dimensional world to the underlying wave mechanics in the higher dimensional space.

Let us begin by considering a relatively simple case. Suppose that we place in a box an electron and a proton, each in a definite momentum state, so that the position amplitude density of each is uniform over the whole box. After a time we would expect a hydrogen atom in the ground state would form, with ensuing radiation. We notice, however that the position amplitude density of each particle is <u>still</u> uniform over the whole box. Nevertheless the amplitude distributions are now no longer independent, but correlated. In particular, the <u>conditional</u> amplitude density for the electron, conditioned by any definite proton (or centroid) position, is <u>not</u> uniform, but is given by the familiar ground state wave function for the hydrogen atom. What we mean by the statement

"A Hydrogen atom has formed in the box" is just that this correlation has taken place--a correlation which insures that the relative configuration for the electron, for a definite proton position, conforms to the customary ground state configuration.

~~We can now open our box, and allow the Hydrogen atom to spread even further (i.e~~

The wave function for the hydrogen atom can be represented as a product of a centroid wave function and a wave function over relative coordinates, where the centroid wave function obeys the wave equation for a particle with mass equal to ~~the~~ total mass of the proton electron system. Therefore, if we now open our box, the centroid wave function will spread with time in the usual manner of wave packets, to eventually occupy a vast region of space. The relative configuration (described by the relative coordinate state function) has, however, a permanent nature, since it represents a bound state, and it is this relative configuration which we usually think of as the object called the hydrogen atom. Therefore, no matter how indefinite the positions of the individual particles become in the total state function (due to the spreading of the centroid), this state can be regarded as ~~as~~ giving (through the centroid wave function) an amplitude distribution over ~~a~~ comparitively definite object, the tightly bound electron-proton system. The general state, then, does not describe any single such definite object, but the superposition of such cases with the object located at different positions.

In a similar fashion larger and more complex objects can be built up through strong correlations which bind together the constituent particles. It is still true that the general state function for such a system may lead to marginal position densities for any single particle (or centroid) which extend over large regions of space. Nevertheless we can speak of the existence of a relatively definite object, since the specification of a single position for a particle, or the centroid, leads to the case where the *relative* position densities of the remaining particles are distributed closely about the specified one, in a manner forming the comparatively definite object spoken of.

~~(To be continued)~~

~~Conditional~~ marginal particle density, for specified centroid position

$$\Psi(x_1, x_2, \ldots)$$

$$\rightarrow P(x_1, x_2, \ldots x_n)$$

$t_{pp}[n/R] =$

Suppose, for example, we begin with a cannonball located at the origin, described by a state function:

$$\psi_{[C,(0,0,0)]}$$

where the subscript indicates that the total state function ψ describes a system of particles bound together so as to form an object of the size and shape of a cannonball, whose centroid is located (approximately) at the origin, [Insert 1]

If we now allow a long lapse of time, the centroid of the system will spread in the usual manner to occupy a large region of space. [Ins 2] Nevertheless, for any specified centroid position the particles, since they remain in bound states, have distributions which again correspond to the fairly well defined size and shape of the cannonball.

(No ¶) ← Thus the total state can be regarded as a (continuous) superposition of states

$$\psi = \int a_{xyz} \, \psi_{[C,(x,y,z)]} \, dx dy dz$$

each of which ($\psi_{[C,(x,y,z)]}$) describes a cannonball at the position x, y, z. The coefficients a_{xyz} of the superposition then correspond to the centroid distribution.

It is not true that each individual particle spreads independently of the rest, in which case ~~so that~~ we would have a final state which is a grand ~~[illegible]~~ superposition of states in which the particles are located independently everywhere. The fact that they are in bound states restricts our final state to a superposition of "cannonball" states. The wave function for the centroid can therefore be taken as a representative wave function for the whole object.

It is thus in this sense of correlations between constituent particles that definite macroscopic objects can exist within the framework of pure wave mechanics. The building up of correlations in a complex system supplies us with a mechanism which also allows us to understand how condensation phenomena (the formation of spatial boundaries which separate phases of different physical or chemical properties) can be controlled by the wave equation, answering a point raised by Schrödinger.[1]

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Since large scale objects, obeying classical laws, have a place in our theory of pure wave mechanics, we have justified the introduction of models ~~ex~~ for observers consisting of classically describable, automatically functioning machinery, and the treatment of observation of chap IV is non-vacuous.

Let us now consider the result of an observation (considered along the lines of Chap IV) performed upon a system of macroscopic bodies in a general state. The observer will not become aware of the fact that the state does not correspond to definite positions and momenta (i.e. he will not see the objects as "smeared out" over large regions of space), but will himself simply become correlated with ~~the~~ the system -- ~~they~~ after the observation the composite system of objects + observer will be in a superposition of states, each element of which describes an observer who has perceived that the objects have nearly definite positions and momenta, and for whom the relative system state is a quasi-classical state in the previous sense, and furthermore to whom the system will appear to behave according to classical mechanics if his observation is continued. We see therefore how the classical appearance of the macroscopic world to us can be explained in the wave theory.

V, §1

cont. part of sentence

Insert 1

, say in the form of a real gaussian wave packet of small dimensions, with variance σ_0^2, for each dimension.

Ins 2

(The spread in each dimension after time t will be given by $\sigma_t^2 = \sigma_0^2 + (\hbar^2 t^2/4\sigma_0^2 m^2)$, where m is the mass.)

$$\psi = \alpha\, e^{-(A - iB)\frac{x^2}{2}}$$

$$A = \frac{(\Delta k)^2}{1 + \frac{\hbar^2 t^2}{m^2}(\Delta k)^4} \qquad B = (\Delta k)^4 \frac{\hbar t}{m} \frac{1}{1 + \frac{\hbar^2 t^2}{m^2}(\Delta k)^4}$$

$$\psi^* \psi = \alpha^* \alpha\, e^{-Ax^2} \quad \Rightarrow \quad e^{-Ax^2} = e^{-\frac{x^2}{2\sigma_x^2}}$$

$$\Rightarrow A = + \frac{1}{2\sigma_x^2}$$

$$\therefore \quad \frac{1}{2\sigma_{x_t}^2} = \frac{(\Delta k)^2}{1 + \frac{\hbar^2 t^2}{m^2}(\Delta k)^4}$$ now, at $t = 0$ $$\frac{1}{2\sigma_{x_0}^2} = (\Delta k)^2$$